UNIT 1

Geometric Structure

Focus

Undrestand basic geometric terms, such as lines, planes, and angles and how they can be used to prove theorems.

CHAPTER 1
Tools of Geometry

BIG Idea Identify and give examples of undefined terms.

BIG Idea Solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.

CHAPTER 2
Reasoning and Proof

BIG Idea Identify and give examples of axioms, theorems, and inductive and deductive reasoning.

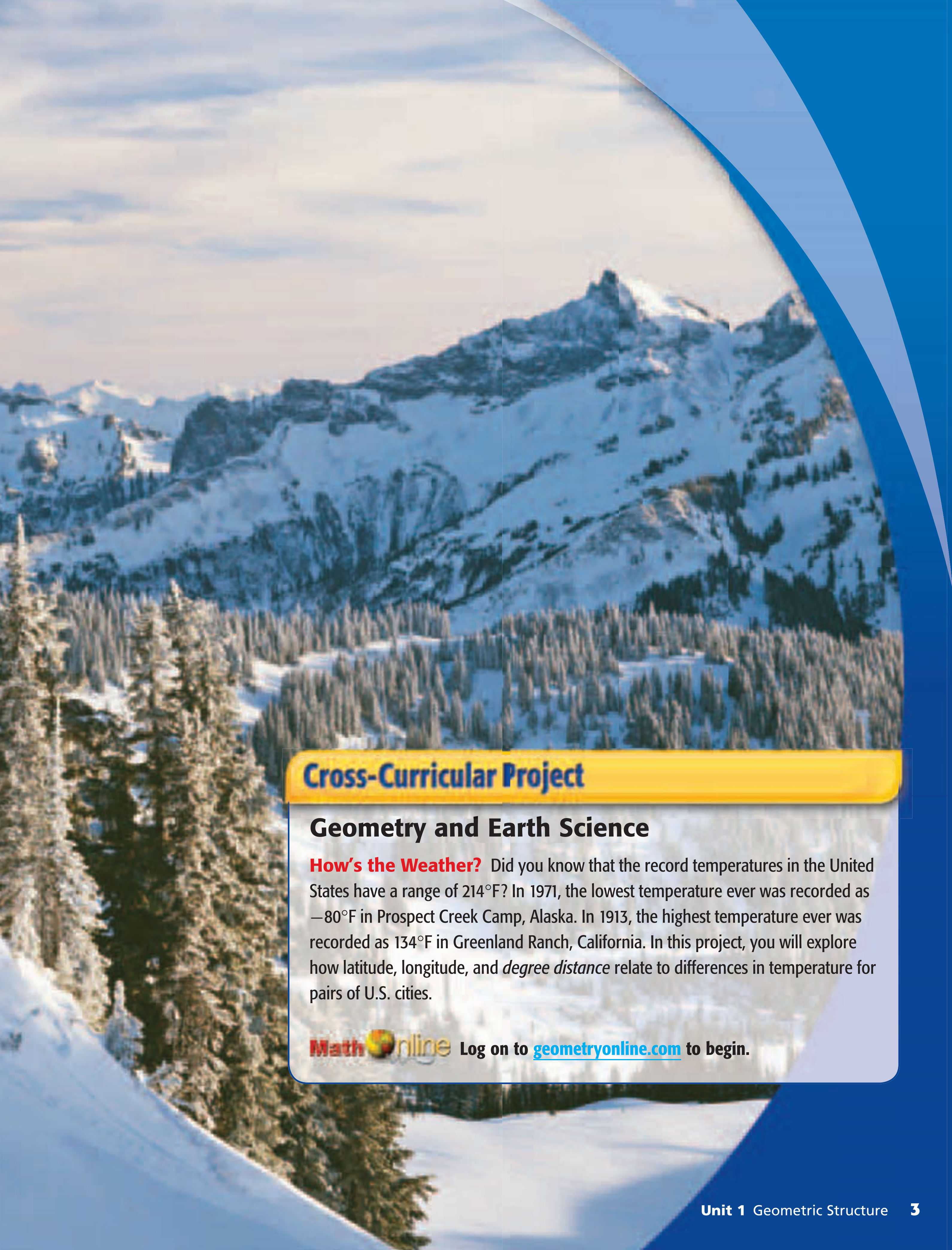
BIG Idea Write geometric proofs, including proofs by contradiction, give counterexamples to disprove a statement, and prove basic theorems involving congruence.

CHAPTER 3
Parallel and Perpendicular Lines

BIG Idea Prove and use theorems involving the properties of parallel lines cut by a transversal.

BIG Idea Perform basic constructions with a straightedge and compass.

Alan Majchrowicz/Getty Images



Cross-Curricular Project

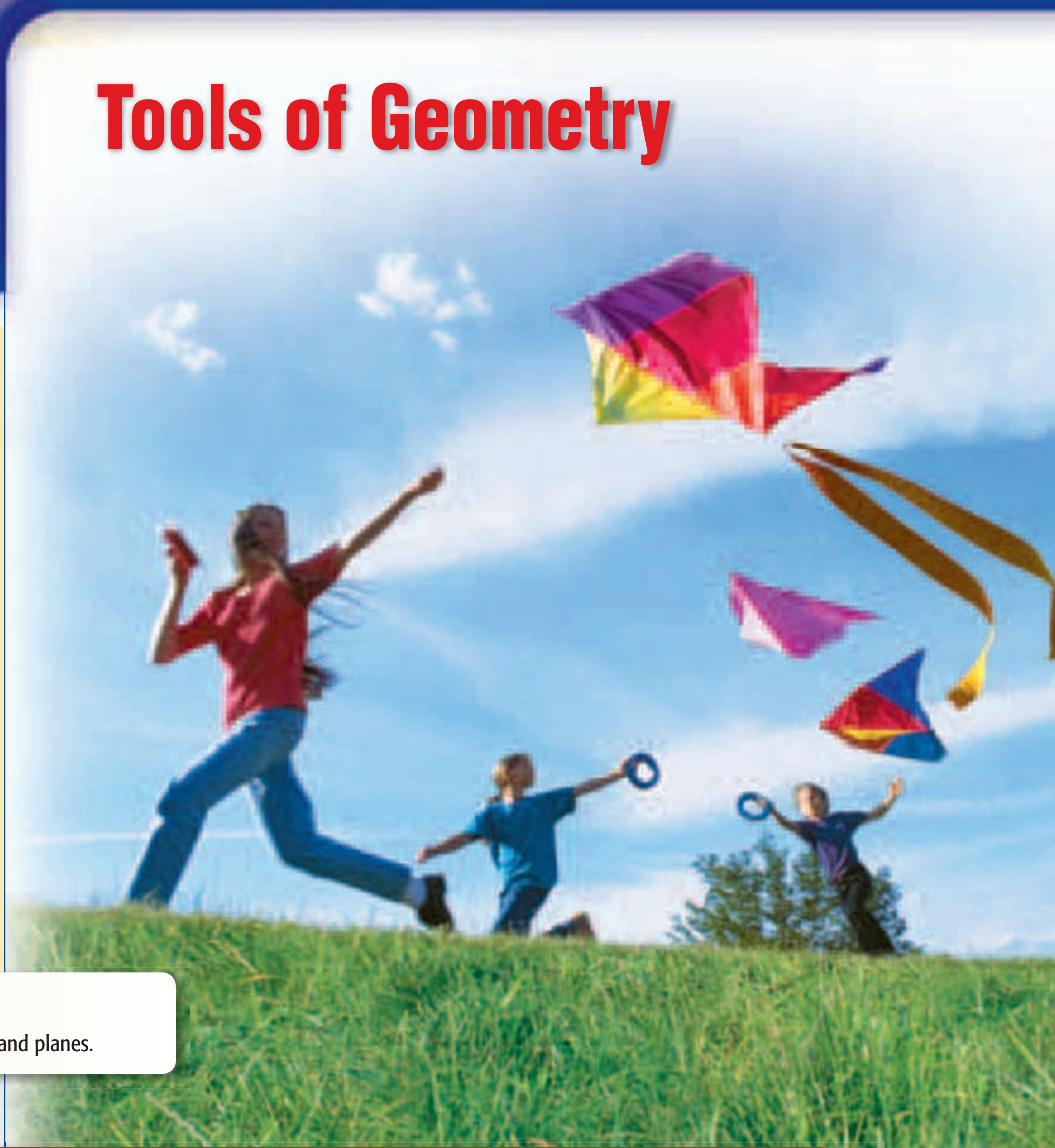
Geometry and Earth Science

How's the Weather? Did you know that the record temperatures in the United States have a range of 214°F? In 1971, the lowest temperature ever was recorded as −80°F in Prospect Creek Camp, Alaska. In 1913, the highest temperature ever was recorded as 134°F in Greenland Ranch, California. In this project, you will explore how latitude, longitude, and *degree distance* relate to differences in temperature for pairs of U.S. cities.

Math Online **Log on to geometryonline.com to begin.**

CHAPTER 1

Tools of Geometry



BIG Ideas

- Measure segments and determine accuracy of measurements.
- Find the distances between points and the midpoints of segments.
- Measure and classify angles and identify angle relationships.
- Identify polygons and find their perimeters.
- Identify three-dimensional figures and find their surface areas and volumes.

Key Vocabulary

line segment (p. 13)
congruent (p. 15)
bisector (pp. 25, 35)
perpendicular (p. 43)

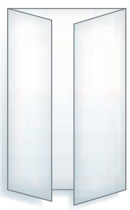
Real-World Link

Kites A kite can model lines, angles, and planes.

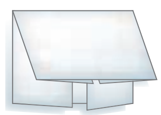
FOLDABLES™ Study Organizer

Lines and Angles Make this Foldable to help you organize your notes. Begin with a sheet of 11" × 17" paper.

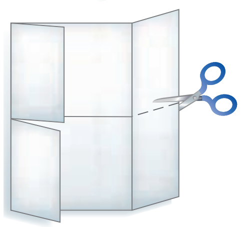
1. **Fold** the short sides to meet in the middle.



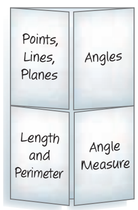
2. **Fold** the top to the bottom.



3. **Open.** Cut flaps along the second fold to make four tabs.



4. **Label** the tabs as shown.



Roy Morsch/CORBIS

GET READY for Chapter 1

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Graph and label each point in the coordinate plane. (Prerequisite Skill)

1. $A(3, -2)$
2. $B(4, 0)$
3. $C(-4, -4)$
4. $D(-1, 2)$
5. **GEOGRAPHY** Joaquin is making a map on a coordinate grid with his school at the center. His house is 4 blocks north and 2 blocks west of the school. The library is 6 blocks east of his house and 2 blocks south. Graph and label the house and library. (Prerequisite Skill)

Find each sum or difference. (Prerequisite Skill)

6. $\frac{3}{4} + \frac{3}{8}$
7. $2\frac{5}{16} + 5\frac{1}{8}$
8. $\frac{7}{8} - \frac{9}{16}$
9. $11\frac{1}{2} - 9\frac{7}{16}$
10. **BAKING** A recipe calls for $\frac{3}{4}$ cup of flour plus $1\frac{1}{2}$ cups of flour. How much flour is needed? (Prerequisite Skill)

Evaluate each expression. (Prerequisite Skill)

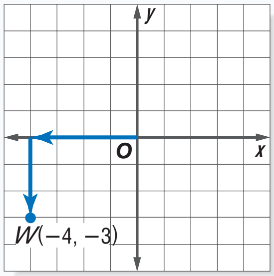
11. $-4 - (-9)$
12. $23 - (-14)$
13. $(18 + 20)^2$
14. $[-7 - (-2)]^2$
15. the sum of 13 and −8 squared

QUICKReview

EXAMPLE 1

Graph and label the point $W(-4, -3)$ in the coordinate plane.

Start at the origin. Move 4 units left, since the x-coordinate is −4. Then move 3 units down, since the y-coordinate is −3. Draw a dot, and label it W.



EXAMPLE 2

Find the sum of $\frac{3}{7} + \frac{5}{6}$.

$$\frac{3}{7} + \frac{5}{6} = \frac{3}{7}\left(\frac{6}{6}\right) + \frac{5}{6}\left(\frac{7}{7}\right) \quad \text{Rename.}$$
$$= \frac{18}{42} + \frac{35}{42} \quad \text{Multiply.}$$
$$= \frac{53}{42} \text{ or } 1\frac{11}{42} \quad \text{Simplify.}$$

EXAMPLE 3

Evaluate the expression $[8 - (-2)^4]^2$.

Follow the order of operations.

$$[8 - (-2)^4]^2 = [8 - 16]^2 \quad (-2)^4 = 16$$
$$= [-8]^2 \quad \text{Subtract.}$$
$$= 64 \quad \text{Square } -8.$$

1-1 Points, Lines, and Planes

GET READY for the Lesson

Have you ever noticed that a four-legged chair sometimes wobbles, but a three-legged stool never wobbles? This is an example of points and how they lie in a plane. All geometric shapes are made of points. In this book, you will learn about those shapes and their characteristics.

Main Ideas

- Identify and model points, lines, and planes.
- Identify collinear and coplanar points and intersecting lines and planes in space.

New Vocabulary

undefined term
point
line
collinear
plane
coplanar
space
locus

Name Points, Lines, and Planes You are familiar with the terms *plane*, *line*, and *point* from algebra. You graph on a coordinate *plane*, and ordered pairs represent *points* on *lines*. In geometry, these terms have similar meanings.

Unlike objects in the real world that model these shapes, points, lines, and planes do not have any actual size. In geometry, *point*, *line*, and *plane* are considered **undefined terms** because they are only explained using examples and descriptions.

- A **point** is simply a location.
- A **line** is made up of points and has no thickness or width. Points on the same line are said to be **collinear**.
- A **plane** is a flat surface made up of points. Points that lie on the same plane are said to be **coplanar**. A plane has no depth and extends infinitely in all directions.

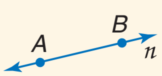
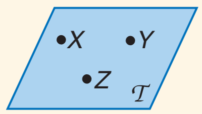
Points are often used to name lines and planes.

Reading Math

Noncollinear and Noncoplanar

The word *noncollinear* means not collinear or not lying on the same line. Likewise, *noncoplanar* means not lying in the same plane.

KEY CONCEPT — *Points, Lines, and Planes*

	Point	Line	Plane
Model			
Drawn	as a dot	with an arrowhead at each end	as a shaded, slanted 4-sided figure
Named by	a capital letter	the letters representing two points on the line or a lowercase script letter	a capital script letter or by the letters naming three noncollinear points
Facts	A point has neither shape nor size.	There is exactly one line through any two points.	There is exactly one plane through any three noncollinear points.
Words/ Symbols	point *P*	line *n*, line *AB* or $\overleftrightarrow{AB}$, line *BA* or $\overleftrightarrow{BA}$	plane $\mathcal{T}$, plane *XYZ*, plane *XZY*, plane *YXZ*, plane *YZX*, plane *ZXY*, plane *ZYX*

C Squared Studios/Getty Images

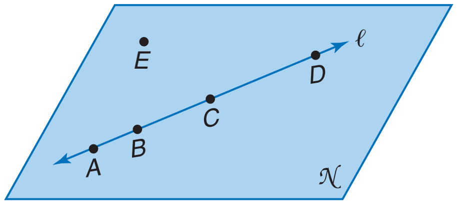
Study Tip

Dimension

A point has no dimension. A line exists in one dimension. However, a square is two-dimensional, and a cube is three-dimensional.

EXAMPLE 1 Name Lines and Planes

Use the figure to name each of the following.



a. a line containing point *A*

The line can be named as line ℓ. There are four points on the line. Any two of the points can be used to name the line.

$\overleftrightarrow{AB}$ $\overleftrightarrow{BA}$ $\overleftrightarrow{AC}$ $\overleftrightarrow{CA}$ $\overleftrightarrow{AD}$ $\overleftrightarrow{DA}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{CB}$ $\overleftrightarrow{BD}$ $\overleftrightarrow{DB}$ $\overleftrightarrow{CD}$ $\overleftrightarrow{DC}$

b. a plane containing point *C*

The plane can be named as plane $\mathcal{N}$. You can also use the letters of any three *noncollinear* points to name the plane.

plane *ABE* plane *ACE* plane *ADE* plane *BCE* plane *BDE* plane *CDE*

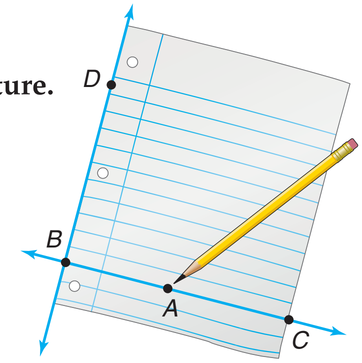
The letters of each name can be reordered to create other names for this plane. For example, *ABE* can be written as *AEB*, *BEA*, *BAE*, *EBA*, and *EAB*.

CHECK Your Progress

1. Use the figure to name a plane containing points *A* and *D*.

EXAMPLE 2 Model Points, Lines, and Planes

Name the geometric shapes modeled by the picture.



The pencil point models point *A*.

The blue rule on the paper models line *BC*.

The edge of the paper models line *BD*.

The sheet of paper models plane *ADC*.

CHECK Your Progress

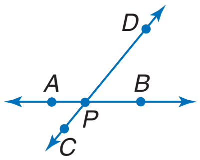
2. Name the geometric shape modeled by stripes on a sweater.

Study Tip

Naming Points

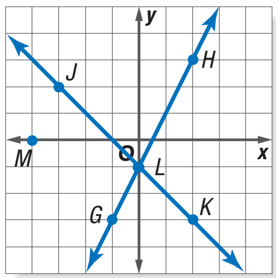
Recall that points on the coordinate plane are named using *rectangular coordinates* or *ordered pairs*. Point *G* can be named as *G*(−1, −3).

Two lines intersect in a point. In the figure at the right, point *P* represents the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Lines can intersect planes, and planes can intersect each other.



EXAMPLE 3 Draw Geometric Figures

Draw and label a figure for each relationship.



a. ALGEBRA Lines *GH* and *JK* intersect at *L* for *G*(−1, −3), *H*(2, 3), *J*(−3, 2), and *K*(2, −3) on a coordinate plane. Point *M* is coplanar with these points, but not collinear with $\overleftrightarrow{GH}$ or $\overleftrightarrow{JK}$.

Graph each point and draw $\overleftrightarrow{GH}$ and $\overleftrightarrow{JK}$.

Label the intersection point as *L*.

An infinite number of points are coplanar with *G*, *H*, *J*, *K*, and *L*, but not collinear with $\overleftrightarrow{GH}$ or $\overleftrightarrow{JK}$. In the graph, one such point is *M*(−4, 0).

Study Tip

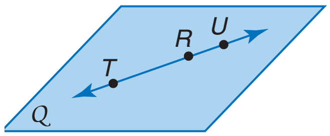
Three-Dimensional Drawings

Because it is impossible to show space or an entire plane in a figure, edged shapes with different shades of color are used to represent planes. If the lines are hidden from view, the lines or segments are shown as dashed lines or segments.

b. $\overleftrightarrow{TU}$ lies in plane Q and contains point R.

Draw a surface to represent plane Q and label it.
Draw a line anywhere on the plane.
Draw dots on the line for points T and U.
Since $\overleftrightarrow{TU}$ contains R, point R lies on $\overleftrightarrow{TU}$.
Draw a dot on $\overleftrightarrow{TU}$ and label it R.

The locations of points T, R, and U are totally arbitrary.



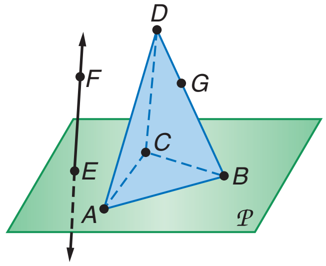
CHECK Your Progress

3. Draw and label a figure in which points A, B, and C are coplanar and B and C are collinear.

Online Personal Tutor at geometryonline.com

Points, Lines, and Planes in Space **Space** is a boundless, three-dimensional set of all points. Space contains lines and planes.

EXAMPLE 4 Interpret Drawings



a. How many planes appear in this figure?

There are four planes: plane $\mathcal{P}$, plane ADB, plane BCD, plane ACD.

b. Name three points that are collinear.

Points D, B, and G are collinear.

c. Are points G, A, B, and E coplanar? Explain.

Points A, B, and E lie in plane $\mathcal{P}$, but point G does not lie in plane $\mathcal{P}$. Thus, they are not coplanar. Points A, G, and B lie in a plane, but point E does not lie in plane AGB.

d. At what point do $\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ intersect?

$\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ do not intersect. $\overleftrightarrow{AB}$ lies in plane $\mathcal{P}$, but only point E lies in $\mathcal{P}$.

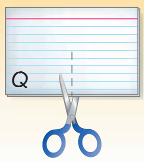
CHECK Your Progress

4. Name the intersection of plane BCD and plane $\mathcal{P}$.

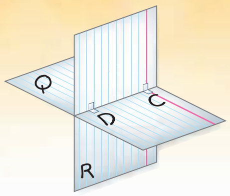
GEOMETRY LAB

Modeling Intersecting Planes

- Label one index card as Q and another as R.
- Hold the two index cards together and cut a slit halfway through both cards.



- Hold the cards so that the slits meet and insert one card into the slit of the other. Use tape to hold the cards together.
- Where the two cards meet models a line. Draw the line and label two points, C and D, on the line.



ANALYZE

1. Draw a point F on your model so that it lies in Q but not in R. Can F lie on $\overleftrightarrow{DC}$? Explain.

2. If point H lies in both Q and R, where would it lie? Draw point H on your model.

3. Draw a sketch of your model on paper. Label all points, lines, and planes.

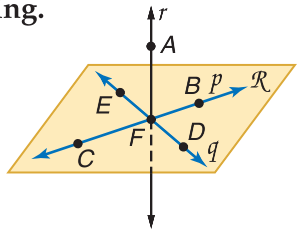
CHECK Your Understanding

Example 1 (p. 7)

Use the figure at the right to name each of the following.

1. a line containing point *B*

2. a plane containing points *D* and *C*



Example 2 (p. 7)

Name the geometric term modeled by each object.

3. the beam from a laser

4. a ceiling

Example 3 (pp. 7–8)

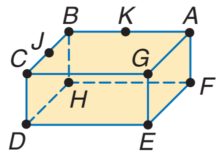
Draw and label a figure for each relationship.

5. A line in a coordinate plane contains *X*(3, −1), *Y*(−3, −4), and *Z*(−1, −3) and a point *W* that does not lie on $\overleftrightarrow{XY}$.

6. Plane *Q* contains lines *r* and *s* that intersect in *P*.

Example 4 (p. 8)

For Exercises 7–9, refer to the figure.



7. How many planes are shown in the figure?

8. Name three points that are collinear.

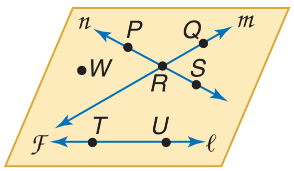
9. Are points *A*, *C*, *D*, and *J* coplanar? Explain.

Exercises

HOMEWORK HELP

For Exercises	See Examples
10–15	1
16–22	2
23–30	3
31–34	4

Refer to the figure.



10. Name a line that contains point *P*.

11. Name the plane containing lines *n* and *m*.

12. Name the intersection of lines *n* and *m*.

13. Name a point not contained in lines *ℓ*, *m*, or *n*.

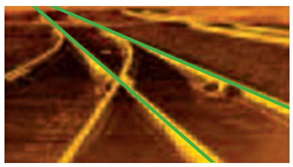
14. What is another name for line *n*?

15. Does line *ℓ* intersect line *m* or line *n*? Explain.

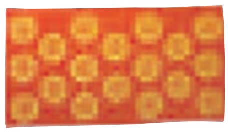
Name the geometric term(s) modeled by each object.

16.

17.



18.



19. a tablecloth

20. a partially-opened newspaper

21. woven threads in a piece of cloth

22. a knot in a string

Draw and label a figure for each relationship.

23. Line *AB* intersects plane *Q* at *W*.

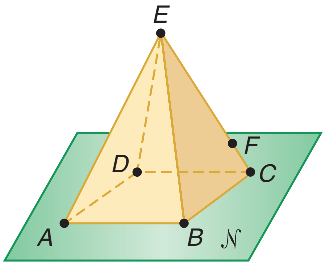
24. Point *T* lies on $\overleftrightarrow{WR}$.

25. Points *Z*(4, 2), *R*(−4, 2), and S are collinear, but points *Q*, *Z*, *R*, and *S* are not.

26. The coordinates for points *C* and *R* are (−1, 4) and (6, 4), respectively. $\overleftrightarrow{RS}$ and $\overleftrightarrow{CD}$ intersect at *P*(3, 2).

(l) Aaron Haupt, (c) Donovan Reese/Getty Images, (r)Laura Sifferlin

Refer to the figure.



27. How many planes are shown in the figure?

28. How many planes contain points *B*, *C*, and *E*?

29. Name three collinear points.

30. Where could you add point *G* on plane *N* so that *A*, *B*, and *G* would be collinear?

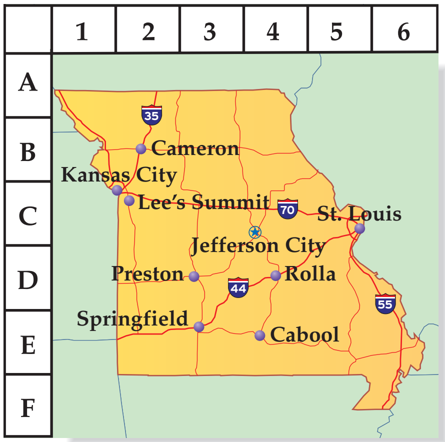
MAPS For Exercises 31–34, refer to the map, and use the following information. A map represents a plane. Locations on this plane are named using a letter/number combination.

31. Name the letter/number combination where St. Louis is located.

32. Name the letter/number combination where Springfield is located.

33. What city is located at (B, 2)?

34. What city is located at (D, 4)?



Real-World Career

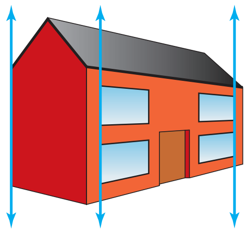
Engineering Technician

Engineering technicians or drafters use perspective to create drawings used in construction, and manufacturing. Technicians must have knowledge of math, science, and engineering.

Math Online

For more information, go to geometryonline.com.

ONE-POINT PERSPECTIVE One-point perspective drawings use lines to convey depth in a picture. Lines representing horizontal lines in the real object can be extended to meet at a single point called the *vanishing point*.



35. Trace the figure at the right. Draw all of the vertical lines. Three are already drawn for you.

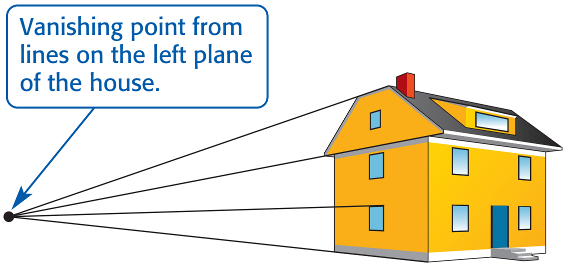
36. Draw and extend the horizontal lines to locate the vanishing point and label it.

37. Draw a one-point perspective of your classroom or a room in your house.

38. RESEARCH Use the Internet or other research resources to investigate one-point perspective drawings in which the vanishing point is in the center of the picture. How do they differ from the drawing for Exercises 35–37?

TWO-POINT PERSPECTIVE Two-point perspective drawings also use lines to convey depth, but two sets of lines can be drawn to meet at two vanishing points.

39. Trace the outline of the house. Draw all of the vertical lines.



40. Draw and extend the lines on your sketch representing horizontal lines in the real house to identify the vanishing point on the right plane in this figure.

41. Which type of lines seems to be unaffected by any type of perspective drawing?

EXTRA PRACTICE
See pages 800, 828.

Math Online
Self-Check Quiz at geometryonline.com

Another way to describe a group of points is called a locus. A locus is a set of points that satisfy a particular condition.

42. Find five points that satisfy the equation $4 - x = y$. Graph them on a coordinate plane and describe the geometric figure they suggest.

43. Find ten points that satisfy the inequality $y > -2x + 1$. Graph them on a coordinate plane and describe the geometric figure they suggest.

H.O.T. Problems

44. OPEN ENDED Fold a sheet of paper. Open the paper and fold it again in a different way. Open the paper and label the geometric figures you observe. Describe the figures.

45. FIND THE ERROR Raymond and Micha were looking for patterns to determine how many ways there are to name a plane given a certain number of points. Who is correct? Explain your reasoning.

Raymond	Micha
If there are 4 points, then there are 4 • 3 • 2 ways to name the plane.	If there are 5 noncollinear points, then there are 5 • 4 • 3 ways to name the plane.

46. CHALLENGE Describe a real-life example of three lines in space that do not intersect each other and no two of which lie in the same plane.

47. *Writing in Math* Refer to the information about chairs on page 6. Explain how the chair legs relate to points in a plane. Include how many legs would create a chair that does not wobble.

STANDARDIZED TEST PRACTICE

48. Four lines are coplanar. What is the *greatest* number of intersection points that can exist?

A 4

B 5

C 6

D 7

49. REVIEW What is the value of x if $-5x + 4 = -6$?

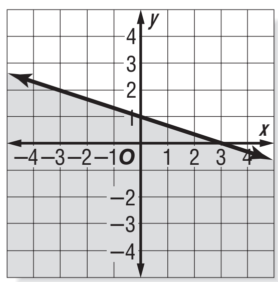
F −5

G −2

H 2

J 5

50. REVIEW Which inequality is shown on the graph below?



A $y > -\frac{1}{3}x + 1$

B $y < -\frac{1}{3}x + 1$

C $y \leq -\frac{1}{3}x + 1$

D $y \geq -\frac{1}{3}x + 1$

GET READY for the Next Lesson

PREREQUISITE SKILL Replace each ● with >, <, or = to make a true statement.

51. $\frac{1}{2}$ in. ● $\frac{3}{8}$ in.

52. $\frac{4}{16}$ in. ● $\frac{1}{4}$ in.

53. $\frac{4}{5}$ in. ● $\frac{6}{10}$ in.

54. 10 mm ● 1 cm

55. 2.5 cm ● 28 mm

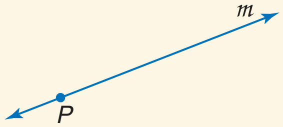
56. 0.025 cm ● 25 mm

READING MATH

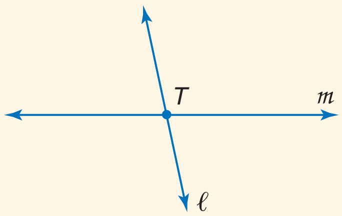
Describing What You See

Figures play an important role in understanding geometric concepts. It is helpful to know what words and phrases can be used to describe figures. Likewise, it is important to know how to read a geometric description and be able to draw the figure it describes.

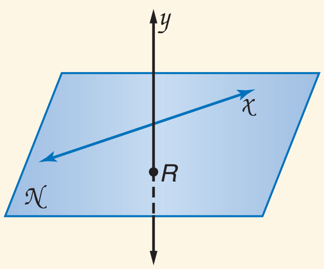
The figures and descriptions below help you visualize and write about points, lines, and planes.



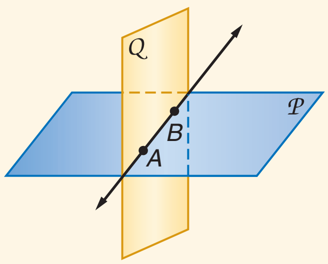
Point P is on line m.
Line m contains P.
Line m passes through P.



Lines ℓ and m intersect in T.
Point T is the interesection of lines ℓ and m.
Point T is on line m. Point T is on line ℓ.



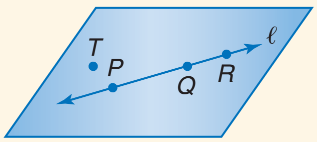
Line x and point R are in $\mathcal{N}$.
Point R lies in $\mathcal{N}$.
Plane $\mathcal{N}$ contains R and line x.
Line y intersects $\mathcal{N}$ at R.
Point R is the intersection of line y with $\mathcal{N}$.
Lines y and x do not intersect.

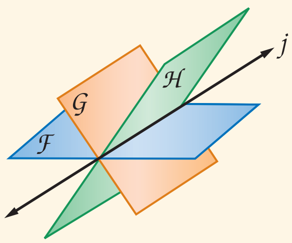


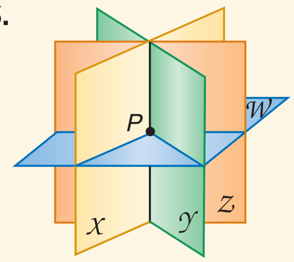
$\overleftrightarrow{AB}$ is in $\mathcal{P}$ and $\mathcal{Q}$.
Points A and B lie in both $\mathcal{P}$ and $\mathcal{Q}$.
Planes $\mathcal{P}$ and $\mathcal{Q}$ both contain $\overleftrightarrow{AB}$.
Planes $\mathcal{P}$ and $\mathcal{Q}$ intersect in $\overleftrightarrow{AB}$.
$\overleftrightarrow{AB}$ is the intersection of $\mathcal{P}$ and $\mathcal{Q}$.

Reading to Learn

Write a description for each figure.

1. 

2. 

3. 

4. Draw and label a figure for the statement *Planes $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ do not intersect.*

1-2 Linear Measure and Precision

GET READY for the Lesson

With the abundance of rulers, yardsticks, laser measuring devices, and other instruments for measurement, it is easy to take our system of measurement for granted. When the ancient Egyptians found a need for a measurement system, they used the human body as a guide. The cubit was the length of an arm from the elbow to the fingertips. Eventually the Egyptians standardized the length of a cubit.

Main Ideas

- Measure segments and determine accuracy of measurement.
- Compute with measures.

New Vocabulary

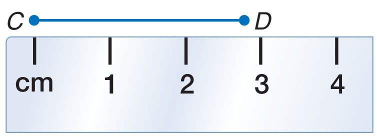
line segment
precision
betweenness of points
between
congruent
construction
absolute error
relative error

Measure Line Segments Unlike a line, a **line segment**, or *segment*, can be measured because it has two endpoints. A segment with endpoints *A* and *B* can be named as $\overline{AB}$ or $\overline{BA}$. The *measure* of $\overline{AB}$ is written as AB. The length or measure of a segment always includes a unit of measure, such as meter or inch.

EXAMPLE 1 Length in Metric Units

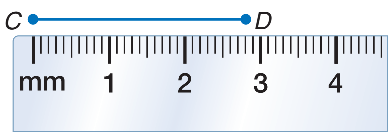
Find the length of $\overline{CD}$ using each ruler.

a.



The ruler is marked in centimeters. Point *D* is closer to the 3-centimeter mark than to 2 centimeters. Thus, $\overline{CD}$ is about 3 centimeters long.

b.



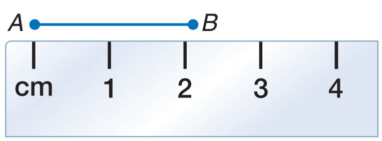
The long marks are centimeters, and the shorter marks are millimeters. There are 10 millimeters for each centimeter. Thus, $\overline{CD}$ is about 28 millimeters long.

CHECK Your Progress

1A. Measure the length of a dollar bill in centimeters.

1B. Measure the length of a pencil in millimeters.

1C. Find the length of $\overline{AB}$.

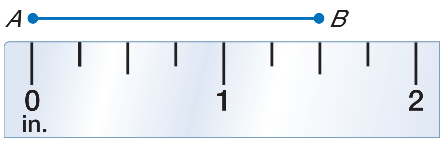


Nebamum Hunting in the Marshes with His Wife and Daughter. c. B.C.E. 1350, Egyptian. Wall painting from the tomb of Nebamum, Thebes/British Museum, London/Bridgeman Art Library

EXAMPLE 2 Length in Customary Units

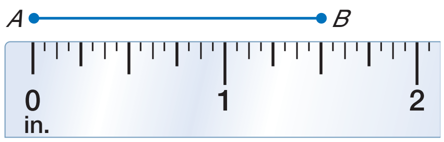
Find the length of $\overline{AB}$ using each ruler.

a.



Each inch is divided into fourths. Point B is closer to the $1\frac{2}{4}$-inch mark. Thus, $\overline{AB}$ is about $1\frac{2}{4}$ or $1\frac{1}{2}$ inches long.

b.



Each inch is divided into sixteenths. Point B is closer to the $1\frac{8}{16}$-inch mark. Thus, $\overline{AB}$ is about $1\frac{8}{16}$ or $1\frac{1}{2}$ inches long.

CHECK Your Progress

2A. Measure the length of a dollar bill in inches.

2B. Measure the length of a pencil in inches.

Study Tip

Using a Ruler

The zero point on a ruler may not be clearly marked. For some rulers, zero is the left edge of the ruler. On others, it may be a line farther in on the scale. If it is not clear where zero is, align one endpoint on 1 and subtract 1 from the measurement at the other endpoint.

The **precision** of any measurement depends on the smallest unit available on the measuring tool. The measurement should be precise to within 0.5 unit of measure. For example, in part **a** of Example 1, 3 centimeters means that the actual length is no less than 2.5 centimeters, but no more than 3.5 centimeters.

Measurements of 28 centimeters and 28.0 centimeters indicate different precision in measurement. A measurement of 28 centimeters means that the ruler is divided into centimeters. However, a measurement of 28.0 centimeters indicates that the ruler is divided into millimeters.

The precision of a measurement in customary units is determined before reducing the fraction. For example, if you measure the length of an object to be $2\frac{2}{4}$ inches, then the measurement is precise to within $\frac{1}{8}$ inch. In this book, the given measurements are indicative of the precision of the measuring instrument.

EXAMPLE 3 Precision

Find the precision for each measurement. Explain its meaning.

a. 5 millimeters

The measurement is precise to within 0.5 millimeter. So, a measurement of 5 millimeters could be 4.5 to 5.5 millimeters.

b. $8\frac{1}{2}$ inches

The measurement is precise to within $\frac{1}{2}\left(\frac{1}{2}\right)$ or $\frac{1}{4}$ inch. Therefore, the measurement could be between $8\frac{1}{4}$ inches and $8\frac{3}{4}$ inches.

CHECK Your Progress

3A. Find the precision for the measure $10\frac{1}{4}$ inches. Explain its meaning.

3B. Find the precision for the measure 12.2 centimeters. Explain its meaning.

Study Tip

Units of Measure

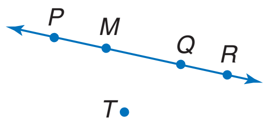
A measurement of 38.0 centimeters on a ruler with millimeter marks means a measurement of 380 millimeters. So the actual measurement is between 379.5 millimeters and 380.5 millimeters, not 37.5 centimeters and 38.5 centimeters. The range of error in the measurement is called the **tolerance** and can be expressed as ± 0.5.

Study Tip

Comparing Measures

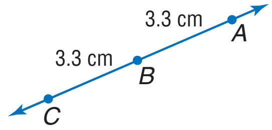
Because measures are real numbers, you can compare them. If X, Y, and Z are collinear in that order, then one of these statements is true. $XY = YZ$, $XY > YZ$, or $XY < YZ$.

Calculate Measures Measures are real numbers, so all arithmetic operations can be used with them. Recall that for any two real numbers a and b, there is a real number n between a and b such that $a < n < b$. This relationship also applies to points on a line and is called **betweenness of points**. Point M is **between** points P and Q if and only if P, Q, and M are collinear and $PM + MQ = PQ$. Notice that points R and T are not between points P and Q.



EXAMPLE 4 Find Measurements

a. Find AC.



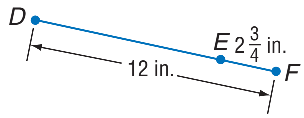
Point B is between A and C. AC can be found by adding AB and BC.

$AB + BC = AC$ Betweenness of points

$3.3 + 3.3 = AC$ Substitution

$6.6 = AC$ So, AC is 6.6 centimeters long.

b. Find DE.



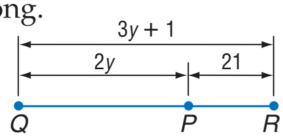
$DE + EF = DF$ Betweenness of points

$DE + 2\frac{3}{4} = 12$ Substitution

$DE + 2\frac{3}{4} - 2\frac{3}{4} = 12 - 2\frac{3}{4}$ Subtract $2\frac{3}{4}$ from each side.

$DE = 9\frac{1}{4}$ So, DE is $9\frac{1}{4}$ inches long.

c. Find y and QP if P is between Q and R, $QP = 2y$, $QR = 3y + 1$, and $PR = 21$.



Draw a figure to represent this information.

$QR = QP + PR$ Betweenness of points

$3y + 1 = 2y + 21$ Substitute known values.

$3y + 1 - 1 = 2y + 21 - 1$ Subtract 1 from each side.

$3y = 2y + 20$ Simplify.

$3y - 2y = 2y + 20 - 2y$ Subtract $2y$ from each side.

$y = 20$ Simplify.

Now find QP.

$QP = 2y$ Given

$= 2(20)$ or 40 $y = 20$

CHECK Your Progress

4. Find x and BC if B is between A and C, $AC = 4x - 12$, $AB = x$, and $BC = 2x + 3$.

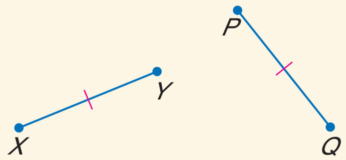
Look at the figure in part **a** of Example 4. Notice that $\overline{AB}$ and $\overline{BC}$ have the same measure. When segments have the same measure, they are said to be **congruent**.

KEY CONCEPT — Congruent Segments

Words Two segments having the same measure are congruent.

Symbol $\cong$ is read *is congruent to*.
Red slashes on the figure also indicate that segments are congruent.

Model $\overline{XY} \cong \overline{PQ}$

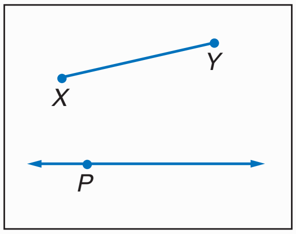


Constructions are methods of creating geometric figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used. You can construct a segment that is congruent to a given segment.

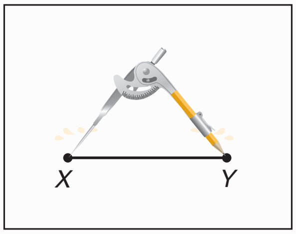
CONSTRUCTION

Copy a Segment

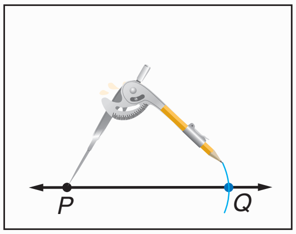
Step 1 Draw a segment $\overline{XY}$. Elsewhere on your paper, draw a line and a point on the line. Label the point P.



Step 2 Place the compass at point X and adjust the compass setting so that the pencil is at point Y.

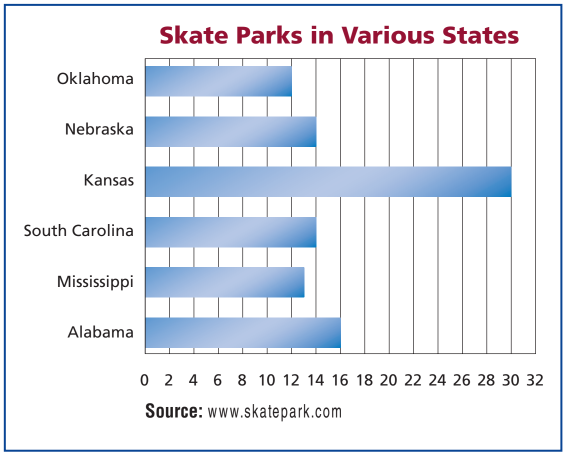


Step 3 Using that setting, place the compass point at P and draw an arc that intersects the line. Label the point of intersection Q. Because of identical compass settings, $\overline{PQ} \cong \overline{XY}$.



Real-World EXAMPLE — Congruent Segments

5 SKATE PARKS In the graph at the right, suppose a segment was drawn along the top of each bar. Which states would have segments that are congruent? Explain.



The segments on the bars for Nebraska and South Carolina would be congruent because they both represent the same number of skate parks.

CHECK Your Progress

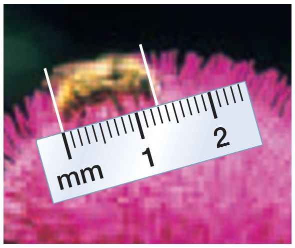
5. Suppose Oklahoma added another skate park. The segment drawn along the bar representing Oklahoma would be congruent to which other segment?

Check Your Understanding

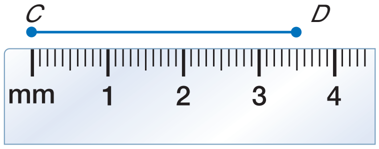
Example 1 (p. 13)

Find the length of each line segment or object.

1.

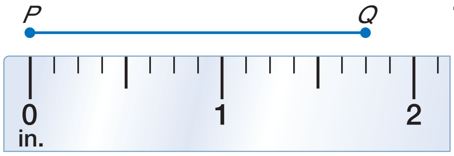


2.

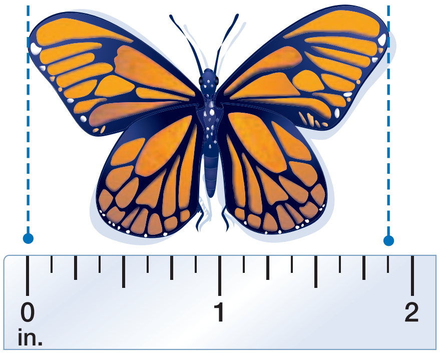


Example 2 (p. 14)

3.



4.



Example 3 (p. 14)

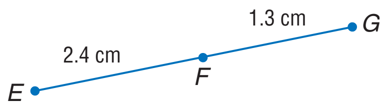
5. Find the precision for a measurement of 14 meters. Explain its meaning.

6. Find the precision for a measurement of $3\frac{1}{4}$ inches. Explain its meaning.

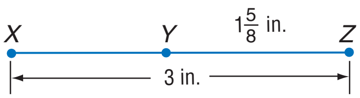
Example 4 (p. 15)

Find the measurement of each segment. Assume that each figure is not drawn to scale.

7. $\overline{EG}$



8. $\overline{XY}$



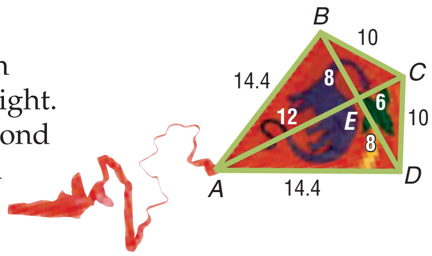
Example 5 (p. 16)

Find the value of x and LM if L is between N and M.

9. $NL = 5x$, $LM = 3x$, and $NL = 15$

10. $NL = 6x - 5$, $LM = 2x + 3$, and $NM = 30$

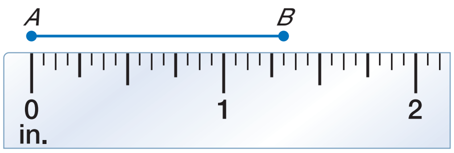
11. KITES Kite making has become an art form using numerous shapes and designs for flight. The figure at the right is known as a diamond kite. The measures are in inches. Name all of the congruent segments in the figure.



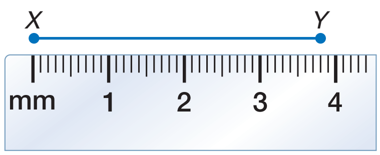
Exercises

Find the length of each line segment or object.

12.



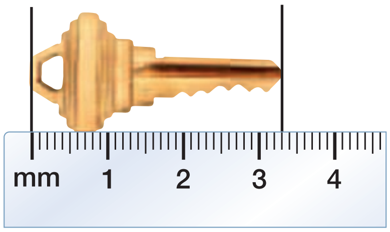
13.



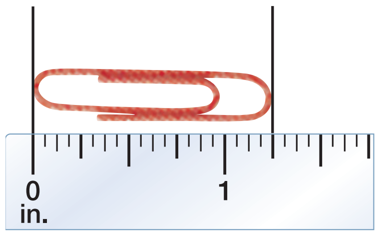
HOMEWORK HELP	
For Exercises	See Examples
13, 14	1
12, 15	2
16–21	3
22–27	4
28–33	5

Find the length of each line segment or object.

14.



15.



Find the precision for each measurement. Explain its meaning.

16. 80 in. **17.** 22 mm **18.** $16\frac{1}{2}$ in.

19. 308 cm **20.** 3.75 meters **21.** $3\frac{1}{4}$ ft

Study Tip

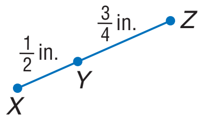
Information from Figures

Segments that have the same measure are congruent. Congruence marks are used to indicate this.

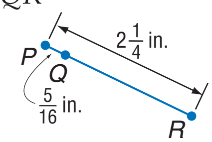
Find the measurement of each segment.

22. AC

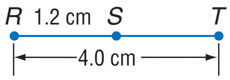
23. XZ



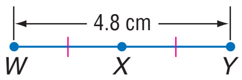
24. QR



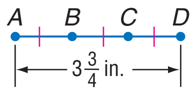
25. ST



26. WX



27. BC



Find the value of the variable and ST if S is between R and T.

28. $RS = 7a$, $ST = 12a$, $RT = 76$

29. $RS = 12$, $ST = 2x$, $RT = 34$

30. $RS = 2x$, $ST = 3x$, $RT = 25$

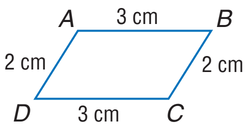
31. $RS = 16$, $ST = 2x$, $RT = 5x + 10$

32. $RS = 3y + 1$, $ST = 2y$, $RT = 21$

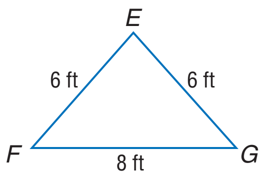
33. $RS = 4y - 1$, $ST = 2y - 1$, $RT = 5y$

Use the figures to determine whether each pair of segments is congruent.

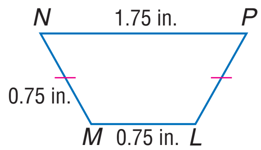
34. $\overline{AB}$, $\overline{CD}$



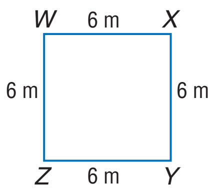
35. $\overline{EF}$, $\overline{FG}$



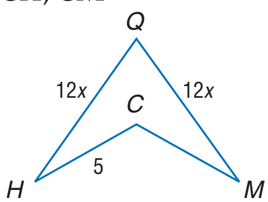
36. $\overline{NP}$, $\overline{LM}$



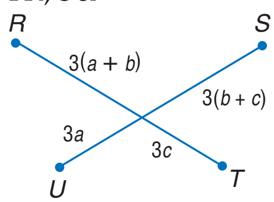
37. $\overline{WX}$, $\overline{XY}$



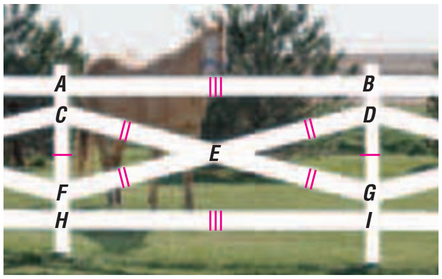
38. $\overline{CH}$, $\overline{CM}$



39. $\overline{TR}$, $\overline{SU}$



40. FENCES Name all segments in the crossbuck pattern in the picture that appear to be congruent.



41. MUSIC A CD has a single spiral track of data, circling from the inside of the disc to the outside. Use a metric ruler to determine the full width of a music CD.

(tl) PhotoLink/Getty Images, (tr) Amanita Pictures, (b) Courtesy Kroy Building Products, Inc.

Real-World Link

There are more than 3300 state parks, historic sites, and natural areas in the United States. Most of the parks are open to visitors year round.

Source: *Parks Directory of the United States*

42. JEWELRY Roxanna sells the jewelry that she makes. One necklace is made from a $14\frac{3}{4}$-inch length of cord. What are the greatest and least possible lengths for the cords to make these necklaces? Assume the same measuring device is used to measure each cord. Explain.

PERIMETER For Exercises 43 and 44, use the following information.
The *perimeter* of a geometric figure is the sum of the lengths of its sides. Pablo used a ruler divided into centimeters and measured the sides of a triangle as 3 centimeters, 5 centimeters, and 6 centimeters. Use what you know about the precision of any measurement to answer each question.

43. What is the least possible perimeter of the triangle? Explain.

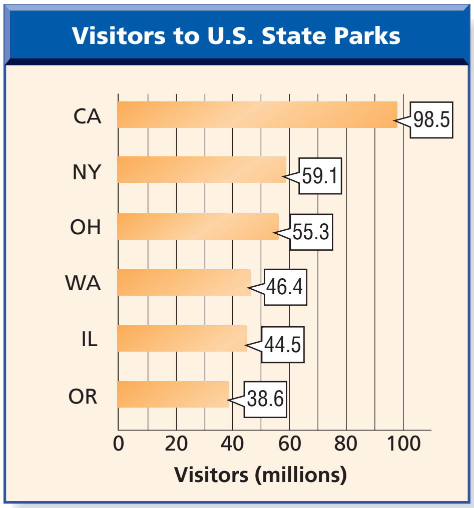
44. What is the greatest possible perimeter of the triangle? Explain.

RECREATION For Exercises 45–47, refer to the graph that shows the states with the greatest number of visitors to state parks in a recent year.

45. To what number can the precision of the data be measured?

46. Find the precision for the California data.

47. Can you be sure that 1.9 million more people visited Washington state parks than Illinois state parks? Explain.



Source: National Association of Park Directors

ERROR Accuracy is an indication of error. The absolute value of the difference between the actual measure of an object and the allowable measure is the **absolute error**. The **relative error** is the ratio of the absolute error to the actual measure. The relative error is expressed as a percent. For a length of 11 inches and an allowable error of 0.5 inch, the absolute error and relative error can be found as follows.

$$\frac{\text{absolute error}}{\text{measure}} = \frac{|11 \text{ in.} - 11.5 \text{ in.}|}{11 \text{ in.}} = \frac{0.5 \text{ in.}}{11 \text{ in.}} \approx 0.045 \text{ or } 4.5\%$$

EXTRA PRACTICE
See pages 800, 828.

Math Online
Self-Check Quiz at
geometryonline.com

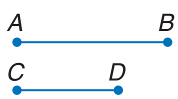
Determine the relative error for each measurement.

48. 27 ft **49.** $14\frac{1}{2}$ in. **50.** 42.3 cm **51.** 63.7 km

CONSTRUCTION For Exercises 52 and 53, refer to the figure.

52. Construct a segment with a measure of $4(CD)$.

53. Construct a segment that has length $3(AB) - 2(CD)$.



H.O.T. Problems

54. REASONING Explain how to measure a segment with a ruler divided into eighths of an inch.

55. OPEN ENDED Give two examples of some geometric figures that have congruent segments.

CHALLENGE **Significant digits** represent the accuracy of a measurement.

- Nonzero digits are always significant.
- In whole numbers, zeros are significant if they fall between nonzero digits.
- In decimal numbers greater than or equal to 1, every digit is significant.
- In decimal numbers less than 1, the first nonzero digit and every digit to its right are significant.

For example, 600.070 has six significant digits, but 0.0210 has only three. How many significant digits are there in each measurement below?

56. 83,000 miles **57.** 33,002 miles **58.** 450.0200 liters

59. *Writing in Math* Why is it important to have a standard of measure? Refer to the information on ancient Egyptian measurement on page 13. Include an advantage and disadvantage to the ancient Egyptian measurement system.

STANDARDIZED TEST PRACTICE

60. A 36-foot-long ribbon is cut into three pieces. The first piece of ribbon is half as long as the second piece of ribbon. The third piece of ribbon is 1 foot longer than twice the length of the second piece of ribbon. What is the length of the longest piece of ribbon?

A 10 feet **C** 21 feet

B 12 feet **D** 25 feet

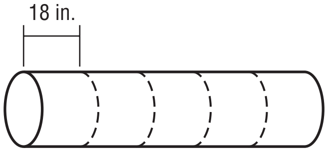
61. **REVIEW** The pipe shown is divided into five equal sections. How long is the pipe in feet (ft) and inches (in.)?

F 6 ft 0 in.

G 6 ft 6 in.

H 7 ft 5 in.

J 7 ft 6 in.



Spiral Review

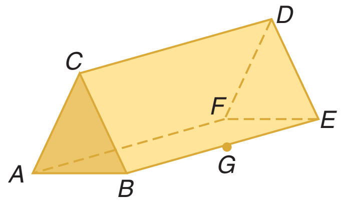
Refer to the figure at the right. (Lesson 1-1)

62. Name three collinear points.

63. Name two planes that contain points B and C.

64. Name another point in plane DFA.

65. How many planes are shown?



TRAVEL **The Hernandez family is driving from Portland, Oregon, to Seattle, Washington. They are using maps to navigate a route. Name the geometric term modeled by each object.** (Lesson 1-1)

66. map **67.** highway **68.** city

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $a = 3$, $b = 8$, and $c = 2$.** (Page 780)

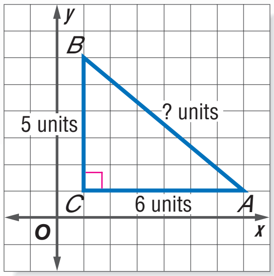
69. $2a + 2b$ **70.** $ac + bc$

71. $\frac{a - c}{2}$ **72.** $\sqrt{(c - a)^2}$

1-3 Distance and Midpoints

GET READY for the Lesson

When you connect two points on a number line or on a plane, you have graphed a line segment. Distance on a number line is determined by counting the units between the two points. On a coordinate plane, you can use the Pythagorean Theorem to find the distance between two points. In the figure, to find the distance from A to B, use $(AC)^2 + (CB)^2 = (AB)^2$.



Main Ideas

- Find the distance between two points.
- Find the midpoint of a segment.

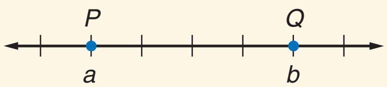
New Vocabulary

midpoint
segment bisector

Distance Between Two Points The coordinates of the endpoints of a segment can be used to find the length of the segment. Because the distance from A to B is the same as the distance from B to A, the order in which you name the endpoints makes no difference.

KEY CONCEPT — *Distance Formulas*

Number Line



$PQ = |b - a|$ or $|a - b|$

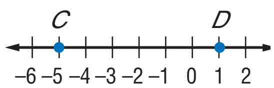
Coordinate Plane

The distance d between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 1 Find Distance on a Number Line

Use the number line to find CD.



The coordinates of C and D are –5 and 1.

$CD = |-5 - 1|$ Distance Formula

$= |-6|$ or 6 Simplify.

Study Tip

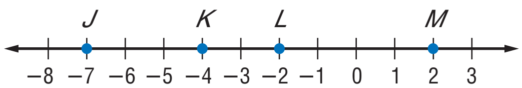
Alternative Method

$CD = |1 - (-5)|$

$= |6|$ or 6

CHECK Your Progress

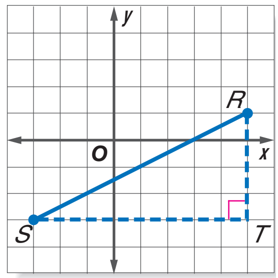
Use the number line to find each measure.



1A. KM **1B.** JM **1C.** KL **1D.** JL

EXAMPLE 1 Find Distance on a Coordinate Plane

Find the distance between $R(5, 1)$ and $S(-3, -3)$.



Method 1 Pythagorean Theorem

Use the gridlines to form a triangle.

$(RS)^2 = (RT)^2 + (ST)^2$ Pythagorean Theorem

$(RS)^2 = 4^2 + 8^2$ $RT = 4$ units, $ST = 8$ units

$(RS)^2 = 80$ Simplify.

$RS = \sqrt{80}$ Take the square root of each side.

Method 2 Distance Formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$RS = \sqrt{(-3 - 5)^2 + (-3 - 1)^2}$ $(x_1, y_1) = (5, 1)$ and $(x_2, y_2) = (-3, -3)$

$RS = \sqrt{(-8)^2 + (-4)^2}$ Subtract.

$RS = \sqrt{64 + 16}$ or $\sqrt{80}$ Simplify.

The distance from R to S is $\sqrt{80}$ units. Use a calculator to find that $\sqrt{80}$ is approximately 8.94.

CHECK Your Progress

2. Find the distance between $D(-5, 6)$ and $E(8, -4)$.

Study Tip

Pythagorean Theorem

Recall that the Pythagorean Theorem is often expressed as $a^2 + b^2 = c^2$, where a and b are the measures of the shorter sides (legs) of a right triangle and c is the measure of the longest side (hypotenuse) of a right triangle.

Midpoint of a Segment The **midpoint** of a segment is the point on the segment that divides the segment into two congruent segments. If X is the midpoint of $\overline{AB}$, then $AX = XB$. In the lab you will derive the Midpoint Formula.

GEOMETRY LAB

Midpoint of a Segment

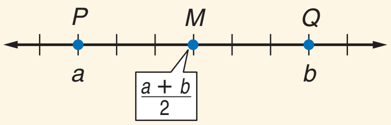
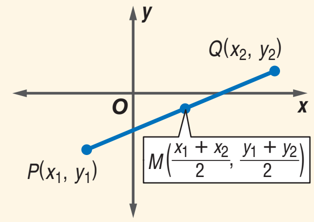
MODEL

- Graph points $A(5, 5)$ and $B(-1, 5)$ on grid paper. Draw $\overline{AB}$.
- Hold the paper up to the light and fold the paper so that points A and B match exactly. Crease the paper slightly. Then open the paper.
- Put a point where the crease intersects $\overline{AB}$. Label this midpoint as C.
- Repeat using endpoints $X(-4, 3)$ and $Y(2, 7)$. Label the midpoint Z.

MAKE A CONJECTURE

1. What are the coordinates of point C? What are the measures of $\overline{AC}$ and $\overline{CB}$?
2. What are the coordinates of point Z? What are the measures of $\overline{XZ}$ and $\overline{ZY}$?
3. Study the coordinates of points A, B, and C. Write a rule that relates these coordinates. Then use points X, Y, and Z to verify your conjecture.

KEY CONCEPT — Midpoint

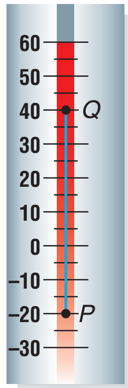
Words	The midpoint M of $\overline{PQ}$ is the point between P and Q such that $PM = MQ$.	
Symbols	**Number Line**	**Coordinate Plane**
	The coordinate of the midpoint of a segment with endpoints that have coordinates a and b is $\frac{a+b}{2}$.	The coordinates of the midpoint of a segment with endpoints that have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.
Models		

Cross-Curricular Project

Math Online Latitude and longitude form another coordinate system. The latitude, longitude, degree distance, and monthly high temperature can be used to create several different scatter plots. Visit geometryonline.com to continue work on your project.

Real-World EXAMPLE — Find Coordinates of Midpoint

3 **TEMPERATURE** **Find the coordinate of the midpoint of $\overline{PQ}$.**



The coordinates of P and Q are -20 and 40.

Let M be the midpoint of $\overline{PQ}$.

$$M = \frac{-20 + 40}{2} \qquad a = -20,\ b = 40$$

$$= \frac{20}{2} \text{ or } 10 \qquad \text{Simplify.}$$

The midpoint is 10.

CHECK Your Progress

3. TEMPERATURE The temperature dropped from 25° to -8°. Find the midpoint of these temperatures.

EXAMPLE — Find Coordinates of Midpoint

4 **Find the coordinates of M, the midpoint of $\overline{JK}$, for $J(-1, 2)$ and $K(6, 1)$.**

Let J be (x_1, y_1) and K be (x_2, y_2).

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{-1+6}{2}, \frac{2+1}{2}\right) \qquad (x_1, y_1) = (-1, 2),\ (x_2, y_2) = (6, 1)$$

$$= M\left(\frac{5}{2}, \frac{3}{2}\right) \text{ or } M\left(2\frac{1}{2}, 1\frac{1}{2}\right) \qquad \text{Simplify.}$$

CHECK Your Progress

4. Find the coordinates of the midpoint of $\overline{AB}$ for $A(5, 12)$ and $B(-4, 8)$.

You can also find the coordinates of an endpoint of a segment if you know the coordinates of its other endpoint and its midpoint.

EXAMPLE Find Coordinates of Endpoint

5 Find the coordinates of X if $Y(-1, 6)$ is the midpoint of $\overline{XZ}$ and Z has coordinates $(2, 8)$.

Let Z be (x_2, y_2) in the Midpoint Formula.

$$Y(-1, 6) = Y\left(\frac{x_1 + 2}{2}, \frac{y_1 + 8}{2}\right) \quad (x_2, y_2) = (2, 8)$$

Write two equations to find the coordinates of X.

$-1 = \frac{x_1 + 2}{2}$		$6 = \frac{y_1 + 8}{2}$	
$-2 = x_1 + 2$	Multiply each side by 2.	$12 = y_1 + 8$	Multiply each side by 2.
$-4 = x_1$	Subtract 2 from each side.	$4 = y_1$	Subtract 8 from each side.

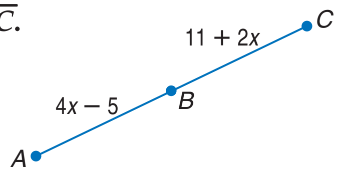
The coordinates of X are $(-4, 4)$.

CHECK Your Progress

5. Find the coordinates of G if $P(-5, 10)$ is the midpoint of $\overline{EG}$ and E has coordinates $(-8, 6)$.

EXAMPLE Use Algebra to Find Measures

6 Find the measure of $\overline{BC}$ if B is the midpoint of $\overline{AC}$.



You know that B is the midpoint of $\overline{AC}$, and the figure gives algebraic measures for $\overline{AB}$ and $\overline{BC}$. You are asked to find the measure of $\overline{BC}$.

Because B is the midpoint, you know that $AB = BC$. Use this equation and the algebraic measures to find a value for x.

$AB = BC$	Definition of midpoint
$4x - 5 = 11 + 2x$	$AB = 4x - 5$, $BC = 11 + 2x$
$4x = 16 + 2x$	Add 5 to each side.
$2x = 16$	Subtract $2x$ from each side.
$x = 8$	Divide each side by 2.

Now substitute 8 for x in the expression for BC.

$BC = 11 + 2x$	Original measure
$= 11 + 2(8)$	$x = 8$
$= 11 + 16$ or 27	The measure of $\overline{BC}$ is 27.

CHECK Your Progress

6. Find the measure of $\overline{XY}$ if Y is the midpoint of $\overline{XZ}$, and $XY = 2x + 3$, and $YZ = 6 - 4x$.

Online **Personal Tutor at** geometryonline.com

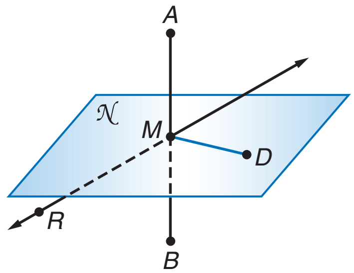
Concepts in Motion
Interactive Lab
geometryonline.com

Study Tip

Segment Bisectors

There can be an infinite number of bisectors and each must contain the midpoint of the segment.

Any segment, line, or plane that intersects a segment at its midpoint is called a **segment bisector**. In the figure at the right, M is the midpoint of $\overline{AB}$. Plane $\mathcal{N}$, $\overline{MD}$, $\overleftrightarrow{RM}$, and point M are all bisectors of $\overline{AB}$. We say that they *bisect* $\overline{AB}$.

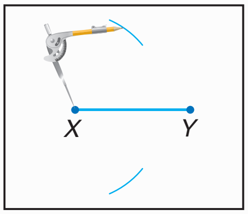


You can construct a line that bisects a segment without measuring to find the midpoint of the given segment.

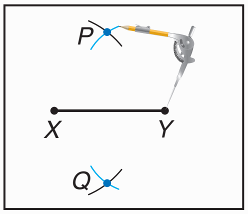
CONSTRUCTION

Bisect a Segment

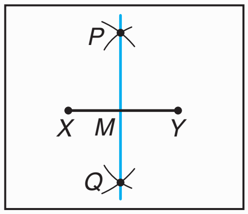
Step 1 Draw a segment and name it $\overline{XY}$. Place the compass at point X. Adjust the compass so that its width is greater than $\frac{1}{2}XY$. Draw arcs above and below $\overline{XY}$.



Step 2 Using the same compass setting, place the compass at point Y and draw arcs above and below $\overline{XY}$ that intersect the two arcs previously drawn. Label the points of intersection as P and Q.



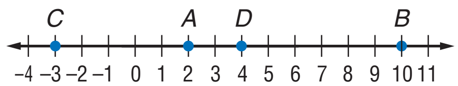
Step 3 Use a straightedge to draw $\overleftrightarrow{PQ}$. Label the point where it intersects $\overline{XY}$ as M. Point M is the midpoint of $\overline{XY}$, and $\overleftrightarrow{PQ}$ is a bisector of $\overline{XY}$. Also $XM = MY = \frac{1}{2}XY$.



✓ CHECK Your Understanding

Example 1 (p. 21)

Use the number line to find each measure.



1. AB

2. CD

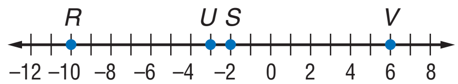
Example 2 (p. 22)

3. Use the Pythagorean Theorem to find the distance between $X(7, 11)$ and $Y(-1, 5)$.

4. Use the Distance Formula to find the distance between $D(2, 0)$ and $E(8, 6)$.

Example 3 (p. 23)

Use the number line to find the coordinate of the midpoint of each segment.



5. $\overline{RS}$

6. $\overline{UV}$

Example 4 (p. 23)

Find the coordinates of the midpoint of a segment having the given endpoints.

7. $X(-4, 3), Y(-1, 5)$

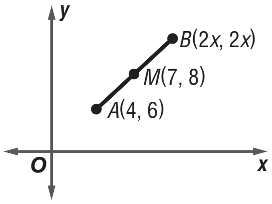
8. $A(2, 8), B(-2, 2)$

Example 5 (p. 24)

9. Find the coordinates of A if $B(0, 5.5)$ is the midpoint of $\overline{AC}$ and C has coordinates $(-3, 6)$.

Example 6 (p. 24)

10. Point M is the midpoint of $\overline{AB}$. What is the value of x in the figure?



Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–16	1
17–28	2
29–34	3
35–42	4
43–45	5
46–49	6

Use the number line to find each measure.

11. DE

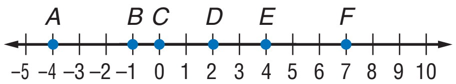
12. CF

13. AB

14. AC

15. AF

16. BE



Use the Pythagorean Theorem to find the distance between each pair of points.

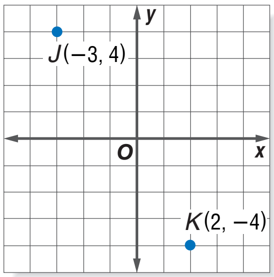
17. $A(0, 0), B(8, 6)$

18. $C(-10, 2), D(-7, 6)$

19. $E(-2, -1), F(3, 11)$

20. $G(-2, -6), H(6, 9)$

21.



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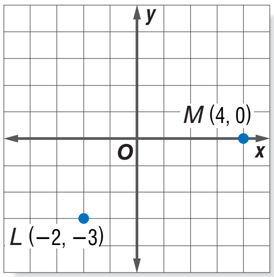
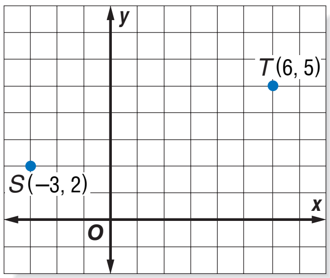


Use the Distance Formula to find the distance between each pair of points.

23. $J(0, 0), K(12, 9)$

24. $L(3, 5), M(7, 9)$

25.



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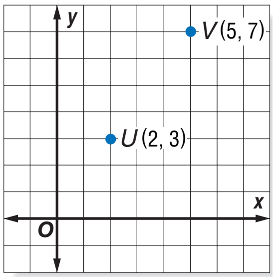

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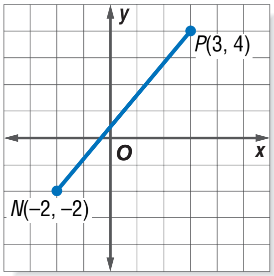
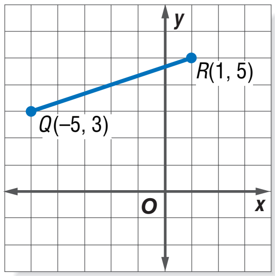
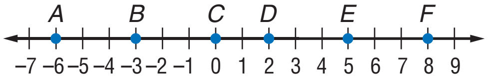


28.



Use the number line to find the coordinate of the midpoint of each segment.

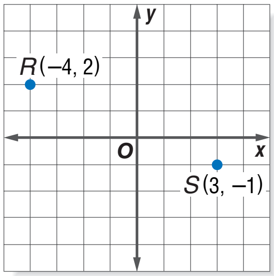


29. $\overline{AC}$ **30.** $\overline{DF}$ **31.** $\overline{CE}$

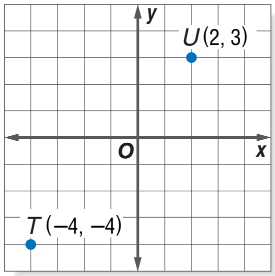
32. $\overline{BD}$ **33.** $\overline{AF}$ **34.** $\overline{BE}$

Find the coordinates of the midpoint of a segment having the given endpoints.

35.



36.



37. $A(8, 4), B(12, 2)$ **38.** $C(9, 5), D(17, 4)$

39. $E(-11, -4), F(-9, -2)$ **40.** $G(4, 2), H(8, -6)$

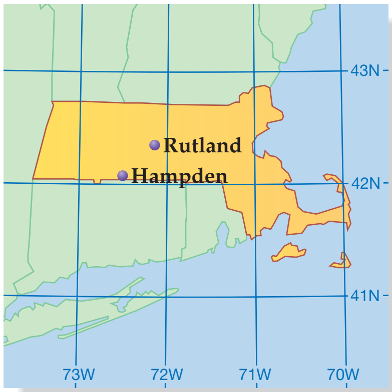
41. $J(3.4, 2.1), K(7.8, 3.6)$ **42.** $L(-1.4, 3.2), M(2.6, -5.4)$

Find the coordinates of the missing endpoint given that S is the midpoint of $\overline{RT}$.

43. $T(-4, 3), S(-1, 5)$ **44.** $T(2, 8), S(-2, 2)$ **45.** $R\left(\frac{2}{3}, -5\right), S\left(\frac{5}{3}, 3\right)$

GEOGRAPHY **For Exercises 46–49, use the following information.**

The geographic center of Massachusetts is in Rutland at $(42.4°, 71.9°)$, which represents north latitude and west longitude. Hampden is near the southern border of Massachusetts at $(42.1°, 72.4°)$.



Study Tip

Distance on Earth

Actual distances on Earth are calculated along the curve of the Earth's surface. This uses *spherical geometry*. When points are close together, you can use plane geometry to approximate the distance.

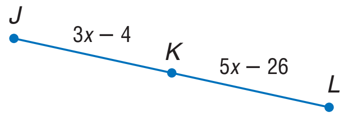
46. If Hampden is one endpoint of a segment and Rutland is its midpoint, find the latitude and longitude of the other endpoint.

47. Use an atlas or the Internet to find a city near the location of the other endpoint.

48. If Hampden is the midpoint of a segment with one endpoint at Rutland, find the latitude and longitude of the other endpoint.

49. Use an atlas or the Internet to find a city near the location of the other endpoint.

50. **ALGEBRA** Find the measure of $\overline{JK}$ if K is the midpoint of $\overline{JL}$.



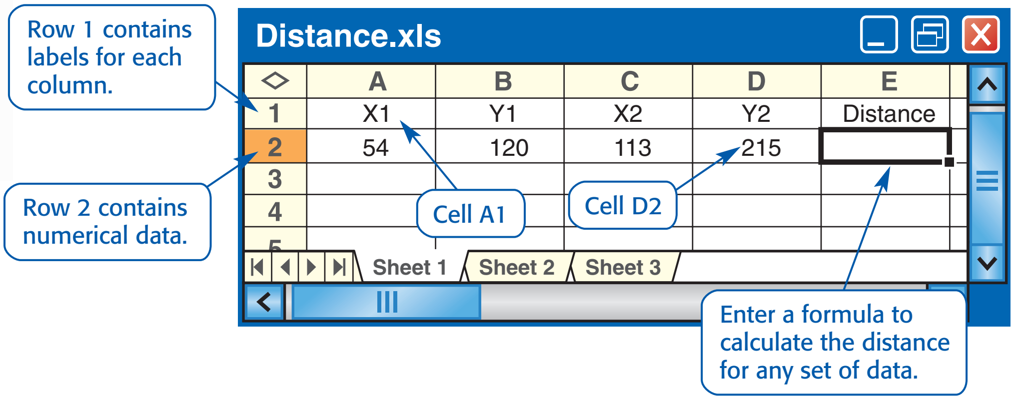
Study Tip

Spreadsheets

Spreadsheets often use special commands to perform operations. For example, $\sqrt{x_1 - x_2}$ would be written as = SQRT(A2 – C2). Use the ^ symbol to raise a number to a power. For example, x^2 is written as x^2.

SPREADSHEETS For Exercises 51–55, refer to the information at the left and below.

Spreadsheets can be used to perform calculations quickly. The spreadsheet below can be used to calculate the distance between two points. Values are used in formulas by using a specific cell name. The value of x_1 is used in a formula using its cell name, A2.



51. Write a formula for cell E2 that could be used to calculate the distance between (x_1, y_1) and (x_2, y_2).

EXTRA PRACTICE

See pages 800, 828.

Math Online

Self-Check Quiz at geometryonline.com

Find the distance between each pair of points to the nearest tenth.

52. (54, 120), (113, 215)

53. (68, 153), (175, 336)

54. (421, 454), (502, 798)

55. (837, 980), (612, 625)

H.O.T. Problems

56. **REASONING** Explain three ways to find the midpoint of a segment.

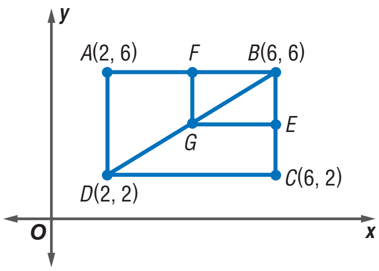
57. **OPEN ENDED** Draw a segment. Construct the bisector of the segment and use a millimeter ruler to check the accuracy of your construction.

58. **CHALLENGE** In the figure, $\overline{GE}$ bisects $\overline{BC}$, and $\overline{GF}$ bisects $\overline{AB}$. $\overline{GE}$ is a horizontal segment, and $\overline{GF}$ is a vertical segment.

a. Find the coordinates of points F and E.

b. Name the coordinates of G and explain how you calculated them.

c. Describe what relationship, if any, exists between $\overline{DG}$ and $\overline{GB}$. Explain.

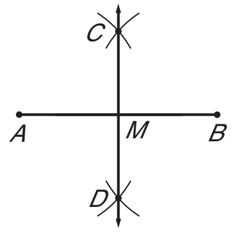


59. **CHALLENGE** $\overline{WZ}$ has endpoints $W(-3, -8)$ and $Z(5, 12)$. Point X lies between W and Z, such that $WX = \frac{1}{4}WZ$. Find the coordinates of X.

60. ***Writing in Math*** Explain how you can find the distance between two points without a ruler. Include how to use the Pythagorean Theorem and the Distance Formula to find the distance between two points, and the length of $\overline{AB}$ from the figure on page 21.

STANDARDIZED TEST PRACTICE

61. Which of the following *best* describes the first step in bisecting segment AB?



A From point A, draw equal arcs on $\overline{CD}$ using the same compass width.

B From point A, draw equal arcs above and below $\overline{AB}$ using a compass width of $\frac{1}{3}AB$.

C From point A, draw equal arcs above and below $\overline{AB}$ using a compass width greater than $\frac{1}{2}AB$.

D From point A, draw equal arcs above and below $\overline{AB}$ using a compass width less than $\frac{1}{2}AB$.

62. Madison paid \$138.16 for 4 pairs of jeans. All 4 pairs of jeans were the same price. How much did each pair of jeans cost?

F \$34.54

G \$42.04

H \$135.16

J \$142.16

63. **REVIEW** Simplify $(3x^2 - 2)(2x + 4) - 2x^2 + 6x + 7$.

A $4x^2 + 14x - 1$

B $4x^2 - 14x + 15$

C $6x^3 + 12x^2 + 2x - 1$

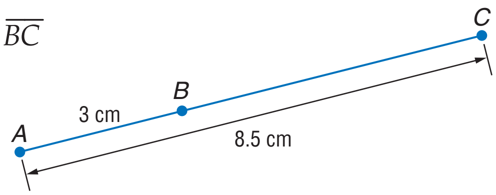
D $6x^3 + 10x^2 + 2x - 1$

Spiral Review

Find the measurement of each segment. (Lesson 1-2)

64. $\overline{WY}$

65. $\overline{BC}$



Draw and label a figure for each relationship. (Lesson 1-1)

66. four noncollinear points A, B, C, and D that are coplanar

67. line m that intersects plane A and line n in plane A

68. Lines a, b, and c are coplanar, but do not intersect.

69. Lines a, b, and c are coplanar and meet at point F.

70. Point C and line r lie in M. Line r intersects line s at D. Point C, line r, and line s are not coplanar.

71. Planes A and B intersect in line s. Plane C intersects A and B, but does not contain line s.

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** (Pages 781–782)

72. $2k = 5k - 30$

73. $14x - 31 = 12x + 8$

74. $180 - 8t = 90 + 2t$

75. $12m + 7 = 3m + 52$

76. $8x + 7 = 5x + 20$

77. $13n - 18 = 5n + 32$

EXTEND 1-3

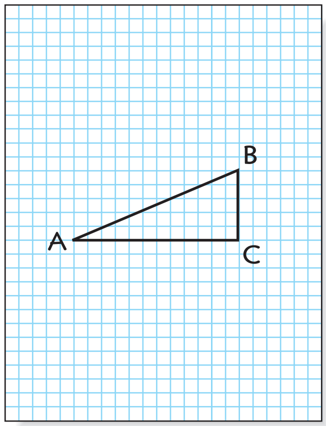
Geometry Lab

Modeling the Pythagorean Theorem

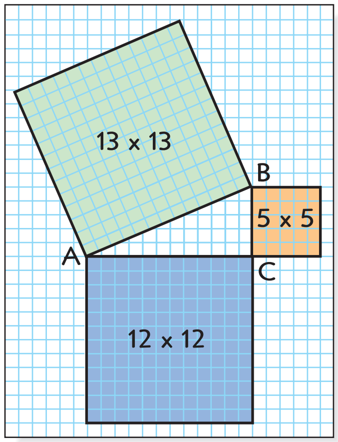
In Chapter 8, you will prove the Pythagorean Theorem, but this activity will suggest that the Pythagorean Theorem holds for any right triangle.

ACTIVITY

- Draw right triangle *ABC* in the center of a piece of grid paper.



- On another piece of grid paper, draw a square that is 5 units on each side, a square that is 12 units on each side, and a square that is 13 units on each side. Use colored pencils to shade each of these squares. Cut out the squares. Label them as 5×5, 12×12, and 13×13, respectively.
- Place the squares so that a side of the square matches up with a side of the right triangle.



ANALYZE THE RESULTS

1. Determine the number of grid squares in each square you drew.
2. How do the numbers of grid squares relate?
3. If $AB = c$, $BC = a$, and $AC = b$, write an expression to describe each of the squares.
4. Compare this expression with what you know about the Pythagorean Theorem.
5. **MAKE A CONJECTURE** Repeat the activity for triangles with each of the side measures listed below. What do you find is true of the relationship of the squares on the sides of the triangle?

   a. 3, 4, 5 **b.** 8, 15, 17 **c.** 6, 8, 10
6. Repeat the activity with a right triangle with shorter sides that are both 5 units long. How could you determine the number of grid squares in the larger square?

1-4 Angle Measure

Main Ideas

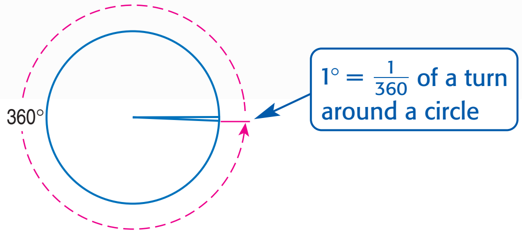
- Measure and classify angles.
- Identify and use congruent angles and the bisector of an angle.

New Vocabulary

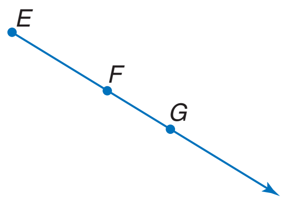
degree
ray
opposite rays
angle
sides
vertex
interior
exterior
right angle
acute angle
obtuse angle
angle bisector

GET READY for the Lesson

Astronomer Claudius Ptolemy based his observations of the solar system on a unit that resulted from dividing the circumference, or the distance around, a circle into 360 parts. This later became known as a **degree**. In this lesson, you will learn to measure angles in degrees.



Measure Angles A **ray** is part of a line. It has one endpoint and extends indefinitely in one direction. Rays are named stating the endpoint first and then any other point on the ray. The figure at the right shows ray *EF*, which can be symbolized as $\overrightarrow{EF}$. This ray could also be named as $\overrightarrow{EG}$, but not as $\overrightarrow{FE}$ because *F* is not the endpoint of the ray.



If you choose a point on a line, that point determines exactly two rays called **opposite rays**. Line *m*, shown below, is separated into two opposite rays, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Point *P* is the common endpoint of those rays. $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are collinear rays.

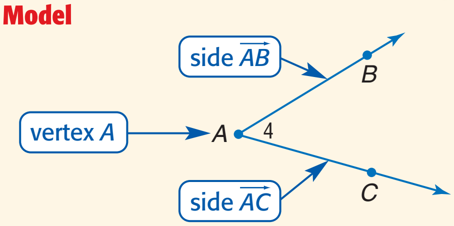
An **angle** is formed by two *noncollinear* rays that have a common endpoint. The rays are called **sides** of the angle. The common endpoint is the **vertex**.

KEY CONCEPT — Angle

Words An angle is formed by two noncollinear rays that have a common endpoint.

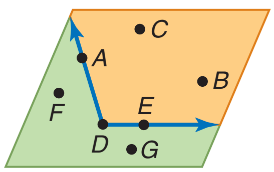
Symbols $\angle A$
$\angle BAC$
$\angle CAB$
$\angle 4$

Model



An angle divides a plane into three distinct parts.

- Points *A*, *D*, and *E* lie on the angle.
- Points *C* and *B* lie in the **interior** of the angle.
- Points *F* and *G* lie in the **exterior** of the angle.

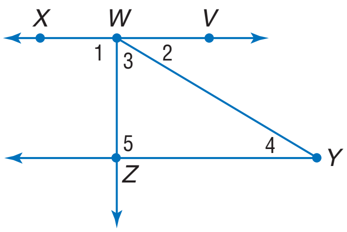


Study Tip

Naming Angles

You can name an angle by a single letter *only* when there is one angle shown at that vertex.

EXAMPLE 1 Angles and Their Parts



a. Name all angles that have *W* as a vertex.

$\angle 1, \angle 2, \angle 3, \angle XWY, \angle ZWV, \angle YWV$

b. Name the sides of ∠1.

$\overrightarrow{WZ}$ and $\overrightarrow{WX}$ are the sides of $\angle 1$.

c. Write another name for ∠*WYZ*.

$\angle 4$, $\angle Y$, and $\angle ZYW$ are other names for $\angle WYZ$.

CHECK Your Progress **1.** Name a pair of opposite rays.

To measure an angle, you can use a *protractor*. Angle *PQR* is a 65 degree (65°) angle. We say that the *degree measure* of $\angle PQR$ is 65, or simply $m\angle PQR = 65$.

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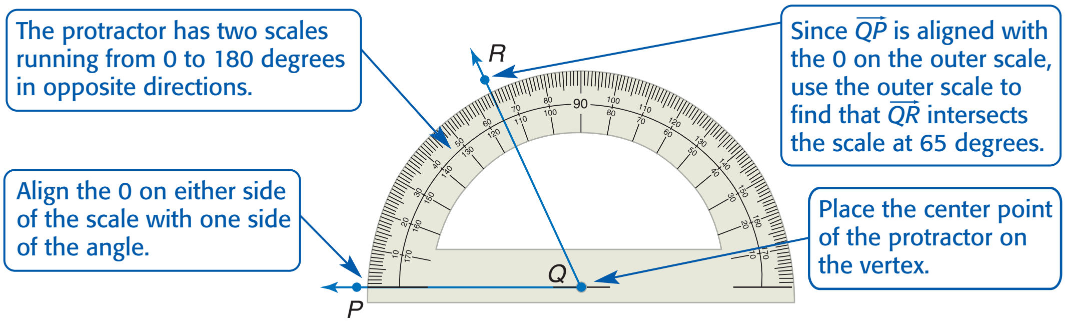
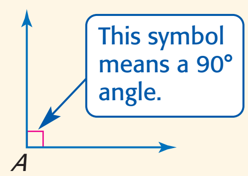
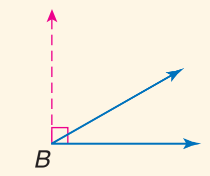
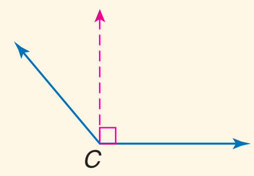


Angles can be classified by their measures.

Reading Math

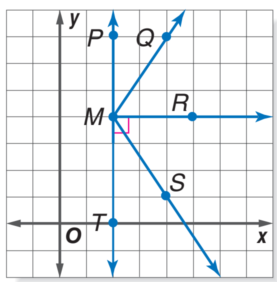
Angles Opposite rays are also known as a *straight angle*. Its measure is 180°. Unless otherwise specified in this book, the term *angle* means a nonstraight angle.

KEY CONCEPT *Classify Angles*

Name	right angle	acute angle	obtuse angle
Measure	$m\angle A = 90$	$m\angle B < 90$	$180 > m\angle C > 90$
Model			

EXAMPLE 2 Measure and Classify Angles

Measure each angle and classify as *right*, *acute*, or *obtuse*.



a. ∠*PMQ*

Use a protractor to find that $m\angle PMQ = 30$.
$30 < 90$, so $\angle PMQ$ is an acute angle.

b. ∠*TMR*

$\angle TMR$ is marked with a right angle symbol, so measuring is not necessary; $m\angle TMR = 90$.

CHECK Your Progress

2. Measure $\angle QMT$ and classify it as *right*, *acute*, or *obtuse*.

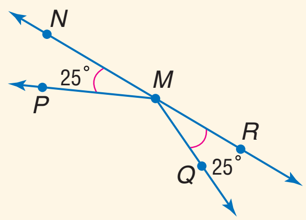
Congruent Angles Just as segments that have the same measure are congruent, angles that have the same measure are congruent.

KEY CONCEPT — Congruent Angles

Words Angles that have the same measure are congruent angles. Arcs on the figure indicate which angles are congruent.

Symbols $\angle NMP \cong \angle QMR$

Model



You can construct an angle congruent to a given angle without knowing the measure of the angle.

CONSTRUCTION

Copy an Angle

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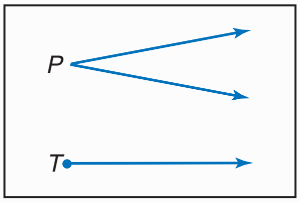
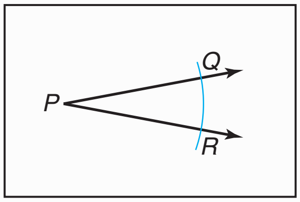
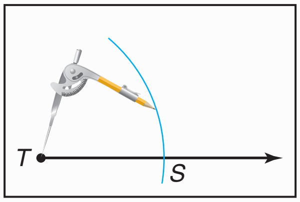
Step 1
Draw an angle like $\angle P$ on your paper. Use a straightedge to draw a ray on your paper. Label its endpoint *T*.



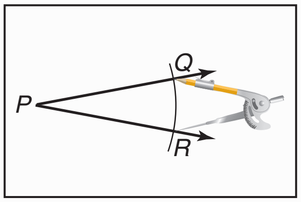
Step 2
Place the tip of the compass at point *P* and draw a large arc that intersects both sides of $\angle P$. Label the points of intersection *Q* and *R*.



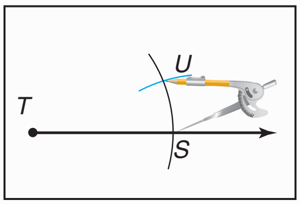
Step 3
Using the same compass setting, put the compass at *T* and draw a large arc that intersects the ray. Label the point of intersection *S*.



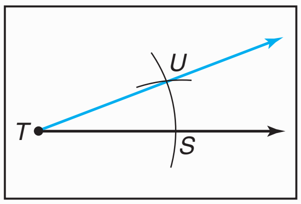
Step 4
Place the point of your compass on *R* and adjust so that the pencil tip is on *Q*.



Step 5
Without changing the setting, place the compass at *S* and draw an arc to intersect the larger arc you drew in Step 3. Label the point of intersection *U*.

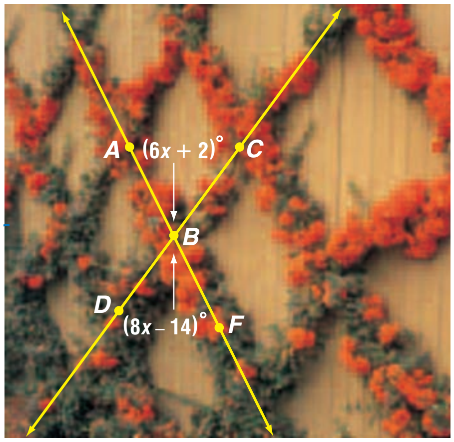


Step 6
Use a straightedge to draw $\overrightarrow{TU}$.



EXAMPLE 3 Use Algebra to Find Angle Measures

GARDENING **A trellis is often used to provide a frame for vining plants. Some of the angles formed by the slats of the trellis are congruent angles. In the figure, $\angle ABC \cong \angle DBF$. If $m\angle ABC = 6x + 2$ and $m\angle DBF = 8x - 14$, find the actual measurements of $\angle ABC$ and $\angle DBF$.**



$\angle ABC \cong \angle DBF$	Given
$m\angle ABC = m\angle DBF$	Definition of congruent angles
$6x + 2 = 8x - 14$	Substitution
$6x + 16 = 8x$	Add 14 to each side.
$16 = 2x$	Subtract 6x from each side.
$8 = x$	Divide each side by 2.

Use the value of x to find the measure of one angle.

$m\angle ABC = 6x + 2$	Given
$= 6(8) + 2$	$x = 8$
$= 48 + 2$ or 50	Simplify.

Since $m\angle ABC = m\angle DBF$, $m\angle DBF = 50$. Both $\angle ABC$ and $\angle DBF$ measure 50.

> **Study Tip**
>
> **Checking Solutions**
>
> Check that you have computed the value of x correctly by substituting the value into the expression for $\angle DBF$. If you don't get the same measure as $\angle ABC$, you have made an error.

CHECK Your Progress

3. Suppose $\angle JKL \cong \angle MKN$. If $m\angle JKL = 5x + 4$ and $m\angle MKN = 3x + 12$, find the actual measurements of the two angles.

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GEOMETRY LAB

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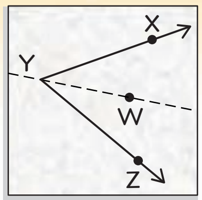
Bisect an Angle

MAKE A MODEL

- Draw any $\angle XYZ$ on patty paper or tracing paper.
- Fold the paper through point Y so that $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are aligned together.
- Open the paper and label a point on the crease in the interior of $\angle XYZ$ as point W.



ANALYZE THE MODEL

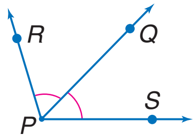
1. What seems to be true about $\angle XYW$ and $\angle WYZ$?

2. Measure $\angle XYZ$, $\angle XYW$, and $\angle WYZ$.

3. You learned about segment bisectors in Lesson 1-3. **Make a conjecture** about the term *angle bisector*.

Red Habegger/Grant Heilman Photography

A ray that divides an angle into two congruent angles is called an **angle bisector**. If $\overrightarrow{PQ}$ is the angle bisector of $\angle RPS$, then point Q lies in the interior of $\angle RPS$, and $\angle RPQ \cong \angle QPS$. A line segment can also bisect an angle.



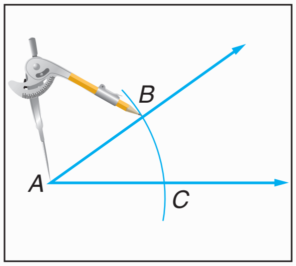
Just as with segments, when a line divides an angle into smaller angles, the sum of the measures of the smaller angles equals the measure of the largest angle. So in the figure, $m\angle RPS = m\angle RPQ + m\angle QPS$.

You can construct the angle bisector of any angle without knowing the measure of the angle.

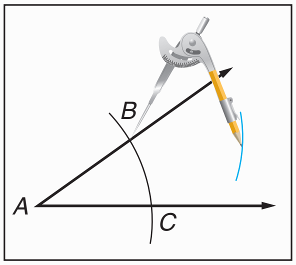
CONSTRUCTION

Bisect an Angle

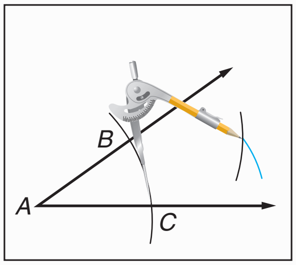
Step 1
Draw an angle and label the vertex as A. Put your compass at point A and draw a large arc that intersects both sides of $\angle A$. Label the points of intersection B and C.



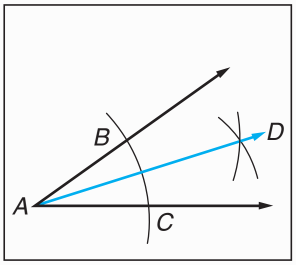
Step 2
With the compass at point B, draw an arc in the interior of the angle.



Step 3
Keeping the same compass setting, place the compass at point C and draw an arc that intersects the arc drawn in Step 2.



Step 4
Label the point of intersection D. Draw $\overrightarrow{AD}$. $\overrightarrow{AD}$ is the bisector of $\angle A$. Thus, $m\angle BAD = m\angle DAC$ and $\angle BAD \cong \angle DAC$.



CHECK Your Understanding

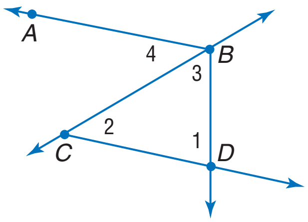
Example 1 (p. 32)

For Exercises 1–3, use the figure at the right.

1. Name the vertex of $\angle 2$.

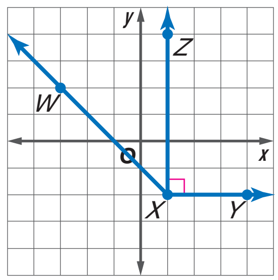
2. Name the sides of $\angle 4$.

3. Write another name for $\angle BDC$.



Example 2 (p. 32)

Measure each angle and classify as *right*, *acute*, or *obtuse*.

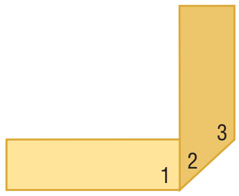


4. $\angle WXY$

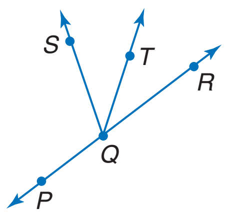
5. $\angle WXZ$

Example 3 (p. 34)

6. **ORIGAMI** The art of origami involves folding paper at different angles to create designs and three-dimensional figures. One of the folds in origami involves folding a strip of paper so that the lower edge of the strip forms a right angle with itself. Identify each numbered angle as *right*, *acute*, or *obtuse*.



ALGEBRA **In the figure, $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are opposite rays, and $\overrightarrow{QT}$ bisects $\angle RQS$.**



7. If $m\angle RQT = 6x + 5$ and $m\angle SQT = 7x - 2$, find $m\angle RQT$.

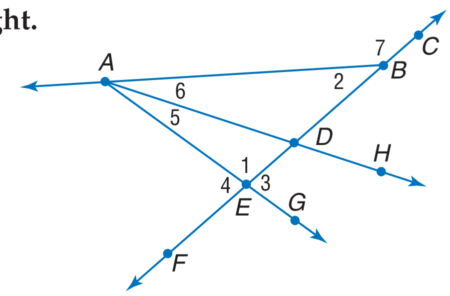
8. Find $m\angle TQS$ if $m\angle RQS = 22a - 11$ and $m\angle RQT = 12a - 8$.

Exercises

HOMEWORK HELP

For Exercises	See Examples
9–24	1
25–30	2
31–36	3

For Exercises 9–24, use the figure on the right. Name the vertex of each angle.



9. $\angle 1$ **10.** $\angle 2$

11. $\angle 6$ **12.** $\angle 5$

Name the sides of each angle.

13. $\angle ADB$ **14.** $\angle 6$

15. $\angle 3$ **16.** $\angle 5$

Write another name for each angle.

17. $\angle 7$ **18.** $\angle AEF$ **19.** $\angle ABD$ **20.** $\angle 1$

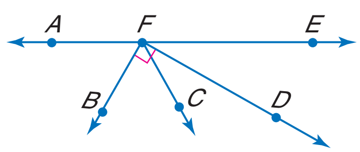
21. Name a point in the interior of $\angle GAB$.

22. Name an angle with vertex B that appears to be acute.

23. Name a pair of angles that share exactly one point.

24. Name a point in the interior of $\angle CEG$.

Measure each angle and classify as *right*, *acute*, or *obtuse*.

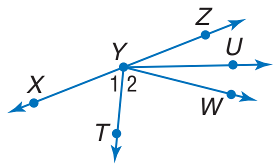


25. $\angle BFD$ **26.** $\angle AFB$

27. $\angle DFE$ **28.** $\angle EFC$

29. $\angle AFD$ **30.** $\angle EFB$

ALGEBRA **In the figure, $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays. $\overrightarrow{YU}$ bisects $\angle ZYW$, and $\overrightarrow{YT}$ bisects $\angle XYW$.**



31. If $m\angle ZYU = 8p - 10$ and $m\angle UYW = 10p - 20$, find $m\angle ZYU$.

32. If $m\angle 1 = 5x + 10$ and $m\angle 2 = 8x - 23$, find $m\angle 2$.

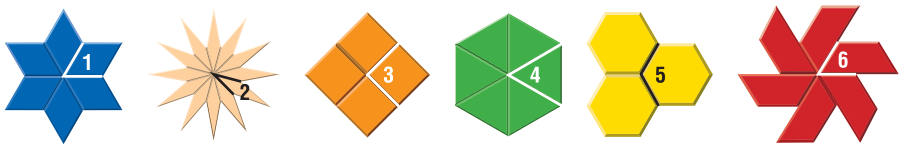
33. If $m\angle 1 = y$ and $m\angle XYW = 6y - 24$, find y.

34. If $m\angle WYZ = 82$ and $m\angle ZYU = 4r + 25$, find r.

35. If $m\angle WYX = 2(12b + 7)$ and $m\angle ZYU = 9b - 1$, find $m\angle UYW$.

36. If $\angle ZYW$ is a right angle and $m\angle ZYU = 13a - 7$, find a.

37. **PATTERN BLOCKS** Pattern blocks can be arranged to fit in a circular pattern without leaving spaces. Remember that the measurement around a full circle is 360°. Determine the degree measure of the numbered angles shown below.



EXTRA PRACTICE
See pages 801, 828.
Math Online
Self-Check Quiz at
geometryonline.com

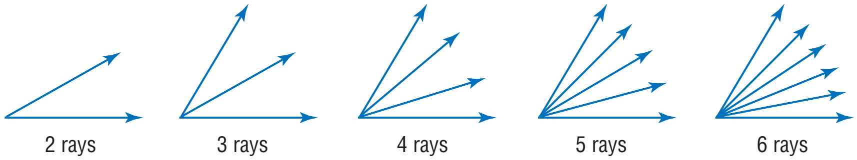
38. **RESEARCH** The words *obtuse* and *acute* have other meanings in the English language. Look these words up in a dictionary and write how the everyday meaning relates to the mathematical meaning.

H.O.T. Problems

39. **OPEN ENDED** Draw and label a figure to show $\overrightarrow{PR}$ that bisects $\angle SPQ$ and $\overrightarrow{PT}$ that bisects $\angle SPR$. Use a protractor to measure each angle.

40. **REASONING** Are all right angles congruent? What information would you use to support your answer?

CHALLENGE **For Exercises 41–44, use the following information.**
Each figure below shows noncollinear rays with a common endpoint.

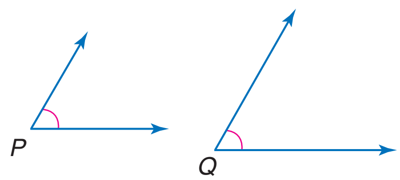


41. Count the number of angles in each figure.

42. Describe the pattern between the number of rays and the number of angles.

43. **Make a conjecture** about the number of angles that are formed by 7 noncollinear rays and by 10 noncollinear rays.

44. Write a formula for the number of angles formed by n noncollinear rays with a common endpoint.

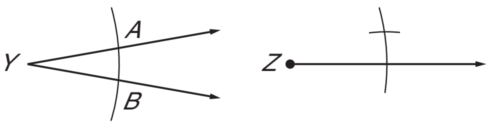


45. **REASONING** How would you compare the sizes of $\angle P$ and $\angle Q$? Explain.

46. ***Writing in Math*** Refer to page 31. Describe the size of a degree. Include how to find degree measure with a protractor.

STANDARDIZED TEST PRACTICE

47. Dominic is using a straightedge and compass to do the construction shown below.



Which *best* describes the construction Dominic is doing?

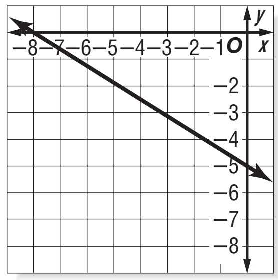
A a line through Z that bisects $\angle AYB$

B a line through Z parallel to $\overrightarrow{YA}$

C a ray through Z congruent to $\overrightarrow{YA}$

D an angle Z congruent to $\angle AYB$

48. REVIEW Which coordinate points represent the x- and y-intercepts of the graph below?



F (–5, –8), (0, 0)

G (0, –8), (–5, 0)

H (–8, 0), (0, –5)

J (0, –5), (0, –8)

Spiral Review

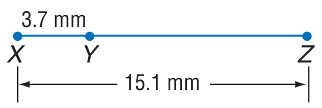
Find the distance between each pair of points. Then find the coordinates of the midpoint of the line segment between the points. (Lesson 1-3)

49. $A(2, 3)$, $B(5, 7)$ **50.** $C(-2, 0)$, $D(6, 4)$ **51.** $E(-3, -2)$, $F(5, 8)$

Find the measurement of each segment. (Lesson 1-2)

52. $\overline{WX}$

53. $\overline{YZ}$



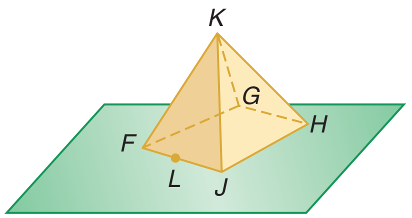
54. Find PQ if Q lies between P and R, $PQ = 6x - 5$, $QR = 2x + 7$, and $PQ = QR$. (Lesson 1-2)

Refer to the figure at the right. (Lesson 1-1)

55. How many planes are shown?

56. Name three collinear points.

57. Name a point coplanar with J, H, and F.



GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. (Pages 781–782)

58. $14x + (6x - 10) = 90$

59. $2k + 30 = 180$

60. $180 - 5y = 90 - 7y$

61. $90 - 4t = \frac{1}{4}(180 - t)$

62. $(6m + 8) + (3m + 10) = 90$

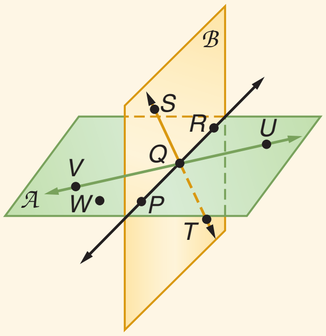
63. $(7n - 9) + (5n + 45) = 180$

CHAPTER 1

Mid-Chapter Quiz

Lessons 1-1 through 1-4

For Exercises 1–2, refer to the figure. (Lesson 1-1)



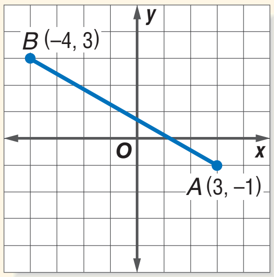
1. Name another point that is collinear with points S and Q.
2. Name a line that is coplanar with $\overleftrightarrow{VU}$ and point W.

Find the value of x and SR if R is between S and T. (Lesson 1-2)

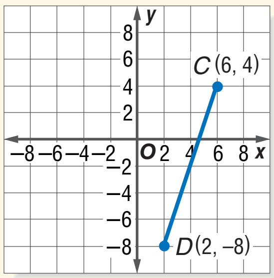
3. $SR = 3x$, $RT = 2x + 1$, $ST = 6x - 1$
4. $SR = 5x - 3$, $ST = 7x + 1$, $RT = 3x - 1$

Find the coordinates of the midpoint of each segment. Then find the distance between the endpoints. (Lesson 1-3)

5.



6.



Find the coordinates of the midpoint of a segment having the given endpoints. Then find the distance between the endpoints. (Lesson 1-3)

7. $E(10, 20)$, $F(-10, -20)$
8. $A(-1, 3)$, $B(5, -5)$
9. $C(4, 1)$, $D(-3, 7)$
10. $F(4, -9)$, $G(-2, -15)$
11. $H(-5, -2)$, $J(7, 4)$

12. **MULTIPLE CHOICE** $\overline{AB}$ has endpoints $A(n, 4n)$ and $B(3n, 6n)$. Which of the following is true?

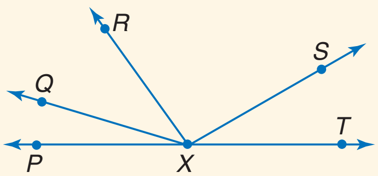
    A $AB = 4n$

    B The midpoint of $\overline{AB}$ is $(2n, 2n)$.

    C $AB = n\sqrt{8}$

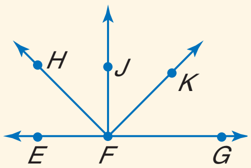
    D The midpoint of $\overline{AB}$ is $(4n, 10n)$.

In the figure, $\overrightarrow{XP}$ and $\overrightarrow{XT}$ are opposite rays. (Lesson 1-4)



13. If $m\angle SXT = 3a - 4$, $m\angle RXS = 2a + 5$, and $m\angle RXT = 111$, find $m\angle RXS$.
14. If $m\angle QXR = a + 10$, $m\angle QXS = 4a - 1$, and $m\angle RXS = 91$, find $m\angle QXS$.

Measure each angle and classify as *right*, *acute*, or *obtuse*. (Lesson 1-4)



15. $\angle KFG$
16. $\angle HFG$
17. $\angle HFK$
18. $\angle JFE$
19. $\angle HFJ$
20. $\angle EFK$

1-5 Angle Relationships

GET READY for the Lesson

When two lines intersect, four angles are formed. In some cities, more than two streets might intersect to form even more angles. All of these angles are related in special ways.

Main Ideas

- Identify and use special pairs of angles.
- Identify perpendicular lines.

New Vocabulary

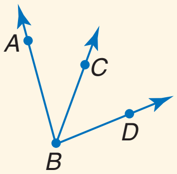
adjacent angles
vertical angles
linear pair
complementary angles
supplementary angles
perpendicular

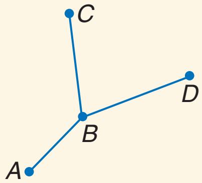
Pairs of Angles Certain pairs of angles have special names.

KEY CONCEPT — Angle Pairs

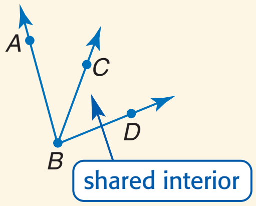
Words **Adjacent angles** are two angles that lie in the same plane, have a common vertex and a common side, but no common interior points.

Examples
$\angle ABC$ and $\angle CBD$

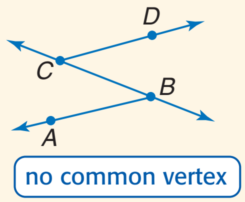




Nonexamples
$\angle ABC$ and $\angle ABD$

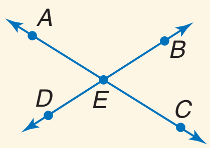


$\angle ABC$ and $\angle BCD$

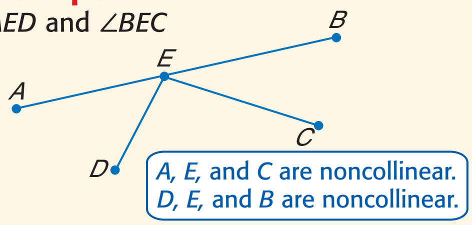


Words **Vertical angles** are two nonadjacent angles formed by two intersecting lines.

Examples
$\angle AEB$ and $\angle CED$
$\angle AED$ and $\angle BEC$

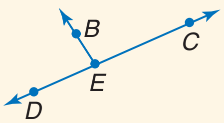


Nonexample
$\angle AED$ and $\angle BEC$

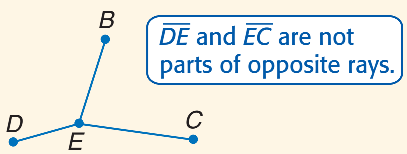


Words A **linear pair** is a pair of adjacent angles with noncommon sides that are opposite rays.

Example
$\angle DEB$ and $\angle BEC$

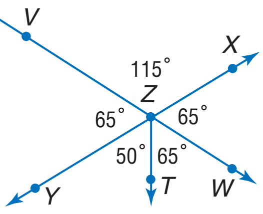


Nonexample
$\angle DEB$ and $\angle BEC$



EXAMPLE 1 Identify Angle Pairs

Name an angle pair that satisfies each condition.



a. two obtuse vertical angles

$\angle VZX$ and $\angle YZW$ are vertical angles.

They each have measures greater than 90, so they are obtuse.

b. two acute adjacent angles

There are four acute angles shown.

Adjacent acute angles are $\angle VZY$ and $\angle YZT$, $\angle YZT$ and $\angle TZW$, and $\angle TZW$ and $\angle WZX$.

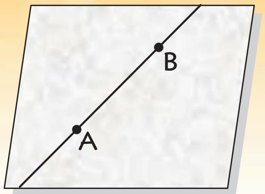
CHECK Your Progress

1. Name an angle pair that is a linear pair.

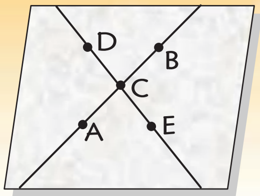
The measures of angles formed by intersecting lines have a special relationship.

GEOMETRY LAB

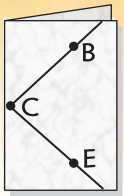
Angle Relationships



Step 1
Fold a piece of patty paper so that it makes a crease across the paper. Open the paper, trace the crease with a pencil, and name two points on the crease *A* and *B*.



Step 2
Fold the paper again so that the new crease intersects $\overleftrightarrow{AB}$ between the two labeled points. Open the paper, trace this crease, and label the intersection *C*. Label two other points, *D* and *E*, on the second crease so that *C* is between *D* and *E*.



Step 3
Fold the paper again through point *C* so that $\overrightarrow{CB}$ aligns with $\overrightarrow{CD}$.

ANALYZE THE MODEL

1. What did you notice about $\angle BCE$ and $\angle DCA$ when you made the last fold?

2. Fold again through *C* so that $\overrightarrow{CB}$ aligns with $\overrightarrow{CE}$. What do you notice?

3. Use a protractor to measure each angle. Label the measures on your model.

4. Name pairs of vertical angles and their measures.

5. Name linear pairs of angles and their measures.

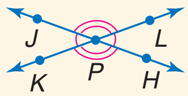
6. Compare your results with those of your classmates. Write a "rule" about the measures of vertical angles and another about the measures of linear pairs.

The Geometry Lab suggests that all vertical angles are congruent. It also supports the concept that the sum of the measures of a linear pair is 180.

KEY CONCEPT *Vertical Angles*

Words Vertical angles are congruent.

Examples $\angle JPK \cong \angle HPL$
$\angle JPL \cong \angle HPK$

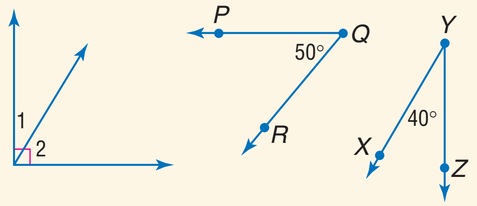


There are other angle relationships that you may remember from previous math courses. These are complementary angles and supplementary angles.

KEY CONCEPT *Angle Relationships*

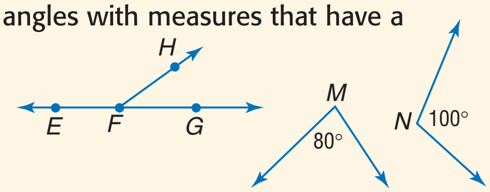
Words **Complementary angles** are two angles with measures that have a sum of 90.

Examples
$\angle 1$ and $\angle 2$ are complementary.
$\angle PQR$ and $\angle XYZ$ are complementary.



Words **Supplementary angles** are two angles with measures that have a sum of 180.

Examples
$\angle EFH$ and $\angle HFG$ are supplementary.
$\angle M$ and $\angle N$ are supplementary.



Study Tip

Complementary and Supplementary Angles

While the other angle pairs in this lesson share at least one point, complementary and supplementary angles need not share any points.

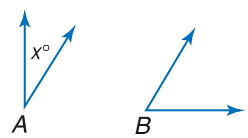
Remember that angle measures are real numbers. So, the operations for real numbers and algebra can be used with angle measures.

EXAMPLE Angle Measure

2 **ALGEBRA** **Find the measures of two complementary angles if the difference in the measures of the two angles is 12.**

Explore The problem relates the measures of two complementary angles. You know that the sum of the measures of complementary angles is 90.

Plan Draw two figures to represent the angles.
Let the measure of one angle be x.
If $m\angle A = x$, then, because $\angle A$ and $\angle B$ are complementary, $m\angle B + x = 90$ or $m\angle B = 90 - x$.



The problem states that the difference of the two angle measures is 12, or $m\angle B - m\angle A = 12$.

Online **Problem Solving Handbook at** geometryonline.com

Solve

$m\angle B - m\angle A = 12$ Given

$(90 - x) - x = 12$ $m\angle A = x$, $m\angle B = 90 - x$

$90 - 2x = 12$ Simplify.

$-2x = -78$ Subtract 90 from each side.

$x = 39$ Divide each side by -2.

Use the value of x to find each angle measure.

$m\angle A = x$ $m\angle B = 90 - x$

$m\angle A = 39$ $m\angle B = 90 - 39$ or 51

Check Add the angle measures to verify that the angles are complementary.

$m\angle A + m\angle B = 90$

$39 + 51 = 90$

$90 = 90$

CHECK Your Progress

2. Find the measures of two supplementary angles if the difference in the measures of the two angles is 32.

Online Personal Tutor at geometryonline.com

Perpendicular Lines

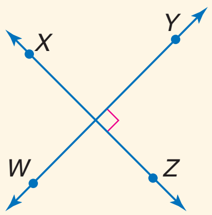
Lines, segments, or rays that form right angles are **perpendicular**.

KEY CONCEPT *Perpendicular Lines*

Words

- Perpendicular lines intersect to form four right angles.
- Perpendicular lines intersect to form congruent adjacent angles.
- Segments and rays can be perpendicular to lines or to other line segments and rays.
- The right angle symbol in the figure indicates that the lines are perpendicular.

Example



$\overleftrightarrow{XZ} \perp \overleftrightarrow{WY}$

Symbol $\perp$ is read *is perpendicular to*.

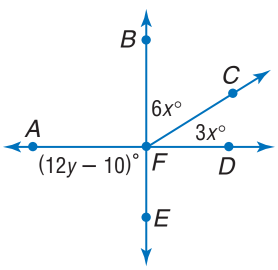
Study Tip

Interpreting Figures

Never assume that two lines are perpendicular because they appear to be so in the figure. The only sure way to know if they are perpendicular is if the right angle symbol is present or if the problem states angle measures that allow you to make that conclusion.

EXAMPLE Perpendicular Lines

3 ALGEBRA Find x and y so that $\overrightarrow{BE}$ and $\overrightarrow{AD}$ are perpendicular.



If $\overrightarrow{BE} \perp \overrightarrow{AD}$, then $m\angle BFD = 90$ and $m\angle AFE = 90$. To find x, use $\angle BFC$ and $\angle CFD$.

$m\angle BFD = m\angle BFC + m\angle CFD$ Sum of parts = whole

$90 = 6x + 3x$ Substitution

$90 = 9x$ Add.

$10 = x$ Divide each side by 9.

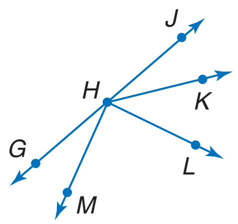
To find y, use $\angle AFE$.

$m\angle AFE = 12y - 10$	Given
$90 = 12y - 10$	Substitution
$100 = 12y$	Add 10 to each side.
$\frac{25}{3} = y$	Divide each side by 12, and simplify.

CHECK Your Progress

3. Suppose $m\angle D = 3x - 12$. Find x so that $\angle D$ is a right angle.

While two lines may appear to be perpendicular in a figure, you cannot assume this is true unless other information is given. In geometry, figures are used to depict a situation. They are not drawn to reflect total accuracy of the situation. There are certain relationships you can assume to be true, but others that you cannot. Study the figure at the right and then compare the lists below.



Study Tip

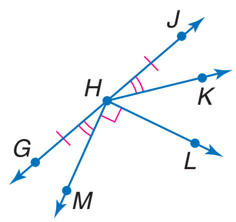
Naming Figures

The list of statements that can be assumed is not a complete list. There are more special pairs of angles than those listed. Also remember that all figures except points usually have more than one way to name them.

Can Be Assumed	Cannot Be Assumed
All points shown are coplanar.	Perpendicular segments: $\overline{HL} \perp \overline{GJ}$
G, H, and J are collinear.	Congruent angles: $\angle JHK \cong \angle GHM$
$\overrightarrow{HM}$, $\overrightarrow{HL}$, $\overrightarrow{HK}$, and $\overleftrightarrow{GJ}$ intersect at H.	$\angle JHK \cong \angle KHL$
H is between G and J.	$\angle KHL \cong \angle GHM$
L is in the interior of $\angle MHK$.	Congruent segments: $\overline{GH} \cong \overline{HJ}$
$\angle GHM$ and $\angle MHL$ are adjacent angles.	$\overline{HJ} \cong \overline{HK}$
$\angle GHL$ and $\angle LHJ$ are a linear pair.	$\overline{HK} \cong \overline{HL}$
$\angle JHK$ and $\angle KHG$ are supplementary.	$\overline{HL} \cong \overline{HG}$

EXAMPLE 4 Interpret Figures

Determine whether each statement can be assumed from the figure at the right.



a. $\angle GHM$ and $\angle MHK$ are adjacent angles.

Yes; they share a common side and vertex and have no interior points in common.

b. $\angle KHJ$ and $\angle GHM$ are complementary.

No; they are congruent, but we do not know anything about their exact measures.

c. $\angle GHK$ and $\angle JHK$ are a linear pair.

Yes; they are adjacent angles whose noncommon sides are opposite rays.

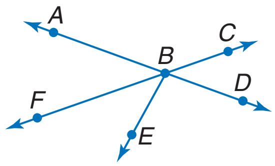
CHECK Your Progress

4. Determine whether the statement *$\angle GHL$ and $\angle LHJ$ are supplementary* can be assumed from the figure.

CHECK Your Understanding

Example 1 (p. 41)

For Exercises 1 and 2, use the figure at the right and a protractor.



1. Name two acute vertical angles.

2. Name two obtuse adjacent angles.

Example 2 (pp. 42–43)

3. **SKIING** Alisa Camplin won a gold medal in the 2002 Winter Olympics with a triple-twisting, double backflip jump in the women's freestyle skiing event. While she is in the air, her skis give the appearance of intersecting lines. If $\angle 4$ measures 60°, find the measures of the other angles.

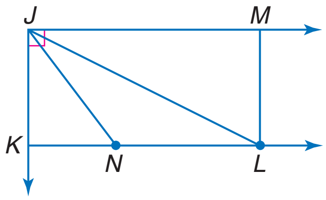
Example 3 (pp. 43–44)

4. The measures of two complementary angles are $16z - 9$ and $4z + 3$. Find the measures of the angles.

5. Find $m\angle T$ if $m\angle T$ is 20 more than four times the measure of its supplement.

Example 4 (p. 44)

Determine whether each statement can be assumed from the figure. Explain.



6. $\angle MLJ$ and $\angle JLN$ are complementary.

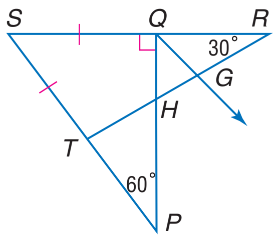
7. $\angle KJN$ and $\angle NJL$ are adjacent, but neither complementary nor supplementary.

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–13	1
14–19	2
20–22	3
23–27	4

For Exercises 8–13, use the figure at the right and a protractor.



8. Name two acute vertical angles.

9. Name two obtuse vertical angles.

10. Name a pair of complementary adjacent angles.

11. Name a pair of complementary nonadjacent angles.

12. Name a linear pair whose vertex is G.

13. Name an angle supplementary to $\angle HTS$.

14. Rays PQ and QR are perpendicular. Point S lies in the interior of $\angle PQR$. If $m\angle PQS = 4 + 7a$ and $m\angle SQR = 9 + 4a$, find $m\angle PQS$ and $m\angle SQR$.

15. The measure of the supplement of an angle is 60 less than three times the measure of the complement of the angle. Find the measure of the angle.

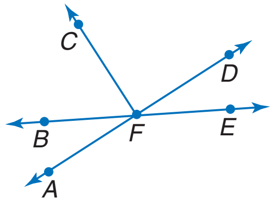
16. Lines p and q intersect to form adjacent angles 1 and 2. If $m\angle 1 = 3x + 18$ and $m\angle 2 = -8y - 70$, find the values of x and y so that p is perpendicular to q.

17. The measure of an angle's supplement is 44 less than the measure of the angle. Find the measure of the angle and its supplement.

18. Two angles are supplementary. One angle measures 12° more than the other. Find the measures of the angles.

19. The measure of $\angle 1$ is five less than four times the measure of $\angle 2$. If $\angle 1$ and $\angle 2$ form a linear pair, what are their measures?

ALGEBRA **For Exercises 20–22, use the figure at the right.**

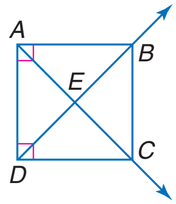


20. If $m\angle CFD = 12a + 45$, find a so that $\overrightarrow{FC} \perp \overrightarrow{FD}$.

21. If $m\angle AFB = 8x - 6$ and $m\angle BFC = 14x + 8$, find the value of x so that $\angle AFC$ is a right angle.

22. If $m\angle BFA = 3r + 12$ and $m\angle DFE = -8r + 210$, find $m\angle AFE$.

Determine whether each statement can be assumed from the figure. Explain.



23. $\angle DAB$ is a right angle.

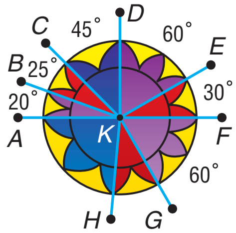
24. $\overline{AB} \perp \overline{BC}$

25. $\angle AEB \cong \angle DEC$

26. $\angle DAE \cong \angle ADE$

27. $\angle ADB$ and $\angle BDC$ are complementary.

28. STAINED GLASS In the stained glass pattern at the right, determine which segments are perpendicular.



Real-World Link

As light from the Sun travels to Earth, it is reflected or refracted by many different surfaces. Light that strikes a smooth, flat surface is very bright because the light is being reflected at the same angle.

Determine whether each statement is *sometimes*, *always*, or *never* true.

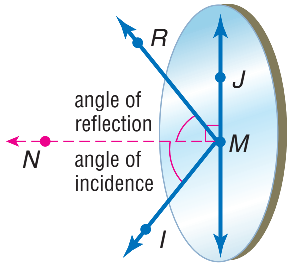
29. If two angles are supplementary and one is acute, the other is obtuse.

30. If two angles are complementary, they are both acute angles.

31. If $\angle A$ is supplementary to $\angle B$, and $\angle B$ is supplementary to $\angle C$, then $\angle A$ is supplementary to $\angle C$.

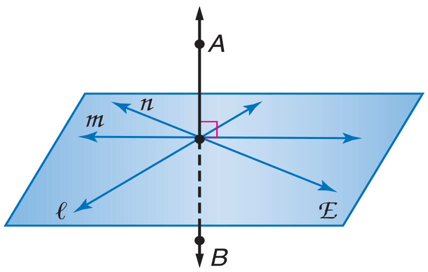
32. If $\overline{PN} \perp \overline{PQ}$, then $\angle NPQ$ is acute.

33. PHYSICS As a ray of light meets a mirror, the light is reflected. The angle that the light strikes the mirror is the *angle of incidence*. The angle that the light is reflected is the *angle of reflection*. The angle of incidence and the angle of reflection are congruent. In the diagram at the right, if $m\angle RMI = 106$, find the angle of reflection and $m\angle RMJ$.



34. RESEARCH Look up the words *complementary* and *complimentary* in a dictionary. Discuss the differences in the terms and determine which word has a mathematical meaning.

35. The concept of perpendicularity can be extended to include planes. If a line, line segment, or ray is perpendicular to a plane, it is perpendicular to every line, line segment, or ray in that plane at the point of intersection. In the figure at the right, $\overleftrightarrow{AB} \perp \mathcal{E}$. Name all pairs of perpendicular lines.



EXTRA PRACTICE

See pages 801, 828.

Math Online

Self-Check Quiz at geometryonline.com

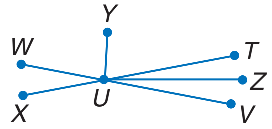
H.O.T. Problems

36. OPEN ENDED Draw two angles that are supplementary, but not adjacent.

37. REASONING Explain the statement *If two adjacent angles form a linear pair, they must be supplementary.*

38. **CHALLENGE** A *counterexample* is used to show that a statement is not necessarily true. Draw a counterexample for the statement *Supplementary angles form linear pairs*.

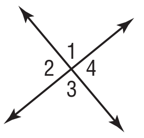
39. **CHALLENGE** In the figure, $\angle WUT$ and $\angle XUV$ are vertical angles, $\overline{YU}$ is the bisector of $\angle WUT$, and $\overline{UZ}$ is the bisector of $\angle TUV$. Write a convincing argument that $\overline{YU} \perp \overline{UZ}$.



40. ***Writing in Math*** Refer to page 40. What kinds of angles are formed when streets intersect? Include the types of angles that might be formed by two intersecting lines, and a sketch of intersecting streets with angle measures and angle pairs identified.

STANDARDIZED TEST PRACTICE

41. In the diagram below, $\angle 1$ is an acute angle.



Which conclusion is *not* true?

A $m\angle 2 > m\angle 3$

B $m\angle 2 = m\angle 4$

C $m\angle 1 < m\angle 4$

D $m\angle 3 > m\angle 4$

42. **REVIEW** Solve: $5(x - 4) = 3x + 18$

Step 1: $5x - 4 = 3x + 18$

Step 2: $2x - 4 = 18$

Step 3: $2x = 22$

Step 4: $x = 11$

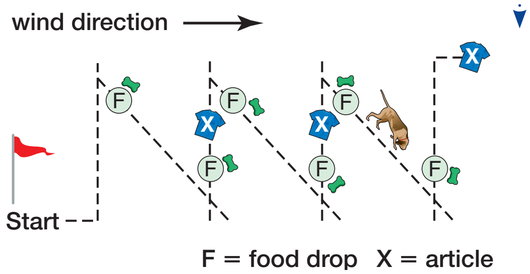
Which is the first *incorrect* step in the solution shown above?

F Step 1 **H** Step 3

G Step 2 **J** Step 4

Spiral Review

43. **DOG TRACKING** A dog is tracking when it is following the scent trail left by a human being or other animal that has passed along a certain route. One of the training exercises for these dogs is a tracking trail. The one shown is called an acute tracking trail. Explain why it might be called this. (Lesson 1-4)



Find the distance between each pair of points. (Lesson 1-3)

44. $A(3, 5), B(0, 1)$

45. $C(5, 1), D(5, 9)$

46. $E(-2, -10), F(-4, 10)$

47. $G(7, 2), H(-6, 0)$

48. $J(-8, 9), K(4, 7)$

49. $L(1, 3), M(3, -1)$

Find the value of the variable and QR, if Q is between P and R. (Lesson 1-2)

50. $PQ = 1 - x, QR = 4x + 17, PR = -3x$

51. $PR = 7n + 8, PQ = 4n - 3, QR = 6n + 2$

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $\ell = 3$, $w = 8$, and $s = 2$.** (Page 780)

52. $2\ell + 2w$

53. ℓw

54. $4s$

55. $\ell w + ws$

56. $s(\ell + w)$

EXTEND 1-5

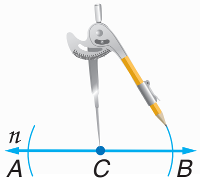
Geometry Lab
Constructing Perpendiculars

You can use a compass and a straightedge to construct a line perpendicular to a given line through a point on the line, or through a point not on the line.

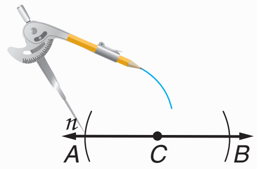
ACTIVITY 1 Perpendicular Through a Point on the Line

Construct a line perpendicular to line n and passing through point C on n.

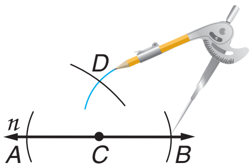
1. Place the compass at point C. Using the same compass setting, draw arcs to the right and left of C, intersecting line n. Label the points of intersection A and B.



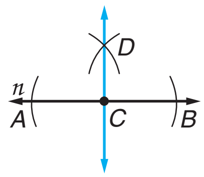
2. Open the compass to a setting greater than AC. Put the compass at point A and draw an arc above line n.



3. Using the same compass setting as in Step 2, place the compass at point B and draw an arc intersecting the arc drawn in Step 2. Label the point of intersection D.



4. Use a straightedge to draw $\overleftrightarrow{CD}$.

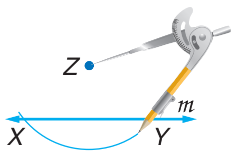


Concepts in Motion
Animation geometryonline.com

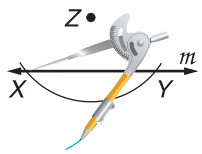
ACTIVITY 2 Perpendicular Through a Point not on the Line

Construct a line perpendicular to line m and passing through point Z *not* on m.

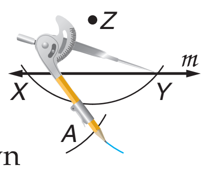
1. Place the compass at point Z. Draw an arc that intersects line m in two different places. Label the points of intersection X and Y.



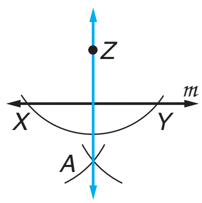
2. Open the compass to a setting greater than $\frac{1}{2}XY$. Put the compass at point X and draw an arc below line m.



3. Using the same compass setting, place the compass at point Y and draw an arc intersecting the arc drawn in Step 2. Label the point of intersection A.



4. Use a straightedge to draw $\overleftrightarrow{ZA}$.



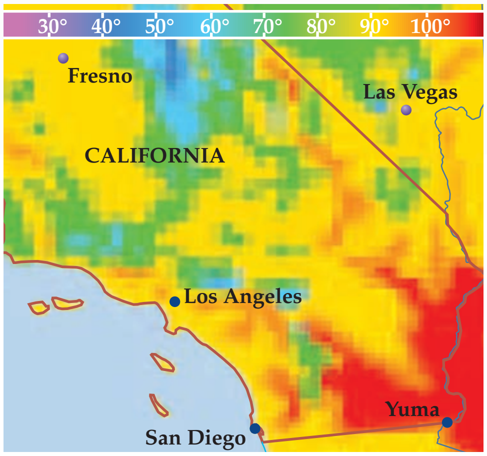
Model and Analyze the Results

1. Draw a line and construct a line perpendicular to it through a point on the line. Repeat with a point not on the line.
2. How is the second construction similar to the first one?

1-6 Two-Dimensional Figures

GET READY for the Lesson

To predict weather conditions, meteorologists divide the surface into small geographical cells. Meteorologists use computer programs and mathematical models to track climate and other weather conditions for each cell. The weather in each cell is affected by the weather in surrounding cells. Maps like the one on the right show the temperature for each cell.



Main Ideas

- Identify and name polygons.
- Find perimeter or circumference and area of two-dimensional figures.

New Vocabulary

polygon
concave
convex
n-gon
regular polygon
perimeter
circumference
area

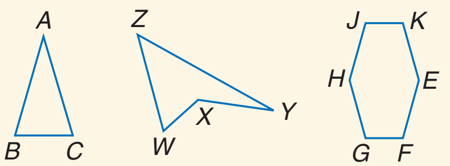
Polygons Each closed figure shown in the map is a **polygon**. A polygon is a closed figure whose sides are all segments. The sides of each angle in a polygon are called *sides* of the polygon, and the vertex of each angle is a *vertex* of the polygon.

KEY CONCEPT — *Polygon*

Words A polygon is a closed figure formed by a finite number of coplanar segments such that
(1) the sides that have a common endpoint are noncollinear, and
(2) each side intersects exactly two other sides, but only at their endpoints.

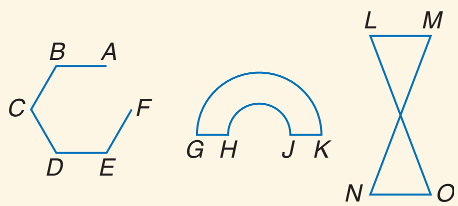
Symbol A polygon is named by the letters of its vertices, written in order of consecutive vertices.

Examples

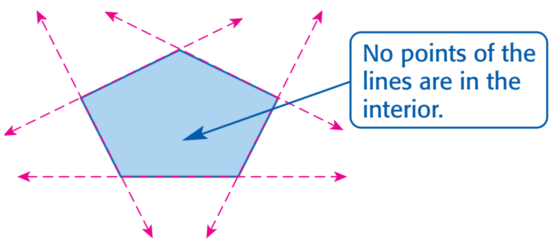


polygons *ABC*, *WXYZ*, *EFGHJK*

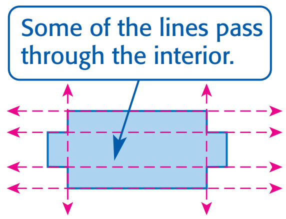
Nonexamples



Polygons can be **concave** or **convex**. Suppose the line containing each side is drawn. If any of the lines contain any point in the interior of the polygon, then it is concave. Otherwise it is convex.



convex polygon



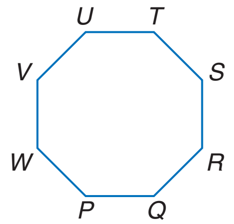
concave polygon

Reading Math

Root Word The term *polygon* is derived from a Greek word meaning *many angles*. Since *hexa-* means 6, *hexagon* means 6 angles. Every polygon has the same number of angles as it does sides.

You are already familiar with many polygon names, such as triangle, square, and rectangle. In general, polygons can be classified by the number of sides they have. A polygon with n sides is an ***n*-gon**. The table lists some common names for various categories of polygon.

A convex polygon in which all the sides are congruent and all the angles are congruent is called a **regular polygon**. Octagon *PQRSTUVW* below is a regular octagon.



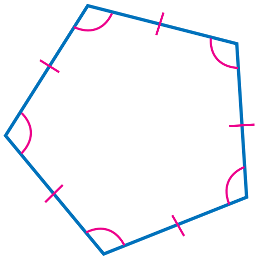
Number of Sides	Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
n	n-gon

Polygons and circles are examples of *simple closed curves*.

EXAMPLE 1 Identify Polygons

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

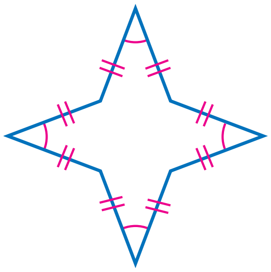
a.



There are 5 sides, so this is a pentagon. No line containing any of the sides will pass through the interior of the pentagon, so it is convex.

The sides are congruent, and the angles are congruent. It is regular.

b.

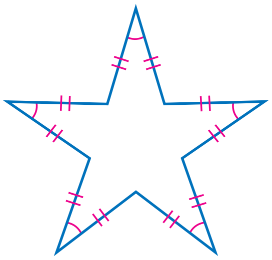


There are 8 sides, so this is an octagon. A line containing any of the sides will pass through the interior of the octagon, so it is concave.

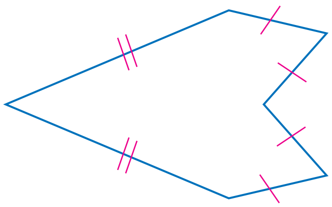
The sides are congruent. However, since it is concave, it cannot be regular.

CHECK Your Progress

1A.



1B.



Perimeter, Circumference, and Area Review the formulas for circles and three common polygons given below. Some shapes have special formulas for perimeter, but all are derived from the basic definition of perimeter. You will derive the area formulas in Chapter 11.

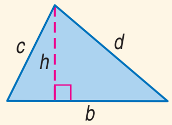
Reading Math

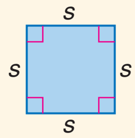
Math Symbols The symbol π is read *pi*. It is an irrational number. In this book, we will use a calculator to evaluate expressions involving π. If no calculator is available, 3.14 is a good estimate for π.

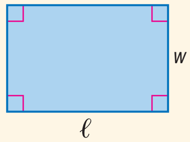
KEY CONCEPT — *Perimeter, Circumference, and Area*

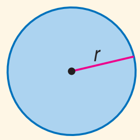
Words The **perimeter** P of a polygon is the sum of the length of the sides of the polygon. The **circumference** C of a circle is the distance around the circle. The **area** A is the number of square units needed to cover a surface.

Examples	**triangle**	**square**	**rectangle**	**circle**
Perimeter/ Circumference	$P = b + c + d$	$P = s + s + s + s$ $= 4s$	$P = \ell + w + \ell + w$ $= 2\ell + 2w$	$C = 2\pi r$
Area	$A = \frac{1}{2}bh$	$A = s^2$	$A = \ell w$	$A = \pi r^2$





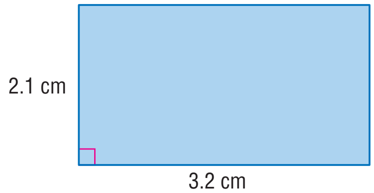




EXAMPLE Find Perimeter and Area

2 **Find the perimeter or circumference and area of each figure to the nearest tenth.**

a.



$P = 2\ell + 2w$ — Perimeter of a rectangle

$= 2(3.2) + 2(2.1)$ — $\ell = 3.2, w = 2.1$

$= 10.6$ — Simplify.

The perimeter is 10.6 centimeters.

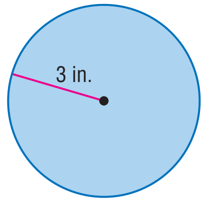
$A = \ell w$ — Area of a rectangle

$= (3.2)(2.1)$ — $\ell = 3.2, w = 2.1$

$= 6.72$ — Simplify.

The area is about 6.7 square centimeters.

b.



$C = 2\pi r$ — Circumference

$= 2\pi(3)$ — $r = 3$

≈ 18.9 — Use a calculator.

The circumference is about 18.85 inches.

$A = \pi r^2$ — Area of a circle

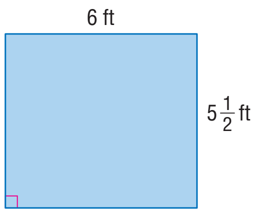
$= \pi(3)^2$ — $r = 3$

≈ 28.3 — Use a calculator.

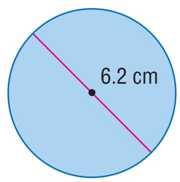
The area is about 28.3 square inches.

CHECK Your Progress

2A.



2B.



STANDARDIZED TEST EXAMPLE Largest Area

Winona has 26 centimeters of cording to frame a photograph in her scrapbook. Which of these shapes would use *most* or all of the cording and enclose the *largest* area?

A right triangle with each leg about 7 centimeters long

B circle with a radius of about 4 centimeters

C rectangle with a length of 8 centimeters and a width of 4.5 centimeters

D square with a side length of 6 centimeters

Test-Taking Tip

Mental Math When you are asked to compare measures for varying figures, it can be helpful to use mental math. By estimating the perimeter or area of each figure, you can check your calculations.

Read the Test Item

You are asked to compare the area and perimeter of four different shapes.

Solve the Test Item

Find the perimeter and area of each shape.

Right Triangle

Use the Pythagorean Theorem to find the length of the hypotenuse.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 7^2 + 7^2$ or 98	$a = 7, b = 7$
$c^2 = 98$	Simplify.
$c = \sqrt{98}$	Take the square root of each side.
$c \approx 9.9$	Use a calculator.
$P = a + b + c$	Perimeter of a triangle
$\approx 7 + 7 + 9.9$ or 23.9 cm	Substitution
$A = \frac{1}{2}bh$	Area of a triangle
$= \frac{1}{2}(7)(7)$ or 24.5 cm^2	Substitution

Circle	Rectangle	Square
$C = 2\pi r$	$P = 2\ell + 2w$	$P = 4s$
$\approx 2\pi(4)$	$= 2(8) + 2(4.5)$	$= 4(6)$
≈ 25.1 cm	$= 25$ cm	$= 24$ cm
$A = \pi r^2$	$A = \ell w$	$A = s^2$
$\approx \pi(4)^2$	$= (8)(4.5)$	$= 6^2$
≈ 50.3 cm^2	$= 36$ cm^2	$= 36$ cm^2

The shape that uses all of the cording and encloses the largest area is the circle. The answer is B.

CHECK Your Progress

3. Danny wants to fence in a play area for his dog. He has 32 feet of fencing. Which shape uses *most* or all of the fencing and encloses the *largest* area?

F circle with radius of about 5 feet

G rectangle with length 6 feet and width 10 feet

H right triangle with legs of length 10 feet

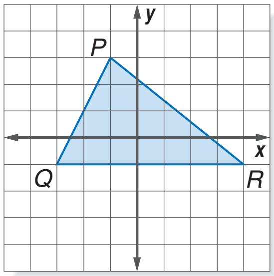
J square with a side length of 8 feet

Online **Personal Tutor at** geometryonline.com

You can use the Distance Formula to find the perimeter of a polygon graphed on a coordinate plane.

EXAMPLE Perimeter and Area on the Coordinate Plane

4 **COORDINATE GEOMETRY** **Refer to $\triangle PQR$ with vertices $P(-1, 3)$, $Q(-3, -1)$, and $R(4, -1)$.**



a. Find the perimeter of $\triangle PQR$.

Since $\overline{QR}$ is a horizontal line, we can count the squares on the grid. The length of $\overline{QR}$ is 7 units. Use the Distance Formula to find PQ and PR.

> **Study Tip**
>
> **Look Back**
>
> To review the **Distance Formula**, see Lesson 1-3.

$$PQ = \sqrt{(-1-(-3))^2 + (3-(-1))^2} \quad \text{Substitute.}$$
$$= \sqrt{2^2 + 4^2} \quad \text{Subtract.}$$
$$= \sqrt{20} \approx 4.5 \quad \text{Simplify.}$$

$$PR = \sqrt{(-1-4)^2 + (3-(-1))^2} \quad \text{Substitute.}$$
$$= \sqrt{(-5)^2 + 4^2} \quad \text{Subtract.}$$
$$= \sqrt{41} \approx 6.4 \quad \text{Simplify.}$$

The perimeter of $\triangle PQR$ is $7 + \sqrt{20} + \sqrt{41}$ or about 17.9 units.

b. Find the area of $\triangle PQR$.

The height is the perpendicular distance from P to $\overline{QR}$. Counting squares on the graph, the height is 4 units. The length of $\overline{QR}$ is 7 units.

$$A = \frac{1}{2}bh \quad \text{Area of a triangle}$$
$$= \frac{1}{2}(7)(4) \quad \text{Substitution}$$
$$= 14 \quad \text{Simplify.}$$

The area of $\triangle PQR$ is 14 square units.

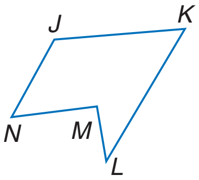
CHECK Your Progress

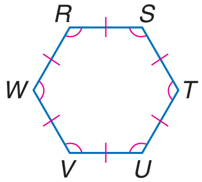
4. Find the perimeter and area of $\triangle ABC$ with vertices $A(-1, 4)$, $B(-1, -1)$, and $C(6, -1)$.

CHECK Your Understanding

Example 1 (p. 50)

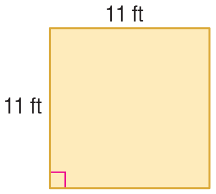
Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

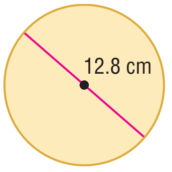
1\. 

2\. 

Example 2 (p. 51)

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.

3\. 

4\. 

Example 3 (p. 52)

5\. **STANDARDIZED TEST PRACTICE** Tara is building a playpen for the children next door. She has 15 square feet of fabric. What shape will use *most* or all of the fabric?

A a square with a side length of 4 feet
B a rectangle with a length of 4 feet and a width of 3.5 feet
C a circle with a radius of about 2.5 feet
D a right triangle with legs of about 5 feet

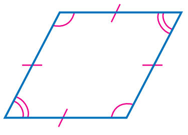
Example 4 (p. 53)

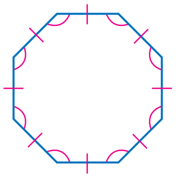
6\. **COORDINATE GEOMETRY** Find the perimeter and area of $\triangle ABC$ with vertices $A(-1, 2)$, $B(3, 6)$, and $C(3, -2)$.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
7–11	1
12–17	2
18, 19	3
20–23	4

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

7\. 

8\. 

TRAFFIC SIGNS **Identify the shape of each traffic sign.**

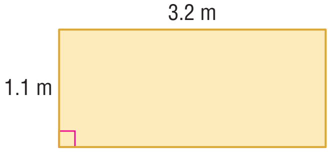
9\. school zone

10\. caution or warning

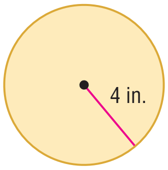
11\. railroad

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.

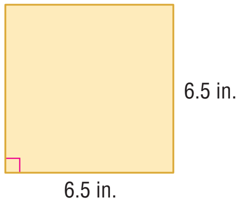
12.



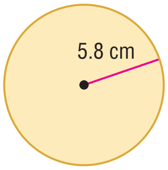
13.



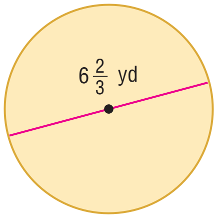
14.



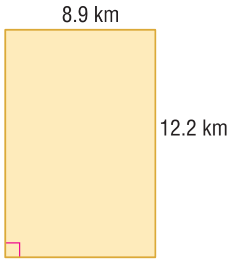
15.



16.



17.



Real-World Link

Some gardens are landscaped with shrubbery that has been sculpted or *topiary*. Columbus, Ohio, is home to a topiary garden that recreates Georges Seurat's painting *A Sunday Afternoon on the Ile De La Grande Jatte*. The garden includes 54 people sculpted in topiary.

Source: topiarygarden.org

18. CRAFTS Candace has a square picture that is 4 inches on each side. The picture is framed with a length of ribbon. She wants to use the same piece of ribbon to frame a circular picture. What is the maximum radius of the circular frame?

19. LANDSCAPING Mr. Hernandez has a circular garden with a diameter of 10 feet surrounded by edging. Using the same length of edging, he is going to create a square garden. What is the maximum side length of the square?

COORDINATE GEOMETRY Find the perimeter and area of each figure with the given vertices.

20. $D(-2, -2)$, $E(-2, 3)$, and $F(2, -1)$

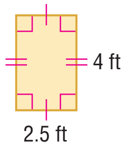
21. $J(-3, -3)$, $K(3, 2)$, and $L(3, -3)$

22. $P(-1, 1)$, $Q(3, 4)$, $R(6, 0)$, and $S(2, -3)$

23. $T(-2, 3)$, $U(1, 6)$, $V(5, 2)$, and $W(2, -1)$

24. HISTORIC LANDMARKS The Pentagon building in Arlington, Virginia, is so named because of its five congruent sides. Find the perimeter of the outside of the Pentagon if one side is 921 feet long.

CHANGING DIMENSIONS For Exercises 25–27, use the rectangle at the right.

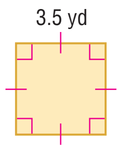


25. Find the perimeter of the rectangle.

26. Find the area of the rectangle.

27. Suppose the length and width of the rectangle are doubled.What effect does this have on the perimeter? Describe the effect on the area.

CHANGING DIMENSIONS For Exercises 28–30, use the square at the right.



28. Find the perimeter of the square.

29. Find the area of the square.

30. Suppose the length of a side of the square is divided by 2. What effect does this have on the perimeter? Describe the effect on the area.

ALGEBRA Find the perimeter or circumference for each figure with the given information.

31. The area of a square is 36 square inches.

32. The length of a rectangle is half the width. The area is 25 square meters.

33. The area of a circle is 25π square units.

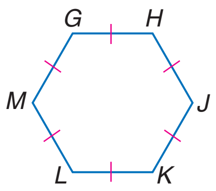
34. The area of a circle is 32π square units.

35. The length of a rectangle is three times the width. The area is 27 square inches.

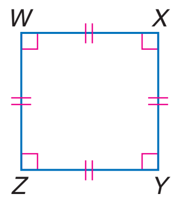
36. The length of a rectangle is twice the width. The area is 48 square inches.

ALGEBRA Find the length of each side of the polygon for the given perimeter.

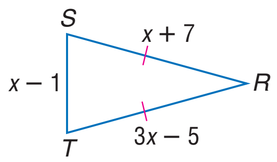
37. $P = 90$ centimeters



38. $P = 14$ miles

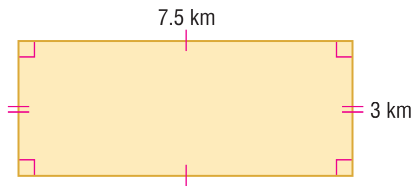


39. $P = 31$ units



40. CHANGING DIMENSIONS The perimeter of an n-gon is 12.5 meters. Find the perimeter of the n-gon if the length of each of its n sides is multiplied by 10.

CHANGING DIMENSIONS For Exercises 41–44, use the rectangle at the right.



41. Find the perimeter of the rectangle.

42. Find the area of the rectangle.

43. Suppose the length and width of the rectangle are doubled. What effect does this have on the perimeter?

44. Describe the effect of doubling the length and width on the area.

ENLARGEMENT For Exercises 45–48, use the following information.
The coordinates of the vertices of a triangle are $A(1, 3)$, $B(9, 10)$, and $C(11, 18)$.

45. Find the perimeter of triangle ABC.

46. Suppose each coordinate is multiplied by 2. What is the perimeter of this triangle?

47. Find the perimeter of the triangle when the coordinates are multiplied by 3.

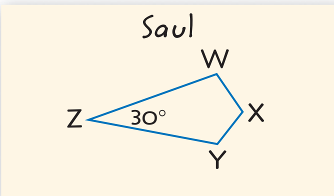
48. Make a conjecture about the perimeter of a triangle when the coordinates of its vertices are multiplied by the same positive factor.

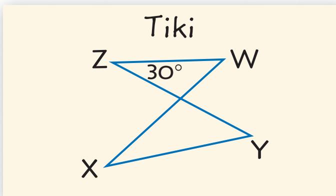
EXTRA PRACTICE
See pages 801, 828.
Math Online
Self-Check Quiz at geometryonline.com

H.O.T. Problems

49. OPEN ENDED Explain how you would find the length of a side of a regular decagon if the perimeter is 120 centimeters.

50. FIND THE ERROR Saul and Tiki were asked to draw quadrilateral $WXYZ$ with $m\angle Z = 30$. Who is correct? Explain your reasoning.





CHALLENGE **Use grid paper to draw all possible rectangles with length and width that are whole numbers and with a perimeter of 12. Record the number of grid squares contained in each rectangle.**

51. What do you notice about the rectangle with the greatest number of squares?

52. The perimeter of another rectangle is 36. What would be the dimensions of the rectangle with the greatest number of squares?

53. **Which One Doesn't Belong?** Identify the term that does not belong with the other three. Explain your reasoning.

square	circle	triangle	pentagon

54. ***Writing in Math*** Refer to page 49. Explain why dividing a state into geographical cells allows meteorologists to more accurately predict weather.

STANDARDIZED TEST PRACTICE

55. The circumferences of two circles are in the ratio of 9 to 16. What is the ratio between the areas of the two circles?

A 3 to 4

B 9 to 16

C 81 to 64

D 81 to 256

56. **REVIEW** A rectangle has an area of 1000 square meters and a perimeter of 140 meters. What are the dimensions of the rectangle?

F 100 m by 100 m

G 50 m by 20 m

H 40 m by 25 m

J 10 m by 100 m

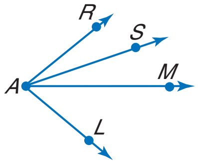
Spiral Review

Determine whether each statement is *always*, *sometimes*, or *never* true. (Lesson 1-5)

57. Two angles that form a linear pair are supplementary.

58. If two angles are supplementary, then one of the angles is obtuse.

In the figure, $\overrightarrow{AM}$ bisects $\angle LAR$, and $\overrightarrow{AS}$ bisects $\angle MAR$. (Lesson 1-4)



59. If $m\angle MAR = 2x + 13$ and $m\angle MAL = 4x - 3$, find $m\angle RAL$.

60. If $m\angle RAL = x + 32$ and $m\angle MAR = x - 31$, find $m\angle LAM$.

61. Find $m\angle LAR$ if $m\angle RAS = 25 - 2x$ and $m\angle SAM = 3x + 5$.

62. **CRAFTS** Martin makes pewter figurines. When a solid object with a volume of 1 cubic centimeter is submerged in water, the water level rises 1 milliliter. Martin pours 200 mL of water into a cup, submerges a figurine in it, and watches it rise to 343 mL. What is the maximum amount of molten pewter needed to make a figurine? Explain. (Lesson 1-2)

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $a = 12$, $b = 16$, $c = 21$, and $d = 18$.** (Page 780)

63. $\frac{1}{2}(b^2) + 2d$

64. $\frac{1}{3}(cd)$

65. $\frac{1}{2}(2a + 3c^2)$

66. $\frac{1}{3}(ac) + d^2$

EXTEND 1-6

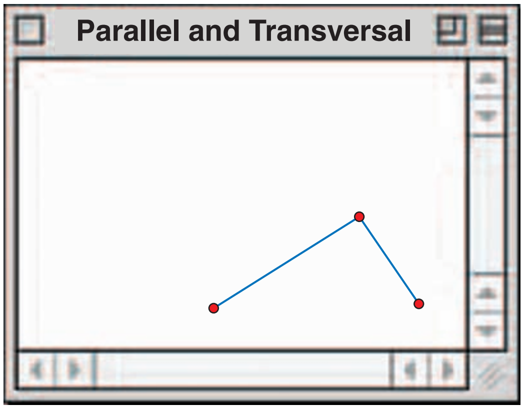
Geometry Software Lab

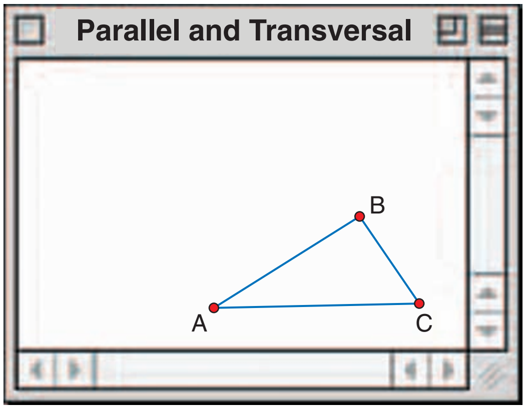
Measuring Two-Dimensional Figures

You can use The Geometer's Sketchpad® to draw and investigate polygons. It can be used to find the measures of the sides and the perimeter of a polygon. You can also find the measures of the angles in a polygon.

ACTIVITY

Step 1 Draw $\triangle ABC$.



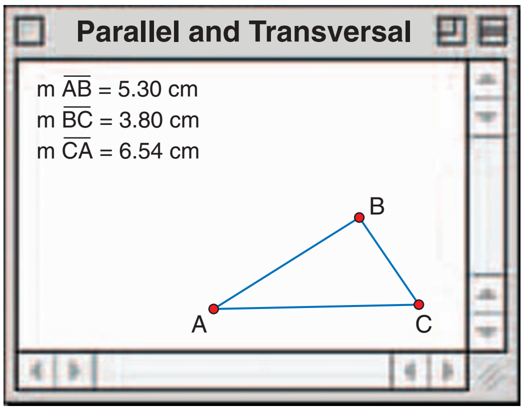


- Select the segment tool from the toolbar, and click to set the first endpoint A of side $\overline{AB}$. Then drag the cursor and click again to set the other endpoint B.
- Click on point B to set the endpoint of $\overline{BC}$. Drag the cursor and click to set point C.
- Click on point C to set the endpoint of $\overline{CA}$. Then move the cursor to highlight point A. Click on A to draw $\overline{CA}$.
- Use the pointer tool to click on points A, B, and C. Under the **Display** menu, select **Show Labels** to label the vertices of your triangle.

Step 2 Find AB, BC, and CA.

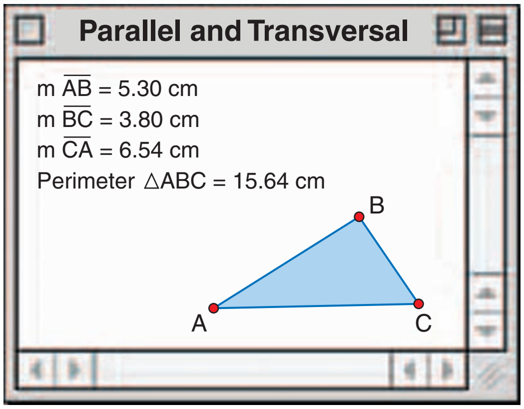
- Use the pointer tool to select $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
- Select the **Length** command under the **Measure** menu to display the lengths of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

$AB = 5.30$
$BC = 3.80$
$CA = 6.54$



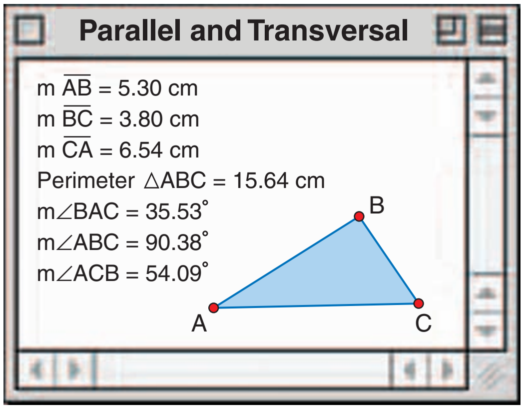
Step 3 Find the perimeter of $\triangle ABC$.

- Use the pointer tool to select points A, B, and C.
- Under the **Construct** menu, select **Triangle Interior**. The triangle will now be shaded.
- Choose the **Perimeter** command under the **Measure** menu to find the perimeter of $\triangle ABC$. The perimeter of $\triangle ABC$ is 15.64 centimeters.



Step 4 Find $m\angle A$, $m\angle B$, and $m\angle C$.

- Recall that $\angle A$ can also be named $\angle BAC$ or $\angle CAB$. Use the pointer to select points B, A, and C in order.
- Select the **Angle** command from the **Measure** menu to find $m\angle A$.
- Select points A, B, and C. Find $m\angle B$.
- Select points A, C, and B. Find $m\angle C$.



Analyze the Results

1. Add the side measures you found in Step 2. Compare this sum to the result of Step 3. How do these compare?
2. What is the sum of the angle measures of $\triangle ABC$?
3. Repeat the activities for each convex polygon.
   a. irregular quadrilateral **b.** square **c.** pentagon **d.** hexagon
4. Draw another quadrilateral and find its perimeter. Then enlarge your figure using the **Dilate** command. How does the change affect the perimeter?
5. Compare your results with those of your classmates.
6. **Make a conjecture** about the sum of the measures of the angles in any triangle.
7. What is the sum of the measures of the angles of a quadrilateral? pentagon? hexagon?
8. Make a conjecture about how the sums of the measures of the angles of polygons are related to the number of sides.
9. Test your conjecture on other polygons. Does your conjecture hold? Explain.
10. When the sides of a polygon are changed by a common factor, does the perimeter of the polygon change by the same factor as the sides? Explain.

1-7 Three-Dimensional Figures

Main Ideas

- Identify three-dimensional figures.
- Find surface area and volume.

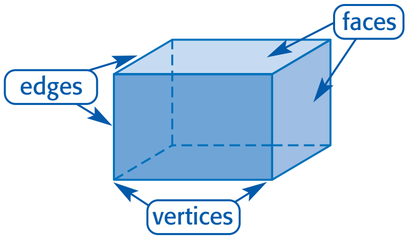
New Vocabulary

polyhedron
face
edges
prism
bases
vertex
regular prism
pyramid
regular polyhedron
Platonic solids
cylinder
cone
sphere
surface area
volume

Archaeologists and Egyptologists continue to study the Great Pyramids of Egypt. Even though some of the exterior materials used to build the pyramids have worn away, scientists can still speculate on the appearance of the pyramids when they were first built.

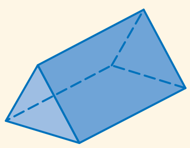
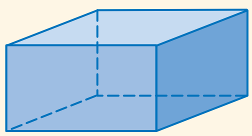
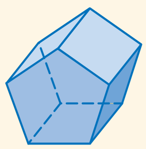
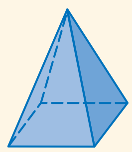
Identify Three-Dimensional Figures

A solid with all flat surfaces that enclose a single region of space is called a **polyhedron**. Each flat surface, or **face**, is a polygon. The line segments where the faces intersect are called **edges**. Edges intersect at a point called a *vertex*.



- A **prism** is a polyhedron with two parallel congruent faces called **bases**. The intersection of three edges is a **vertex**. Prisms are named by the shape of their bases. A **regular prism** is a prism with bases that are regular polygons. A cube is an example of a regular prism.
- A polyhedron with all faces (except for one) intersecting at one vertex is a **pyramid**. Pyramids are named for their bases, which can be any polygon. A polyhedron is a **regular polyhedron** if all of its faces are regular congruent polygons and all of the edges are congruent.

Some common polyhedrons are shown below.

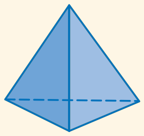
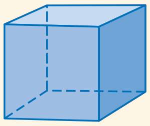
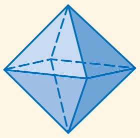
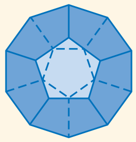
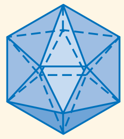
Name	Triangular Prism	Rectangular Prism	Pentagonal Prism	Square Pyramid
Model				
Shape of Base(s)	triangle	rectangle	pentagon	square

Steven Studd/Getty Images

Study Tip

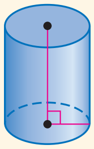
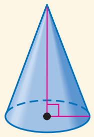
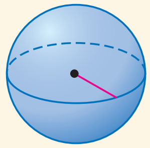
Common Misconception
Prisms can be oriented so the bases are not the top and bottom of the solid.

There are exactly five types of regular polyhedra. These are called the **Platonic solids** because Plato described them extensively in his writings.

Platonic Solids					
Name	**Tetrahedron**	**Hexahedron**	**Octahedron**	**Dodecahedron**	**Icosahedron**
Model					
Faces	4	6	8	12	20
Shape of Face	equilateral triangle	square	equilateral triangle	regular pentagon	equilateral triangle

There are solids that are *not* polyhedrons. Some or all of the faces in each of these types of solids are not polygons.

- A **cylinder** is a solid with congruent circular bases in a pair of parallel planes.
- A **cone** has a circular base and a vertex.
- A **sphere** is a set of points in space that are a given distance from a given point.

Name	**Cylinder**	**Cone**	**Sphere**
Model			
Base(s)	2 circles	1 circle	none

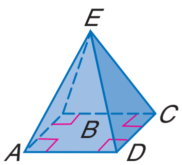
Reading Math

Symbols This symbol ▭ means rectangle. The symbol △ means triangle. The symbol ⊙ means circle.

EXAMPLE 1 Identify Solids

Identify each solid. Name the bases, faces, edges, and vertices.

a.



The base is a rectangle, and the four faces meet in a point. So this solid is a rectangular pyramid.

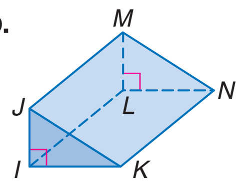
Base: ▭$ABCD$

Faces: ▭$ABCD$, $\triangle AED$, $\triangle DEC$, $\triangle CEB$, $\triangle AEB$

Edges: $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}, \overline{AE}, \overline{DE}, \overline{CE}, \overline{BE}$

Vertices: A, B, C, D, E

b.



The bases are right triangles. It is a triangular prism.

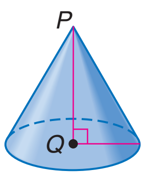
Bases: $\triangle IJK$, $\triangle LMN$

Faces: $\triangle IJK$, $\triangle LMN$, ▭$ILNK$, ▭$KJMN$, ▭$IJML$

Edges: $\overline{IL}, \overline{LN}, \overline{NK}, \overline{IK}, \overline{IJ}, \overline{LM}, \overline{JM}, \overline{MN}, \overline{JK}$

Vertices: I, J, K, L, M, N

c.



The base is a circle and there is one vertex. So it is a cone.

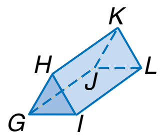
Base: $\odot Q$
Vertex: P

(continued on the next page)

CHECK Your Progress

Identify each solid. Name the bases, faces, edges, and vertices.

1A. 

1B. 

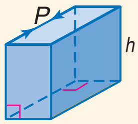
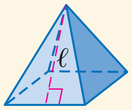
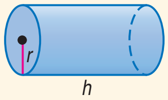
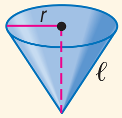
Surface Area and Volume The **surface area** is the sum of the areas of each face of a solid. **Volume** is the measure of the amount of space the solid encloses. You have studied surface area and volume in earlier math classes. The formulas for surface area and volume of four common solids are given below. You will derive these formulas in Chapters 12 and 13.

Study Tip

Formulas

To find the surface area for each solid, find the area of the base(s), and add the area of the face(s).

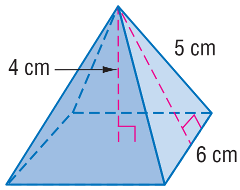
KEY CONCEPT — Surface Area and Volume

Solid	Prism	Pyramid	Cylinder	Cone
				
Surface Area	$T = Ph + 2B$	$T = \frac{1}{2}P\ell + B$	$T = 2\pi rh + 2\pi r^2$	$T = \pi r\ell + \pi r^2$
Volume	$V = Bh$	$V = \frac{1}{3}Bh$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$

P = perimeter of the base; B = area of the base; T = total surface area; h = height of solid; ℓ = slant height; r = radius

EXAMPLE 2 Surface Area and Volume

a. Find the surface area of the square pyramid.



$T = \frac{1}{2}P\ell + B$ — Surface area of pyramid

$= \frac{1}{2}(24)(5) + 36$ — $P = 24$ cm, $\ell = 5$ cm, $B = 36$ cm^2

$= 96$ cm^2 — Simplify.

The surface area of the square pyramid is 96 square centimeters.

b. Find the volume of the square pyramid.

$V = \frac{1}{3}Bh$ — Volume of pyramid

$= \frac{1}{3}(36)(4)$ — Substitution

$= 48$ cm^3 — Simplify.

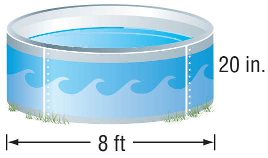
The volume is 48 cubic centimeters.

CHECK Your Progress

2. Find the surface area and volume of a square pyramid that has a base with dimensions 10 centimeters, a height of 12 centimeters, and a slant height of 13 centimeters.

EXAMPLE Surface Area and Volume

3 POOLS The diameter of the pool Mr. Diaz purchased is 8 feet. The height of the pool is 20 inches.



a. What is the surface area of the pool?

The pool's surface consists of the sides and one base. The height of the pool is 20 inches or $1\frac{2}{3}$ feet.

$A = 2\pi rh + \pi r^2$ — Area of sides + Area of base

$= 2\pi(4)\left(1\frac{2}{3}\right) + \pi(4)^2$ — Substitution

≈ 92.2 — Use a calculator to simplify.

The surface area of the pool is about 92.2 square feet.

b. If he fills the pool with water to a depth of 16 inches, what is the volume of the water in the pool, in cubic feet? Round to the nearest tenth.

The pool is a cylinder. The height of the water is 16 inches. To convert this measure to feet, divide by 12 to get $1\frac{1}{3}$ feet.

$V = \pi r^2 h$ — Volume of cylinder

$= \pi(4)^2\left(1\frac{1}{3}\right)$ — Substitution

≈ 67.0 — Use a calculator to simplify.

The volume of water in the pool is approximately 67.0 cubic feet.

CHECK Your Progress

3. **CRAFTS** Jessica is making candles with a mold of a square pyramid. The measure of each side of the base is 2 inches and the height is 4.5 inches. What volume of wax will fill the mold?

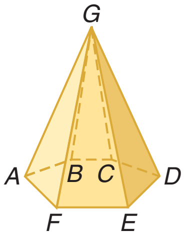
Online Personal Tutor at geometryonline.com

CHECK Your Understanding

Example 1 (p. 61)

Identify each solid. Name the bases, faces, edges, and vertices.

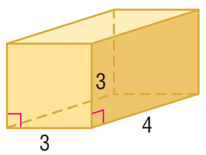
1.



2.

Example 2 (p. 62)

For Exercises 3 and 4, refer to the figure.



3. Find the surface area of the square prism.
4. Find the volume of the square prism.

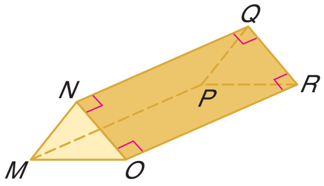
Example 3 (p. 63)

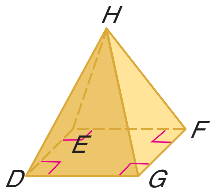
5. **PARTY FAVORS** Latasha is filling cone-shaped hats with candy for party favors. The base of each hat is 4 inches in diameter, and it is 6.5 inches deep. What volume of candy will fill the cone?

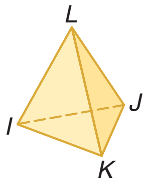
Exercises

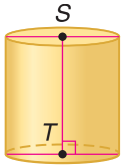
For Exercises	See Examples
6–11	1
12–19	2
20, 21	3

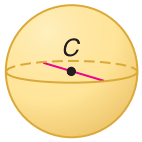
Identify each solid. Name the bases, faces, edges, and vertices.

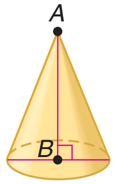
6.


7.


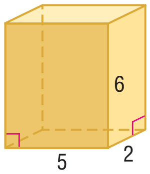
8.


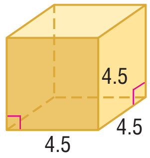
9.


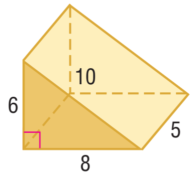
10.


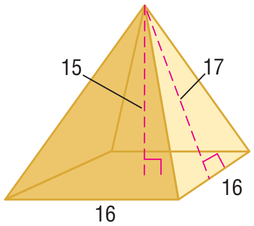
11.


Find the surface area and volume of each solid.

12.


13.


14.


15.


16. SANDBOX A rectangular sandbox is 3 feet by 4 feet. The depth of the box is 8 inches, but the depth of the sand is $\frac{3}{4}$ of the depth of the box. What is the volume of sand in the sandbox? Round to the nearest tenth.

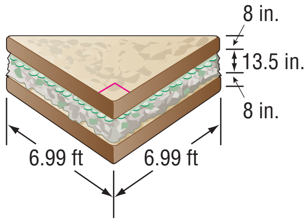
Real-World Link

The Campana brothers started a design workshop in which participants create art using objects that can be inflated, such as tires or balloons.

Source: arango-design.com

17. ART Fernando and Humberto Campana designed the Inflating Table shown at the left. The diameter of the table is $15\frac{1}{2}$ inches. Suppose the height of the cylinder is $11\frac{3}{4}$ inches. What volume of air will fully inflate the table? Round to the nearest tenth. Assume that the side of the table is perpendicular to the bases of the table.

FOOD In 1999, Marks & Spencer, a British department store, created the biggest sandwich ever made. The tuna and cucumber sandwich was in the form of a triangular prism. Suppose each slice of bread was 8 inches thick. Refer to the isometric view of the sandwich.



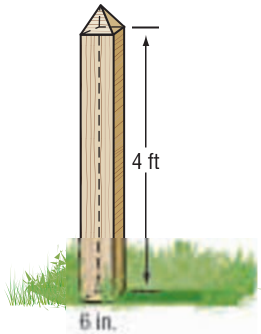
18. Find the surface area in square feet to the nearest tenth.

19. Find the volume, in cubic feet, of filling to the nearest tenth.

20. The surface area of a cube is 54 square inches. Find the length of each edge.

21. The volume of a cube is 729 cubic centimeters. Find the length of each edge.

22. **PAINTING** Desiree is painting her family's fence. Each post is composed of a square prism and a square pyramid. The height of the pyramid is 4 inches. Determine the surface area to be painted and the volume of each post.



23. **CHANGING DIMENSIONS** A rectangular prism has a length of 12 centimeters, width of 18 centimeters, and height of 22 centimeters. Describe the effect on the volume of a rectangular prism when each dimension is doubled.

For Exercises 24–26, use the following table.

24. Name the type of prism or pyramid that has the given number of faces.

Number of Faces	Prism	Pyramid
4	none	tetrahedron
5	a. ___?___	square or rectangular
6	b. ___?___	c. ___?___
7	pentagonal	d. ___?___
8	e. ___?___	heptagonal

25. Analyze the information in the table. Is there a pattern between the number of faces and the bases of the corresponding prisms and pyramids?

26. Is it possible to classify a polyhedron given only the number of faces? Explain.

27. **COLLECT DATA** Use a ruler or tape measure and what you have learned in this lesson to find the surface area and volume of a soup can.

EXTRA PRACTICE
See pages 802, 828.
Math Online
Self-Check Quiz at
geometryonline.com

28. **EULER'S FORMULA** The number of faces F, vertices V, and edges E of a polyhedron are related by Euler's (OY luhrz) Formula: $F + V = E + 2$. Determine whether Euler's Formula is true for each of the figures in Exercises 6–11.

H.O.T. Problems

29. **OPEN ENDED** Draw a rectangular prism.

30. **REASONING** Compare a square pyramid and a square prism.

CHALLENGE Describe the solid that results if the number of sides of each base increases infinitely. The bases of each solid are regular polygons inscribed in a circle.

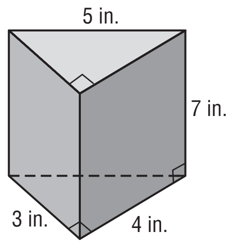
31. pyramid

32. prism

33. ***Writing in Math*** Refer to the information on archaeologists on page 60. Explain how classifying the shape of an ancient structure is helpful to their study.

STANDARDIZED TEST PRACTICE

34. What is the volume of the triangular prism shown?



A 42 in^3

B 60 in^3

C 84 in^3

D 210 in

35. **REVIEW** The data in the table show the cost of renting a moving truck for an in-town move. The cost is based on the miles driven and also includes a one-time fee.

Renting a Moving Truck

Miles (m)	Cost in Dollars (c)
10	45
20	60
30	75

If miles m were graphed on the horizontal axis and cost c were graphed on the vertical axis, what would be the equation of a line that fits the data?

F $c = 1.5m + 30$

G $c = \frac{1}{30}m + 1.5$

H $c = 3m - 15$

J $c = 30m + 15$

Spiral Review

Find the perimeter and area of each figure. (Lesson 1-6)

36. a square with length 12 feet

37. a rectangle with length 4.2 inches and width 15.7 inches

38. a square with length 18 centimeters

39. a rectangle with length 5.3 feet and width 7 feet

Find the measure of the angles. (Lesson 1-5)

40. The measures of two complementary angles are $(3x + 14)^\circ$ and $(5x - 8)^\circ$.

41. The measures of two supplementary angles are $(10x - 25)^\circ$ and $(15x + 50)^\circ$.

Find the value of the variable and MP, if P is between M and N. (Lesson 1-2)

42. $MP = 7x, PN = 3x, PN = 24$

43. $MP = 2c, PN = 9c, PN = 63$

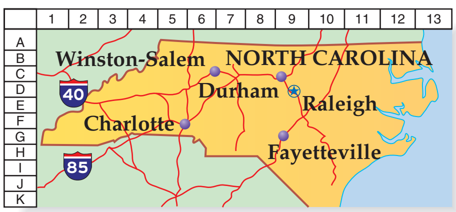
44. $MP = 4x, PN = 5x, MN = 36$

45. $MP = 6q, PN = 6q, MN = 60$

46. $MP = 4y + 3, PN = 2y, MN = 63$

47. $MP = 2b - 7, PN = 8b, MN = 43$

MAPS **For Exercises 48 and 49, refer to the map, and use the following information.** (Lesson 1-1)
A map represents a plane. Points on this plane are named using a letter/number combination.



48. Name the point where Raleigh is located.

49. What city is located at (F, 5)?

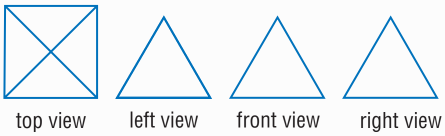
EXTEND 1-7

Geometry Lab

Orthographic Drawings and Nets

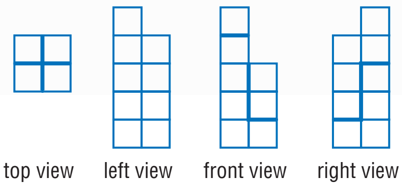
Concepts in Motion
Animation geometryonline.com

If you see a three-dimensional object from only one viewpoint, you may not know its true shape. Here are four views of a square pyramid. The two-dimensional views of the top, left, front, and right sides of an object are called an **orthographic drawing**.

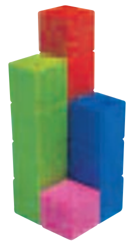


ACTIVITY 1

Make a model of a figure given the orthographic drawing.



- The top view indicates two rows and two columns of different heights.
- The front view indicates that the left side is 5 blocks high and the right side is 3 blocks high. The dark segments indicate breaks in the surface.
- The right view indicates that the right front column is only one block high. The left front column is 4 blocks high. The right back column is 3 blocks high.
- Check the left side of your model. All of the blocks should be flush.

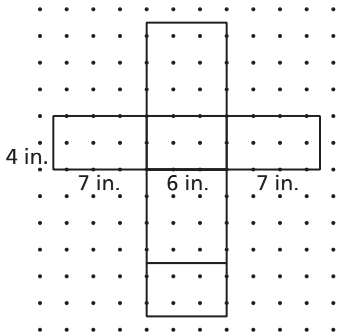


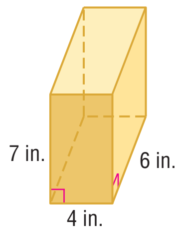
If you cut a cardboard box at the edges and lay it flat, you will have a pattern, or **net**, for the three-dimensional solid.

ACTIVITY 2

Make a model of a figure given the net.

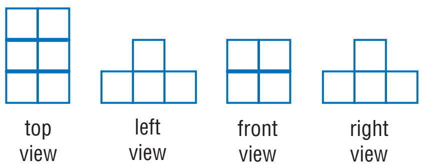
The figure is a rectangular prism. Use a large sheet of paper, a ruler, scissors, and tape. Measure the dimensions on the paper. Cut around the edges. Fold the pattern on the solid lines and secure the edges with tape.



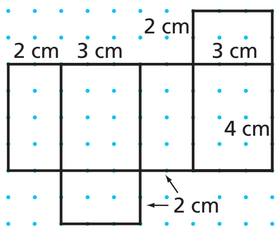


Model and Analyze

1. Make a model of a figure given the orthographic drawing. Then find the volume of the model.



2. Make a model of a figure given the net. Then find the surface area of the model.

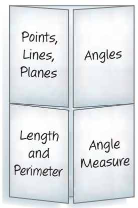


Chapter 1 Study Guide and Review

Download Vocabulary Review from geometryonline.com

Foldables Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Points, Lines, and Planes (Lesson 1-1)

- There is exactly one line through any two points.
- There is exactly one plane through any three noncollinear points.

Distance and Midpoints (Lesson 1-3)

- On a number line, the measure of a segment with endpoint coordinates a and b is given by $|a - b|$.
- In the coordinate plane, the distance between two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- On a number line, the coordinate of the midpoint of a segment with endpoints that have coordinates a and b is $\frac{a+b}{2}$.
- In the coordinate plane, the coordinates of the midpoint of a segment with endpoints that are (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Angles (Lessons 1-4 and 1-5)

- An angle is formed by two noncollinear rays that have a common endpoint. Angles can be classified by their measures.
- Adjacent angles are two angles that lie in the same plane and have a common vertex and a side but no common interior parts.
- Vertical angles are two nonadjacent angles formed by two intersecting lines.
- A linear pair is a pair of adjacent angles with noncommon sides that are opposite rays.
- Complementary angles are two angles with measures that have a sum of 90.
- Supplementary angles are two angles with measures that have a sum of 180.

Key Vocabulary

acute angle (p. 32)
adjacent angles (p. 40)
angle (p. 31)
angle bisector (p. 35)
area (p. 51)
bases (p. 60)
between (p. 15)
circumference (p. 51)
collinear (p. 6)
complementary angles (p. 42)
concave (p. 49)
cone (p. 61)
congruent (p. 15)
construction (p. 16)
convex (p. 49)
coplanar (p. 6)
cylinder (p. 61)
degree (p. 31)
edges (p. 60)
face (p. 60)
line (p. 6)
linear pair (p. 40)
line segment (p. 13)
midpoint (p. 22)
obtuse angle (p. 32)
opposite rays (p. 31)
perimeter (p. 51)
perpendicular (p. 43)
plane (p. 6)
point (p. 6)
polygon (p. 49)
polyhedron (p. 60)
prism (p. 60)
pyramid (p. 60)
ray (p. 31)
right angle (p. 32)
segment bisector (p. 25)
sides (p. 31)
space (p. 8)
sphere (p. 61)
supplementary angles (p. 42)
undefined term (p. 6)
vertex (p. 31)
vertical angles (p. 40)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

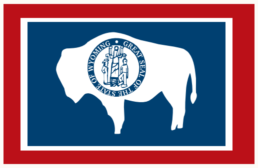
1. A <u>line</u> is determined by points and has no thickness or width.
2. Points that lie on the same plane are said to be <u>collinear</u>.
3. The symbol ≅ is read <u>*is equal to*</u>.
4. Two angles whose measures have a sum of 180° are <u>complementary angles</u>.
5. A <u>ray</u> can be measured because it has two endpoints.

Lesson-by-Lesson Review

1–1 Points, Lines, and Planes (pp. 6–11)

Draw and label a figure for each relationship.

6. Lines ℓ and m are coplanar and meet at point C.
7. Points S, T, and U are collinear, but points S, T, U, and V are not.
8. **FLAGS** The Wyoming state flag is shown below. Identify the geometric figures that could be represented by this flag.



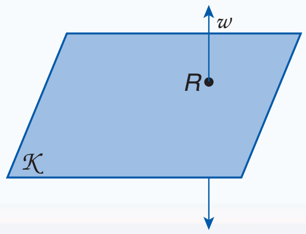
Example 1 **Draw and label a figure for the relationship below.**

Line w intersects plane $\mathcal{K}$ at R.

Draw a surface to represent plane $\mathcal{K}$ and label it.

Draw a line intersecting the plane and label it.

Draw a dot where the line and the plane meet and label it.



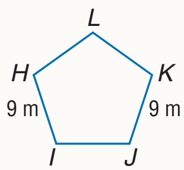
1–2 Linear Measure and Precision (pp. 13–20)

Find the value of the variable and PB, if P is between A and B.

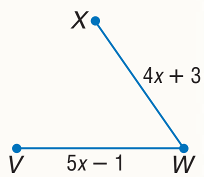
9. $AP = 7, PB = 3x, AB = 25$
10. $AP = s + 2, PB = 4s, AB = 8s - 7$
11. $AP = -2k, PB = k + 6, AB = 11$

Determine whether each pair of segments is congruent.

12. $\overline{HI}, \overline{KJ}$

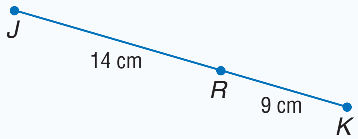


13. $\overline{VW}, \overline{WX}$



14. **HEIGHT** Gregory measured his height to be 71.5 inches. Find and explain the precision for this measurement.

Example 2 **Use the figure to find the measurement $\overline{JK}$.**



$JK = JR + RK$ — Betweenness of points

$= 14 + 9$ — Substitution

$= 23$ — Simplify.

So, $\overline{JK}$ is 23 centimeters long.

Example 3 **Find the precision for 62 miles.**

The measurement is precise to within 0.5 miles. So, a measurement of 62 miles could be 61.5 to 62.5 miles.

CHAPTER 1

Study Guide and Review

1-3 Distance and Midpoints (pp. 21–29)

Use the Pythagorean Theorem to find the distance between each pair of points.

15. $A(1, 0)$, $B(-3, 2)$

16. $G(-7, 4)$, $L(3, 3)$

Use the Distance Formula to find the distance between each pair of points.

17. $J(0, 0)$, $K(4, -1)$

18. $M(-4, 16)$, $P(-6, 19)$

Find the coordinates of the midpoint of each segment.

19. $U(-6, -3)$, $V(12, -7)$

20. $R(3.4, -7.3)$, $S(-2.2, -5.4)$

21. **WALKING** Paul and Susan are standing outside City Hall. Paul walks three blocks north and two blocks west while Susan walks five blocks south and four blocks east. If City Hall represents the origin, find the coordinates of the midpoint of Paul and Susan's locations.

Example 4 **Find the distance between $A(3,-4)$ and $B(-2,10)$.**

Let $(x_1, y_1) = (3, -4)$ and $(x_2, y_2) = (-2, -10)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-2-3)^2 + (-10-(-4))^2}$$
$$= \sqrt{(-5)^2 + (-6)^2}$$
$$= \sqrt{61}$$

The distance from A to B is $\sqrt{61}$ units or about 7.8 units.

Example 5 **Find the coordinates of the midpoint between $G(5, -2)$ and $N(-1, 6)$.**

Let $(x_1, y_1) = (5, -2)$ and $(x_2, y_2) = (-1, 6)$.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= M\left(\frac{5 + (-1)}{2}, \frac{-2 + 6}{2}\right)$$
$$= M(2, 2)$$

The coordinates of the midpoint are (2, 2).

1-4 Angle Measure (pp. 31–38)

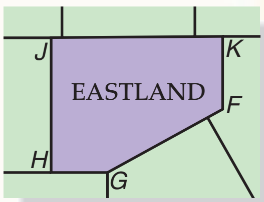
Refer to the figure in Example 6.

22. Name the vertex of ∠4.

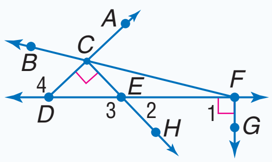
23. Name the sides of ∠2.

24. Write another name for ∠2.

25. **COUNTIES** Refer to the map of Eastland County. Measure each of the five angles and classify them as *right*, *acute*, or *obtuse*.



Example 6 **Refer to the figure. Name all angles that have E as a vertex.**

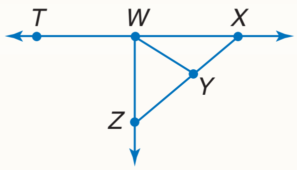


∠3, ∠2, ∠*HEF*, ∠*DEH*, ∠*CED*, ∠*CEF*, ∠*CEH*, ∠*DEF*

Mixed Problem Solving
For mixed problem–solving practice, see page 828.

1–5 Angle Relationships (pp. 40–47)

Refer to the figure.



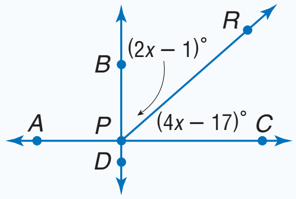
26. Name a linear pair whose vertex is Y.

27. Name an angle supplementary to $\angle XWY$.

28. If $m\angle TWZ = 2c + 36$, find c so that $\overline{TW} \perp \overline{WZ}$.

29. **DRIVING** At the intersection of 3rd and Main Streets, Sareeta makes a 110° turn from Main onto 3rd. Tyrone, behind her, makes a left turn onto 3rd. If 3rd and Main are straight lines, what is the angle measure of his turn?

Example 7 **Refer to the figure. Name a linear pair whose vertex is P.**

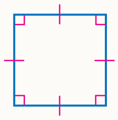


Sample answers: $\angle APR$ and $\angle RPC$, $\angle APD$ and $\angle DPC$, $\angle DPA$ and $\angle APB$, $\angle APB$ and $\angle BPC$, $\angle BPR$ and $\angle RPD$.

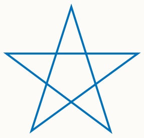
1–6 Two-Dimensional Figures (pp. 49–57)

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

30.

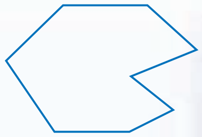


31.



32. **BEES** A honeycomb is formed by repeating regular hexagonal cells, as shown. The length of one side of a cell can range from 5.21 millimeters to 5.375 millimeters. Find the range of perimeters of one cell.

Example 8 **Name the polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.**



There are 8 sides, so this is an octagon. A line containing two of the sides will pass through the interior of the octagon, so it is concave. Since it is concave, it cannot be regular.

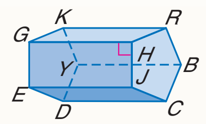
CHAPTER 1

Study Guide and Review

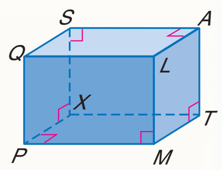
1–7 Three-Dimensional Figures (pp. 60–66)

Identify each solid. Name the bases, faces, edges, and vertices.

33.

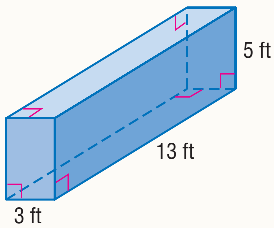


34.

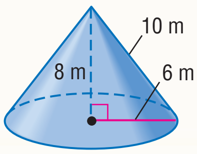


Find the surface area of each solid.

35.

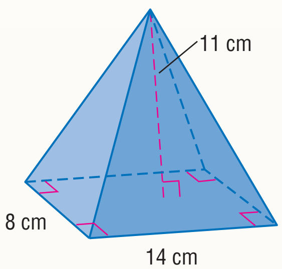


36.

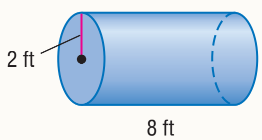


Find the volume of each solid.

37.

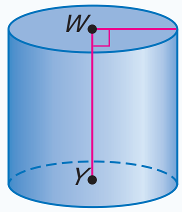


38.



39. CEREAL Company A created a cereal box with dimensions 8.5 by 1.75 by 11.5 inches. Their competitor, company B, created a cereal box with dimensions 8.5 by 2 by 11.25 inches. Which company created the box that would hold more cereal?

Example 9 **Identify the solid below. Name the bases, faces, edges, and vertices.**

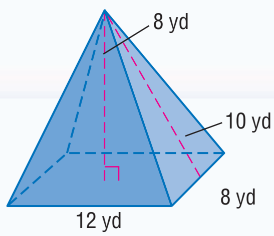


This solid has congruent circular bases in a pair of parallel planes. So, it is a cylinder.

Bases: $\odot W$ and $\odot Y$

A cylinder has no faces, edges, or vertices.

Example 10 **Find the surface area and volume of the rectangular pyramid below.**



$T = \frac{1}{3}P\ell + B$ — Surface area of pyramid

$= \frac{1}{3}(40)(10) + 96$ — Substitution

$\approx 229.3 \text{ yd}^2$

The surface area is 229.3 square yards.

$V = \frac{1}{3}Bh$ — Volume of pyramid

$= \frac{1}{3}(96)(8)$ — Substitution

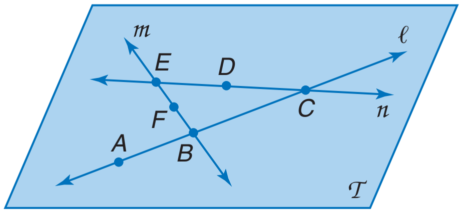
$= 256 \text{ yd}^3$

The volume is 256 cubic yards.

CHAPTER 1

Practice Test

For Exercises 1–3, refer to the figure below.



1. Name the line that contains points B and F.
2. Name a point that is not contained in lines ℓ or m.
3. Name the intersection of lines ℓ and n.

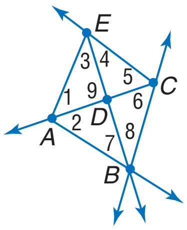
Find the value of the variable and VW if V is between U and W.

4. $UV = 2$, $VW = 3x$, $UW = 29$
5. $UV = r$, $VW = 6r$, $UW = 42$
6. $UV = 4p - 3$, $VW = 5p$, $UW = 15$
7. $UV = 3c + 29$, $VW = -2c - 4$, $UW = -4c$

Find the coordinates of the midpoint of a segment having the given endpoints. Then find the distance between the endpoints.

8. $G(0, 0)$, $H(-3, 4)$
9. $A(-4, -4)$, $W(-2, 2)$
10. $N(5, 2)$, $K(-2, 8)$

For Exercises 11–14, refer to the figure below.



11. Name the vertex of $\angle 6$.
12. Name the sides of $\angle 4$.
13. Write another name for $\angle 7$.
14. Write another name for $\angle ADE$.

15. **ALGEBRA** The measures of two supplementary angles are $(4r + 7)°$ and $(r - 2)°$. Find the measures of the angles.

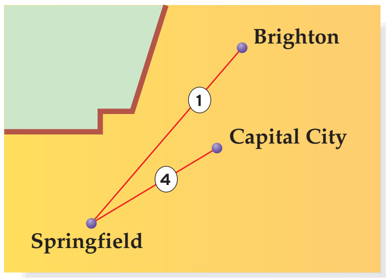
16. **ALGEBRA** Two angles are complementary. One angle measures 26 degrees more than the other. Find the measures of the angles.

Find the perimeter of each polygon.

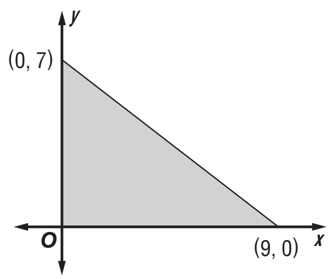
17. triangle PQR with vertices $P(-6, -3)$, $Q(1, -1)$, and $R(1, -5)$
18. pentagon $ABCDE$ with vertices $A(-6, 2)$, $B(-4, 7)$, $C(0, 4)$, $D(0, 0)$, and $E(-4, -3)$

DRIVING For Exercises 19 and 20, use the following information and the diagram.

The city of Springfield is 5 miles west and 3 miles south of Capital City, while Brighton is 1 mile east and 4 miles north of Capital City. Highway 1 runs straight between Brighton and Springfield; Highway 4 connects Springfield and Capital City.



19. Find the length of Highway 1.
20. How long is Highway 4?
21. **MULTIPLE CHOICE** What is the area, in square units, of the triangle shown below?



A 63 **C** 27.4

B 31.5 **D** 8

Standardized Test Practice

Chapter 1

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which of the following describes a plane?

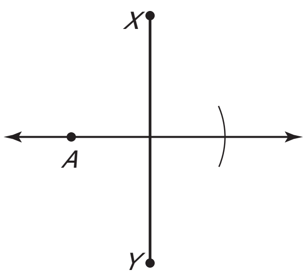
A a location having neither size nor shape

B a flat surface made up of points having no depth

C made up of points and has no thickness or width

D boundless, three-dimensional set of all points

2. Allie is using a straightedge and compass to do the construction shown below.



Which *best* describes the construction Allie is doing?

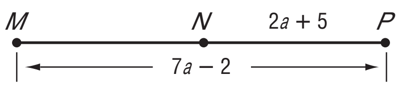
G a line through A parallel to $\overline{XY}$

F a segment starting at A congruent to $\overline{XY}$

H a line through A perpendicular to $\overline{XY}$

J a line through A bisecting $\overline{XY}$

3. What value of a makes N the midpoint of $\overline{MP}$?



4. ALGEBRA $3(x - 2)(x + 4) - 2(x^2 + 3x - 5) =$

A $x^2 - 14$

B $x^2 + 3x + 14$

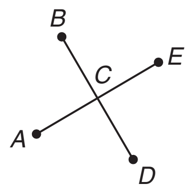
C $x^2 + 12x + 34$

D $5x^2 + 12x - 14$

TEST-TAKING TIP

Question 4 Most standardized tests allow you to write in the test booklet or on scrap paper. To avoid careless errors, work out your answers on paper rather than in your head.

5. In the diagram, $\overline{BD}$ intersects $\overline{AE}$ at C. Which of the following conclusions does *not* have to be true?



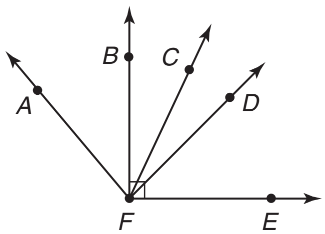
F $\angle ACB \cong \angle ECD$

G $\angle ACB$ and $\angle ACD$ form a linear pair.

H $\angle BCE$ and $\angle ACD$ are vertical angles.

J $\angle BCE$ and $\angle ECD$ are complementary angles.

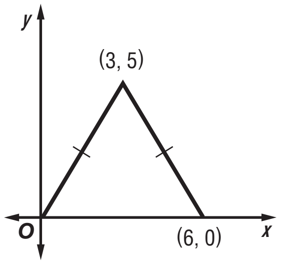
6. In the figure below, $\overrightarrow{FC}$ bisects $\angle AFE$, and $\overrightarrow{FD}$ bisects $\angle BFE$.



If $m\angle AFE = 130°$, what is $m\angle AFD$ in degrees?

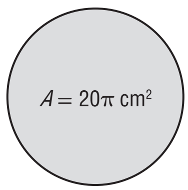
Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

7. What is the perimeter of the triangle shown below?



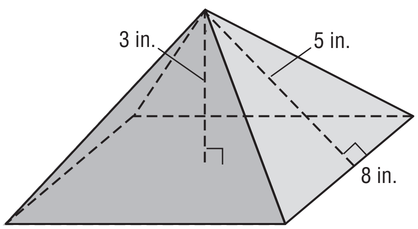
A 11
B 14
C $6 + 2\sqrt{34}$
D $6 + 4\sqrt{2}$

8. The area of a circle is 20π square centimeters. What is its circumference in centimeters?

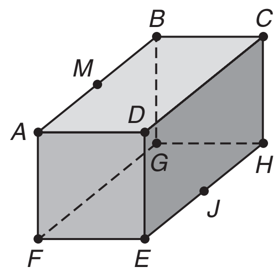


F $\sqrt{5}\pi$
G $2\sqrt{5}\pi$
H $4\sqrt{5}\pi$
J 20π

9. GRIDDABLE What is the volume in cubic inches of the square pyramid shown? (Volume of pyramid $= \frac{1}{3}Bh$, where B = area of base)



10. In the figure below, which of the following points are noncoplanar?

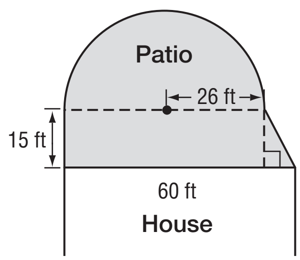


A *A*, *D*, *G*, and *H*
B *E*, *F*, *J*, and *H*
C *A*, *B*, *J*, and *M*
D *C*, *D*, *B*, and *F*

Pre AP

Record your answer on a sheet of paper. Show your work.

11. Suppose you work for a landscaping company and need to give a home owner a cost estimate for laying a patio and putting a stone border around the perimeter of the patio that does not share a side with the house. A scale drawing of the proposed patio is shown below.



The cost (labor and materials) for laying the patio is \$48 per square yard. The cost (labor and materials) for the stone border is \$22 per linear foot. What is your estimate?

NEED EXTRA HELP?											
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson or Page...	1-1	1-2	1-3	792	1-5	1-4	1-6	1-6	1-7	1-1	1-6