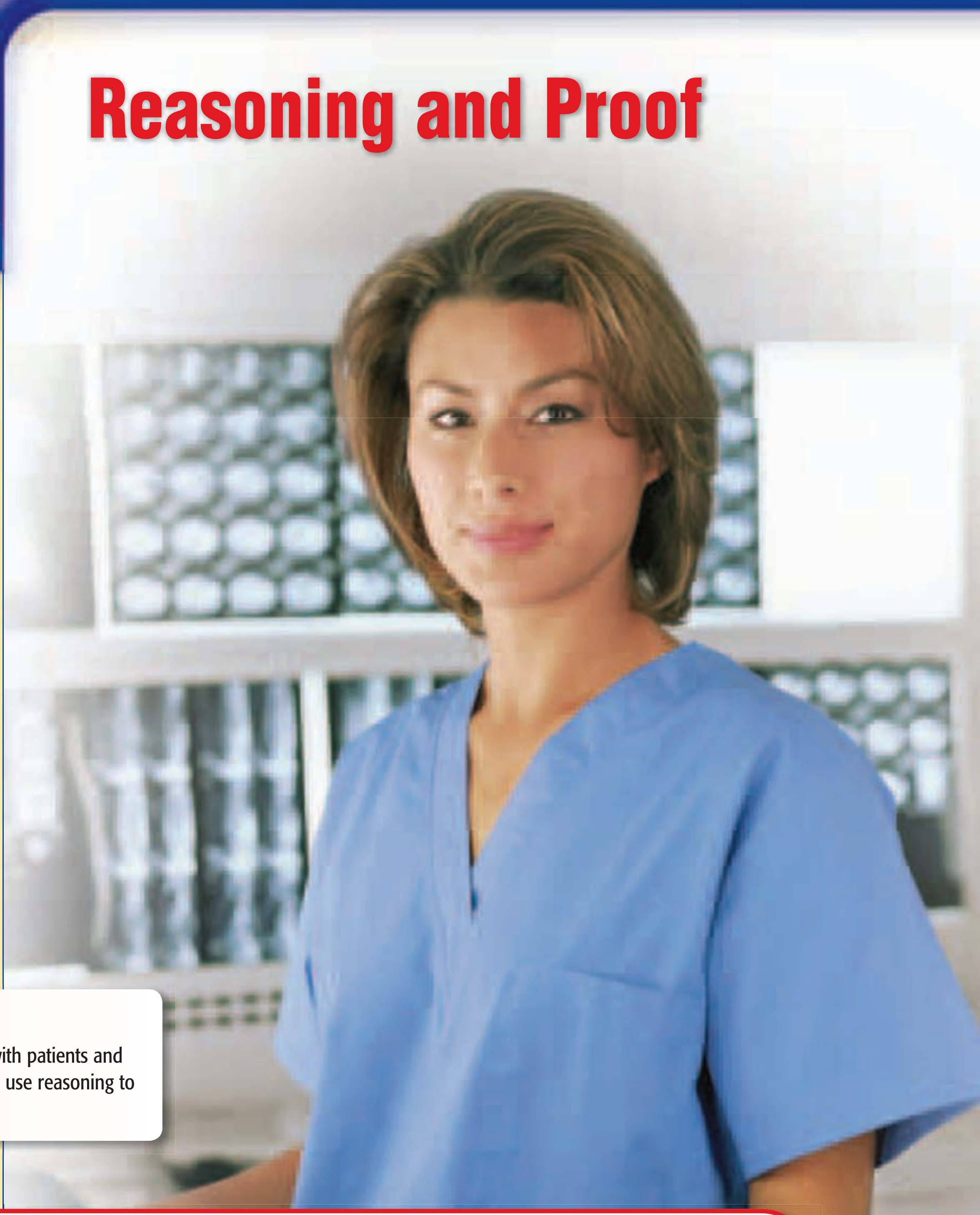
CHAPTER 2

Reasoning and Proof



BIG Ideas

- Make conjectures, determine whether a statement is true or false, and find counterexamples for statements.
- Use deductive reasoning to reach valid conclusions.
- Verify algebraic and geometric conjectures using informal and formal proof.
- Write proofs involving segment and angle theorems.

Key Vocabulary

inductive reasoning (p. 78)

deductive reasoning (p. 99)

postulate (p. 105)

theorem (p. 106)

proof (p. 106)

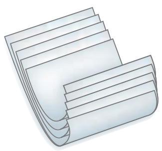
Real-World Link

Health Professionals Doctors talk with patients and run tests. They analyze the results and use reasoning to diagnose and treat patients.

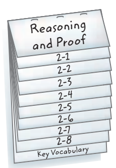
FOLDABLES Study Organizer

Reasoning and Proof Make this Foldable to help you organize your notes. Begin with five sheets of $8\frac{1}{2}$" × 11" plain paper.

1 **Stack** the sheets of paper with edges $\frac{3}{4}$ inch apart. Fold the bottom edges up to create equal tabs.



2 **Staple** along the fold. Label the top tab with the chapter title. Label the next 8 tabs with lesson numbers. The last tab is for Key Vocabulary.



GET READY for Chapter 2

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Evaluate each expression for the given value of n. (Prerequisite Skill)

1. $3n - 2; n = 4$
2. $(n + 1) + n; n = 6$
3. $n^2 - 3n; n = 3$
4. $180(n - 2); n = 5$
5. $n\left(\frac{n}{2}\right); n = 10$
6. $\frac{n(n-3)}{2}; n = 8$
7. Write the expression *three more than the square of a number*.
8. Write the expression *three less than the square of a number and two*.

Solve each equation. (Prerequisite Skill)

9. $6x - 42 = 4x$
10. $8 - 3n = -2 + 2n$
11. $3(y + 2) = -12 + y$
12. $12 + 7x = x - 18$
13. $3x + 4 = \frac{1}{2}x - 5$
14. $2 - 2x = \frac{2}{3}x - 2$
15. **MUSIC** Mark bought 3 CDs and spent \$24. Write and solve an equation for the average cost of each CD. (Prerequisite Skill)

For Exercises 16–19, refer to the figure from Example 3. (Prerequisite Skill)

16. Identify a pair of vertical angles that appear to be acute.
17. Identify a pair of adjacent angles that appear to be obtuse.
18. If $m\angle AGB = 4x + 7$ and $m\angle EGD = 71$, find x.
19. If $m\angle BGC = 45$, $m\angle CGD = 8x + 4$, and $m\angle DGE = 15x - 7$, find x.

QUICKReview

EXAMPLE 1

Evaluate $n^3 - 3n^2 + 3n - 1$ for $n = 1$.

$n^3 - 3n^2 + 3n - 1$	Write the expression.
$= (1)^3 - 3(1)^2 + 3(1) - 1$	Substitute 1 for n.
$= 1 - 3(1) + 3(1) - 1$	Evaluate the exponents.
$= 1 - 3 + 3 - 1$	Multiply.
$= 0$	Simplify.

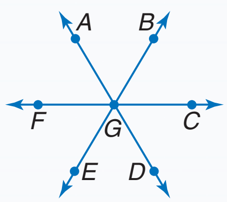
EXAMPLE 2

Solve $70x + 140 = 35x$.

$70x + 140 = 35x$	Write the equation.
$35x + 140 = 0$	Subtract $35x$ from each side.
$35x = -140$	Subtract 140 from each side.
$x = -4$	Divide each side by 35.

EXAMPLE 3

Refer to the figure. If $m\angle AGE = 6x + 2$ and $m\angle BGD = 110$, find x.



$\angle AGE$ and $\angle BGD$ are vertical angles.

$m\angle AGE = m\angle BGD$	Vert. ∠s are ≅.
$6x + 2 = 110$	Substitution
$6x = 108$	Subtract 2 from each side.
$x = 18$	Divide each side by 6.

2-1 Inductive Reasoning and Conjecture

Main Ideas

- Make conjectures based on inductive reasoning.
- Find counterexamples.

New Vocabulary

conjecture
inductive reasoning
counterexample

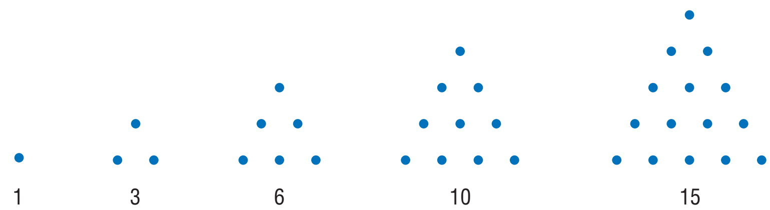
GET READY for the Lesson

People in the ancient Orient developed mathematics to assist in farming, business, and engineering. Documents from that time show that they taught mathematics by showing several examples and looking for a pattern in the solutions. This process is called inductive reasoning.

Make Conjectures A **conjecture** is an educated guess based on known information. Examining several specific situations to arrive at a conjecture is called inductive reasoning. **Inductive reasoning** is reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction.

EXAMPLE 1 Patterns and Conjecture

The numbers represented below are called *triangular numbers*. Make a conjecture about the next triangular number.



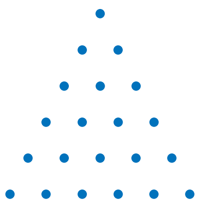
Observe: Each triangle is formed by adding a row of dots.

Find a Pattern: $1 \xrightarrow{+2} 3 \xrightarrow{+3} 6 \xrightarrow{+4} 10 \xrightarrow{+5} 15$

The numbers increase by 2, 3, 4, and 5.

Conjecture: The next number will increase by 6. So, it will be 15 + 6 or 21.

Check: Drawing the next triangle verifies the conjecture.



Study Tip

Conjectures
List your observations and identify patterns before you make a conjecture.

CHECK Your Progress

1. Make a conjecture about the next term in the sequence 20, 16, 11, 5, −2, −10.

Extra Examples at geometryonline.com

Agricultural Scenes. B.C.E. 715-322, Egyptian. Detail from The Book of the Dead of Tsekohns./Egyptian Museum, Turin, Italy/Alinari/Bridgeman Art Library

In Chapter 1, you learned some basic geometric concepts. These concepts can be used to make conjectures in geometry.

EXAMPLE Geometric Conjecture

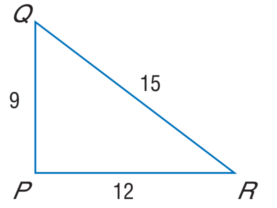
2 **For points P, Q, and R, $PQ = 9$, $QR = 15$, and $PR = 12$. Make a conjecture and draw a figure to illustrate your conjecture.**

Given: points P, Q, and R; $PQ = 9$, $QR = 15$, and $PR = 12$

Examine the measures of the segments.
Since $PQ + PR \neq QR$, the points cannot be collinear.

Conjecture: P, Q, and R are noncollinear.

Check: Draw $\triangle PQR$. This illustrates the conjecture.



CHECK Your Progress

2. K is the midpoint of $\overline{JL}$. Make a conjecture and draw a figure to illustrate your conjecture.

Find Counterexamples A conjecture based on several observations may be true in most circumstances, but false in others. It takes only one false example to show that a conjecture is not true. The false example is called a **counterexample**.

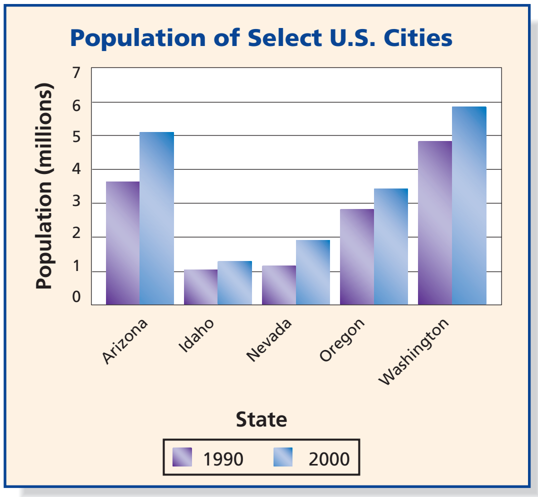
Vocabulary Link
Counterexample
Everyday use: the prefix *counter-* means *the opposite of*
Math use: a counterexample is the opposite of an example

Real-World EXAMPLE

3 **POPULATION** **Find a counterexample for the following statement based on the graph.**

The populations of these U.S. states increased by less than 1 million from 1990 to 2000.

Examine the graph. The statement is true for Idaho, Nevada, and Oregon. However, the populations of Arizona and Washington increased by more than 1 million from 1990 to 2000. Thus, either of these increases is a counterexample to the given statement.



Source: census.gov

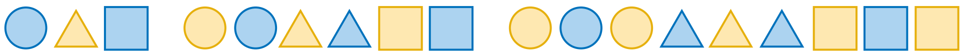
CHECK Your Progress

3. Find a counterexample to the statement *The states with a population increase of less than 1 million people increased their population by more than 25% from 1990 to 2000.*

Online **Personal Tutor at** geometryonline.com

CHECK Your Understanding

Example 1 (p. 78) **Make a conjecture about the next item in each sequence.**

1. 

2. −8, −5, −2, 1, 4

Example 2 (p. 79) **Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.**

3. $PQ = RS$ and $RS = TU$

4. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at P.

Example 3 (p. 79) **FISHING For Exercises 5 and 6, refer to the graphic and find a counterexample for each statement.**

5. The number of youth anglers in a state is less than one-fourth of the total anglers in that state.

6. Each state listed has at least 3,000,000 anglers.

Fishing

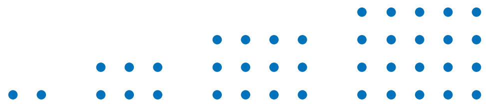
State	Number of Youth Anglers	Percent of Total Anglers per State
California	1,099,000	31
Florida	543,000	15
Michigan	452,000	25
North Carolina	353,000	21.5

Source: American Sportfishing Association

Exercises

HOMEWORK HELP	
For Exercises	See Examples
7–16	1
17–24	2
25–32	3

Make a conjecture about the next item in each sequence.

7. 

8. 

9. 1, 2, 4, 8, 16

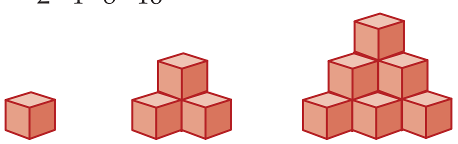
10. 4, 6, 9, 13, 18

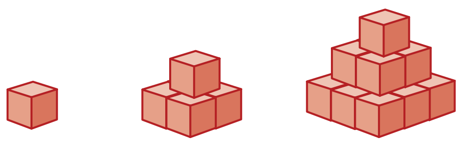
11. $\frac{1}{3}, 1, \frac{5}{3}, \frac{7}{3}, 3$

12. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

13. 2, −6, 18, −54

14. −5, 25, −125, 625

15. 

16. 

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

17. Lines ℓ and m are perpendicular.

18. $A(-2, -11)$, $B(2, 1)$, $C(5, 10)$

19. $\angle 3$ and $\angle 4$ are a linear pair.

20. $\overrightarrow{BD}$ is an angle bisector of $\angle ABC$.

21. $P(-1, 7)$, $Q(6, -2)$, $R(6, 5)$

22. $HIJK$ is a square.

23. $PQRS$ is a rectangle.

24. $\angle B$ is a right angle in $\triangle ABC$.

Cross-Curricular Project

Math Online You can use scatter plots to make conjectures about the relationships among latitude, longitude, degree distance, and the monthly high temperature. Visit geometryonline.com to continue work on your project.

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

25. **Given:** $\angle 1$ and $\angle 2$ are complementary angles.
Conjecture: $\angle 1$ and $\angle 2$ form a right angle.

26. **Given:** $m + y \geq 10$, $y \geq 4$
Conjecture: $m \leq 6$

27. **Given:** points W, X, Y, and Z
Conjecture: W, X, Y, and Z are noncollinear.

28. **Given:** $A(-4, 8)$, $B(3, 8)$, $C(3, 5)$
Conjecture: $\triangle ABC$ is a right triangle.

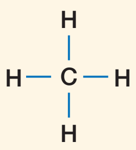
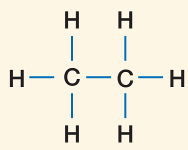
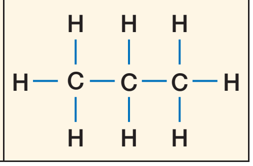
29. **Given:** n is a real number.
Conjecture: n^2 is a nonnegative number.

30. **Given:** $DE = EF$
Conjecture: E is the midpoint of $\overline{DF}$.

31. **HOUSES** Most homes in the northern United States have roofs made with steep angles. In the warmer southern states, homes often have flat roofs. Make a conjecture about why the roofs are different.

32. **MUSIC** Many people learn to play the piano by ear. This means that they first learned how to play without reading music. What process did they use?

CHEMISTRY **For Exercises 33–35, use the following information.**
Hydrocarbons are molecules composed of only carbon (C) and hydrogen (H) atoms. The simplest hydrocarbons are called alkanes. The first three alkanes are shown below.

Alkanes			
Compound Name	Methane	Ethane	Propane
Chemical Formula	CH_4	C_2H_6	C_3H_8
Structural Formula			

EXTRA PRACTICE
See pages 802, 829.
Math Online
Self-Check Quiz at geometryonline.com

33. **MAKE A CONJECTURE** about butane, which is the next compound in the group. Write its structural formula.

34. Write the chemical formula for the 7th compound in the group.

35. Develop a rule you could use to find the chemical formula of the nth substance in the alkane group.

H.O.T. Problems

36. **REASONING** Determine whether the following conjecture is *always*, *sometimes*, or *never* true based on the given information. Justify your reasoning.
Given: collinear points D, E, and F
Conjecture: $DE + EF = DF$

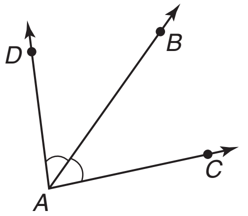
37. **OPEN ENDED** Write a statement. Then find a counterexample for the statement. Justify your reasoning.

38. **CHALLENGE** The expression $n^2 - n + 41$ has a prime value for $n = 1$, $n = 2$, and $n = 3$. Based on this pattern, you might conjecture that this expression always generates a prime number for any positive integral value of n. Try different values of n to test the conjecture. Answer *true* if you think the conjecture is always true. Answer *false* and give a counterexample if you think the conjecture is false. Justify your reasoning.

39. ***Writing in Math*** Refer to the information on page 78. Compare the method used to teach mathematics in the ancient Orient to how you have been taught mathematics. Describe any similarities or differences.

STANDARDIZED TEST PRACTICE

40. In the diagram below, $\overrightarrow{AB}$ is an angle bisector of $\angle DAC$.



Which of the following conclusions does *not* have to be true?

A $\angle DAB \cong \angle BAC$

B $\angle DAC$ is a right angle.

C A and D are collinear.

D $2(m\angle BAC) = m\angle DAC$

41. **REVIEW** A chemistry student mixed some 30%-copper sulfate solution with some 40%-copper sulfate solution to obtain 100 mL of a 32%-copper sulfate solution. How much of the 30%-copper sulfate solution did the student use in the mixture?

F 90 mL

G 80 mL

H 60 mL

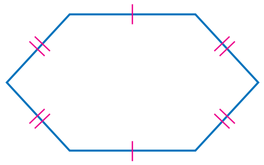
J 20 mL

Spiral Review

42. **FISH TANKS** Brittany purchased a cylindrical fish tank. The diameter of the base is 8 inches, and it is 12 inches tall. What volume of water will fill the tank? (Lesson 1-7)

Name each polygon by its number of sides and then classify it as *convex* or *concave* and *regular* or *not regular*. (Lesson 1-6)

43.



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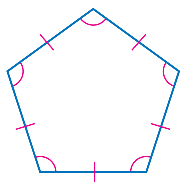
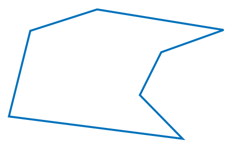


45.



GET READY for the Next Lesson

PREREQUISITE SKILL **Determine which values in the replacement set make the inequality true.**

46. $x + 2 > 5$
$\{2, 3, 4, 5\}$

47. $12 - x < 0$
$\{11, 12, 13, 14\}$

48. $5x + 1 > 25$
$\{4, 5, 6, 7\}$

2-2 Logic

GET READY for the Lesson

When you answer true-false questions on a test, you are using a basic principle of logic. For example, refer to the map, and answer *true* or *false*.

Frankfort is a city in Kentucky.

You know that there is only one correct answer, either true or false.

Main Ideas

- Determine truth values of conjunctions and disjunctions.
- Construct truth tables.

New Vocabulary

statement
truth value
negation
compound statement
conjunction
disjunction
truth table

Determine Truth Values A **statement**, like the true-false example above, is any sentence that is either true or false, but not both. Unlike a conjecture, we know that a statement is either true or false. The truth or falsity of a statement is called its **truth value**.

Statements are often represented using a letter such as p or q. The statement above can be represented by p.

p: Frankfort is a city in Kentucky. This statement is true.

The **negation** of a statement has the opposite meaning as well as an opposite truth value. For example, the negation of the statement above is *not p*.

not p: Frankfort is not a city in Kentucky. In this case, the statement is false.

KEY CONCEPT — **Negation**

Word If a statement is represented by p, then *not p* is the negation of the statement.

Symbols $\sim p$, read *not p*

Two or more statements can be joined to form a **compound statement**. Consider the following two statements.

p: Frankfort is a city in Kentucky.
q: Frankfort is the capital of Kentucky.

The two statements can be joined by the word *and*.

p and q: Frankfort is a city in Kentucky, *and* Frankfort is the capital of Kentucky.

The statement formed by joining p and q is an example of a conjunction.

KEY CONCEPT *Conjunction*

Words A **conjunction** is a compound statement formed by joining two or more statements with the word *and*.

Symbols $p \wedge q$, read *p and q*

A conjunction is true only when both statements in it are true. Since it is true that Frankfort is in Kentucky and it is the capital, the conjunction is also true.

EXAMPLE Truth Values of Conjunctions

1 **Use the following statements to write a compound statement for each conjunction. Then find its truth value.**

p: January 1 is the first day of the year.
q: $-5 + 11 = -6$
r: A triangle has three sides.

a. p **and** q

January is the first day of the year, and $-5 + 11 = -6$.
p and q is false, because p is true and q is false.

b. $\sim q \wedge r$

$-5 + 11 \neq -6$, and a triangle has three sides.
$\sim q \wedge r$ is true because $\sim q$ is true and r is true.

CHECK Your Progress

1A. $r \wedge p$ **1B.** p and not r

Reading Math

Negations The negation of a statement is not necessarily false. It has the opposite truth value of the original statement.

Statements can also be joined by the word *or*. This type of statement is a disjunction. Consider the following statements.

p: Ahmed studies chemistry.

q: Ahmed studies literature.

p or q: Ahmed studies chemistry, *or* Ahmed studies literature.

KEY CONCEPT *Disjunction*

Words A **disjunction** is a compound statement formed by joining two or more statements with the word *or*.

Symbols $p \vee q$, read *p or q*

A disjunction is true if at least one of the statements is true. In the case of p or q above, the disjunction is true if Ahmed either studies chemistry or literature or both. The disjunction is false only if Ahmed studies neither chemistry nor literature.

EXAMPLE 2 Truth Values of Disjunctions

Use the following statements to write a compound statement for each disjunction. Then find its truth value.

p: $100 \div 5 = 20$

q: The length of a radius of a circle is twice the length of its diameter.

r: The sum of the measures of the legs of a right triangle equals the measure of the hypotenuse.

a. **p or q**

$100 \div 5 = 20$, or the length of a radius of a circle is twice the length of its diameter.

p or q is true because p is true. It does not matter that q is false.

b. **$q \vee r$**

The length of a radius of a circle is twice the length of its diameter, or the sum of the measures of the legs of a right triangle equals the measure of the hypotenuse.

$q \vee r$ is false since neither statement is true.

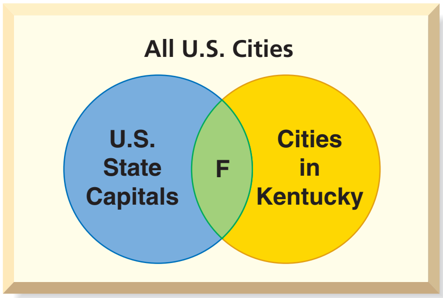
CHECK Your Progress

2. $\sim q \vee r$

Study Tip

Venn Diagrams The size of the overlapping region in a Venn diagram does not indicate how many items fall into that category.

Conjunctions can be illustrated with Venn diagrams. Refer to the statement at the beginning of the lesson. The Venn diagram at the right shows that Frankfort (F) is represented by the *intersection* of the set of cities in Kentucky and the set of state capitals. In other words, Frankfort is in both the set of cities in Kentucky and in the set of state capitals.



A disjunction can also be illustrated with a Venn diagram. Consider the following statements.

p: Jerrica lives in a U.S. state capital.

q: Jerrica lives in a Kentucky city.

$p \vee q$: Jerrica lives in a U.S. state capital, or Jerrica lives in a Kentucky city.

Reading Math

Intersection and Union The word *intersection* means the point at which more than one object overlap. The word *union* means to group together.

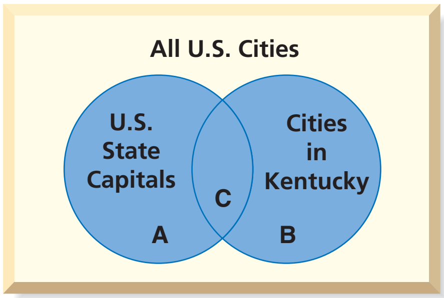
In the Venn diagrams, the disjunction is represented by the *union* of the two sets. The union includes all U.S. capitals and all cities in Kentucky.

The three regions represent

A U.S. state capitals excluding the capital of Kentucky,

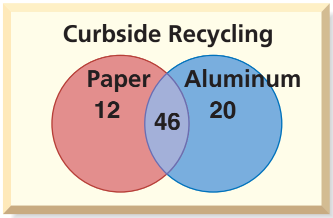
B cities in Kentucky excluding the state capital, and

C the capital of Kentucky, which is Frankfort.



EXAMPLE Use Venn Diagrams

3 **RECYCLING** **The Venn diagram shows the number of neighborhoods that have a curbside recycling program for paper or aluminum.**



a. How many neighborhoods recycle both paper and aluminum?

The neighborhoods that have paper and aluminum recycling are represented by the intersection of the sets. There are 46 neighborhoods that have paper and aluminum recycling.

b. How many neighborhoods recycle paper or aluminum?

The neighborhoods that have paper or aluminum recycling are represented by the union of the sets. There are 12 + 46 + 20 or 78 neighborhoods that have paper or aluminum recycling.

c. How many neighborhoods recycle paper and not aluminum?

The neighborhoods that have paper and not aluminum recycling are represented by the nonintersecting portion of the paper region. There are 12 neighborhoods that have paper and not aluminum recycling.

Real-World Link

Earth could be circled 20 times by the amount of paper produced by American businesses in one day.

Source: Resourcefulschools.org

CHECK Your Progress

3. How many neighborhoods recycle aluminum and not paper?

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Truth Tables A convenient method for organizing the truth values of statements is to use a **truth table**.

Negation	
p	$\sim p$
T	F
F	T

If p is a true statement, then $\sim p$ is a false statement.

If p is a false statement, then $\sim p$ is a true statement.

Truth tables can also be used to determine truth values of compound statements.

Conjunction		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

A conjunction is true only when both statements are true.

Disjunction		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

A disjunction is false only when both statements are false.

You can use the truth values for negation, conjunction, and disjunction to construct truth tables for more complex compound statements.

EXAMPLE Construct Truth Tables

4 **Construct a truth table for each compound statement.**

a. $p \wedge \sim q$

Step 1 Make columns with the headings p, q, $\sim q$, and $p \wedge \sim q$.

Step 2 List the possible combinations of truth values for p and q.

Step 3 Use the truth values of q to determine the truth values of $\sim q$.

Step 4 Use the truth values for p and $\sim q$ to write the truth values for $p \wedge \sim q$.

Step 1 →

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Step 2 Step 3 Step 4

Study Tip

Truth Tables

Use the Fundamental Counting Principle to determine the number of rows necessary. In Example 4b, there are 2 possible values for each of the three statements, p, q, and r. So there should be $2 \cdot 2 \cdot 2$ or 8 rows in the table.

b. $(p \wedge q) \vee r$

Make columns for p, q, $p \wedge q$, r, and $(p \wedge q) \vee r$.

p	q	$p \wedge q$	r	$(p \wedge q) \vee r$
T	T	T	T	T
T	F	F	T	T
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T
F	T	F	F	F
F	F	F	F	F

CHECK Your Progress

4. $\sim p \vee \sim q$

CHECK Your Understanding

Examples 1–2
(pp. 84–85)

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: $9 + 5 = 14$

q: February has 30 days.

r: A square has four sides.

1. p and q **2.** $p \wedge r$ **3.** $q \wedge r$

4. p or $\sim q$ **5.** $q \vee r$ **6.** $\sim p \vee \sim r$

Example 3
(p. 86)

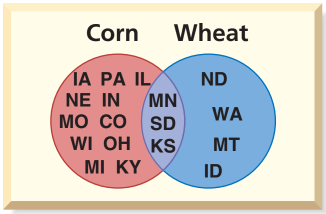
AGRICULTURE **For Exercises 7–9, refer to the Venn diagram that represents the states producing more than 100 million bushels of corn or wheat per year.**

7. How many states produce more than 100 million bushels of corn?

8. How many states produce more than 100 million bushels of wheat?

9. How many states produce more than 100 million bushels of corn and wheat?

Grain Production



Source: U.S. Department of Agriculture

Example 4
(p. 87)

10. Copy and complete the truth table.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	
T	F		
F	T		
F	F		

Construct a truth table for each compound statement.

11. $p \wedge q$

12. $\sim p \wedge r$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–24	1, 2
25–31	3
32–41	4

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: $\sqrt{-64} = 8$

q: **A triangle has three sides.**

r: $0 < 0$

s: **An obtuse angle measures greater than 90 and less than 180.**

13. p and q

14. p or q

15. p and $\sim r$

16. r and s

17. q or r

18. q and s

19. $p \wedge s$

20. $\sim q \wedge r$

21. $r \vee p$

22. $s \vee q$

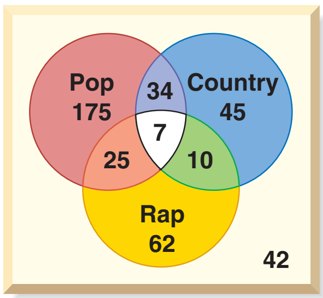
23. $(\sim p \wedge q) \vee s$

24. $s \vee (q \wedge \sim r)$

MUSIC **For Exercises 25–28, use the following information.**

A group of 400 teens were asked what type of music they listened to. They could choose among pop, rap, and country. The results are shown in the Venn diagram.

Music Preference



25. How many said that they listened to none of these types of music?

26. How many said that they listened to all three types of music?

27. How many said that they listened to only pop and rap music?

28. How many said that they listened to pop, rap, or country music?

Real-World Link

Nationwide, approximately 80% of high school seniors participate in extracurricular activities. Athletics, performing arts, and clubs are the most popular.

Source: National Center for Education Statistics

SCHOOL **For Exercises 29–31, use the following information.**
In a school of 310 students, 80 participate in academic clubs, 115 participate in sports, and 20 students participate in both.

29. Make a Venn diagram of the data.

30. How many students participate in either academic clubs or sports?

31. How many students do not participate in either academic clubs or sports?

Copy and complete each truth table.

32.

p	q	$\sim p$	$\sim p \vee q$
T	T		
T	F		
F	T		
F	F		

33.

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T		F	F	
T		F	T	
F		T	F	
F		T	T	

Construct a truth table for each compound statement.

34. q and r **35.** p or q **36.** p or r **37.** p and q

38. $q \wedge \sim r$ **39.** $\sim p \wedge \sim q$ **40.** $\sim p \vee (q \wedge \sim r)$ **41.** $p \wedge (\sim q \vee \sim r)$

GEOGRAPHY **For Exercises 42–44, use the following information.**
A travel agency surveyed their clients about places they had visited. Of the participants, 60 had visited Europe, 45 visited England, and 50 visited France.

42. Make a Venn diagram of the data.

43. Write a conjunction from the data.

44. Write a disjunction from the data.

EXTRA PRACTICE
See pages 802, 829.

Math Online
Self-Check Quiz at
geometryonline.com

RESEARCH **For Exercises 45–47, use the Internet or another resource to determine whether each statement is *true* or *false*.**

45. Dallas is not located on the Gulf of Mexico.

46. Either Cleveland or Columbus is located near Lake Erie.

47. It is false that Santa Barbara is located on the Pacific Ocean.

H.O.T. Problems

OPEN ENDED **Write a compound statement for each condition.**

48. a true disjunction

49. a false conjunction

50. a true statement that includes a negation

CHALLENGE **For Exercises 51 and 52, use the following information.**
All members of Team A also belong to Team B, but only some members of Team B also belong to Team C. Teams A and C have no members in common.

51. Draw a Venn diagram to illustrate the situation.

52. Which statement(s) are true? Justify your reasoning.

p: If a person is a member of Team C, then the person is not a member of Team A.

q: If a person is not a member of Team B, then the person is not a member of Team A.

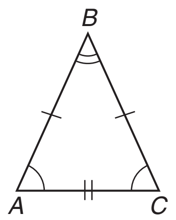
r: No person that is a member of Team A can be a member of Team C.

53. ***Writing in Math*** Refer to page 83. Describe how you can apply logic to taking tests. Include the difference between a conjunction and a disjunction.

Bill Bachmann/PhotoEdit

STANDARDIZED TEST PRACTICE

54. Which statement about $\triangle ABC$ has the same truth value as $AB = BC$?



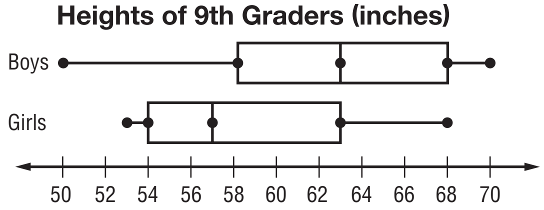
A $m\angle A = m\angle C$

B $m\angle A = m\angle B$

C $AC = BC$

D $AB = AC$

55. REVIEW The box-and-whisker plot below represents the height of 9th graders at a certain high school.



How much greater was the median height of the boys than the median height of the girls?

F 4 inches **H** 6 inches

G 5 inches **J** 7 inches

Spiral Review

Make a conjecture about the next item in each sequence. (Lesson 2-1)

56. 3, 5, 7, 9 **57.** 1, 3, 9, 27 **58.** 6, 3, $\frac{3}{2}$, $\frac{3}{4}$

59. 17, 13, 9, 5 **60.** 64, 16, 4, 1 **61.** 5, 15, 45, 135

62. Rayann has a glass paperweight that is a square pyramid. If the length of each side of the base is 2 inches and the slant height is 2.5 inches, find the surface area. (Lesson 1-7)

COORDINATE GEOMETRY **Find the perimeter of each polygon. Round to the nearest tenth.** (Lesson 1-6)

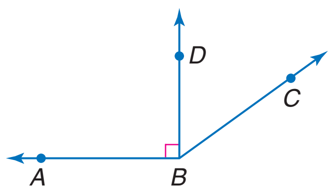
63. triangle ABC with vertices $A(-6, 7)$, $B(1, 3)$, and $C(-2, -7)$

64. square $DEFG$ with vertices $D(-10, -9)$, $E(-5, -2)$, $F(2, -7)$, and $G(-3, -14)$

65. quadrilateral $HIJK$ with vertices $H(5, -10)$, $I(-8, -9)$, $J(-5, -5)$, and $K(-2, -4)$

66. hexagon $LMNPQR$ with vertices $L(2, 1)$, $M(4, 5)$, $N(6, 4)$, $P(7, -4)$, $Q(5, -8)$, and $R(3, -7)$

Measure each angle and classify it as right, acute, or obtuse. (Lesson 1-4)



67. $\angle ABC$ **68.** $\angle DBC$ **69.** $\angle ABD$

70. FENCING Michelle wanted to put a fence around her rectangular garden. The front and back measured 35 feet each, and the sides measured 75 feet each. She plans to buy 5 extra feet of fencing to make sure that she has enough. How much should she buy? (Lesson 1-2)

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression for the given values.** (Page 780)

71. $5a - 2b$ if $a = 4$ and $b = 3$ **72.** $4cd + 2d$ if $c = 5$ and $d = 2$

73. $4e + 3f$ if $e = -1$ and $f = -2$ **74.** $3g^2 + h$ if $g = 8$ and $h = -8$

2-3 Conditional Statements

Main Ideas

- Analyze statements in if-then form.
- Write the converse, inverse, and contrapositive of if-then statements.

New Vocabulary

conditional statement
if-then statement
hypothesis
conclusion
related conditionals
converse
inverse
contrapositive
logically equivalent

GET READY for the Lesson

How are conditional statements used in advertisements? Advertisers often lure consumers into purchasing expensive items by convincing them that they are getting something for free in addition to their purchase.

If-Then Statements The statements above are examples of conditional statements. A **conditional statement** is a statement that can be written in *if-then form*. The second example above can be rewritten to illustrate this.

If you buy a car, *then* you get $1500 cash back.

KEY CONCEPT — *If-Then Statement*

Words An **if-then statement** is written in the form *if p, then q*. The phrase immediately following the word *if* is called the **hypothesis**, and the phrase immediately following the word *then* is called the **conclusion**.

Symbols $p \to q$, read *if p then q*, or *p implies q*.

Reading Math

If and Then The word *if* is not part of the hypothesis. The word *then* is not part of the conclusion.

EXAMPLE 1 Identify Hypothesis and Conclusion

Identify the hypothesis and conclusion of each statement.

a. If points A, B, and C lie on line ℓ, then they are collinear.

If $\underbrace{\text{points } A, B, \text{ and } C \text{ lie on line } \ell}_{\text{hypothesis}}$, then $\underbrace{\text{they are collinear}}_{\text{conclusion}}$.

Hypothesis: points A, B, and C lie on line ℓ

Conclusion: they are collinear

b. The Tigers will play in the tournament if they win their next game.

Hypothesis: the Tigers win their next game

Conclusion: they will play in the tournament

CHECK Your Progress

1A. If a polygon has six sides, then it is a hexagon.

1B. Another performance will be scheduled if the first one is sold out.

Real-World Link

Inline skating is the fastest-growing recreational sport. Participation has increased 630% over the past 15 years.

Source: International Inline Skating Association

Some conditional statements are written without the "if" and "then." You can write these statements in if-then form by first identifying the hypothesis and the conclusion.

EXAMPLE Write a Conditional in If-Then Form

2 **Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.**

a. An angle with a measure greater than 90 is an obtuse angle.

Hypothesis: an angle has a measure greater than 90

Conclusion: it is an obtuse angle

If an angle has a measure greater than 90, then it is an obtuse angle.

b. The length of the course for an inline skating marathon is 26.2 miles.

Hypothesis: a course is for an inline skating marathon

Conclusion: it is 26.2 miles

If a course is for an inline skating marathon, then it is 26.2 miles.

CHECK Your Progress

2A. An angle formed by perpendicular lines is a right angle.

2B. A cheetah has nonretractile claws.

Recall that the truth value of a statement is either true or false. The hypothesis and conclusion of a conditional statement, as well as the conditional statement itself, can also be true or false.

Study Tip

Common Misconception

A true hypothesis does not necessarily mean that a conditional is true. Likewise, a false conclusion does not guarantee that a conditional is false.

Real-World EXAMPLE Truth Values of Conditionals

3 **SCHOOL** **Determine the truth value of the following statement for each set of conditions.**

If you get 100% on your test, then your teacher will give you an A.

a. You get 100%; your teacher gives you an A.

The hypothesis is true since you got 100%, and the conclusion is true because the teacher gave you an A. Since what the teacher promised is true, the conditional statement is true.

b. You get 100%; your teacher gives you a B.

The hypothesis is true, but the conclusion is false. Because the result is not what was promised, the conditional statement is false.

c. You get 98%; your teacher gives you an A.

The hypothesis is false, and the conclusion is true. The statement does not say what happens if you do not get 100% on the test. You could still get an A. It is also possible that you get a B. In this case, we cannot say that the statement is false. Thus, the statement is true.

CHECK Your Progress

3. You get 85%; your teacher gives you a B.

Online **Personal Tutor at** geometryonline.com

Bob Daemmrich/PhotoEdit

The resulting truth values in Example 3 can be used to create a truth table for conditional statements. Notice that a conditional statement is true in all cases except where the hypothesis is true and the conclusion is false.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Converse, Inverse, and Contrapositive Other statements based on a given conditional statement are known as **related conditionals**. Consider the conditional *If you live in San Francisco, then you live in California*. The hypothesis is *you live in San Francisco*, and the conclusion is *you live in California*. If you reverse the hypothesis and conclusion, you form the conditional *If you live in California, then you live in San Francisco*. This is the converse of the conditional. The inverse and the contrapositive are formed using the negations of the hypothesis and the conclusion.

Concepts in Motion

Animation

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KEY CONCEPT *Related Conditionals*

Statement	Formed by	Symbols	Examples
Conditional	given hypothesis and conclusion	$p \rightarrow q$	If two angles have the same measure, then they are congruent.
Converse	exchanging the hypothesis and conclusion of the conditional	$q \rightarrow p$	If two angles are congruent, then they have the same measure.
Inverse	negating both the hypothesis and conclusion of the conditional	$\sim p \rightarrow \sim q$	If two angles do not have the same measure, then they are not congruent.
Contrapositive	negating both the hypothesis and conclusion of the converse statement	$\sim q \rightarrow \sim p$	If two angles are not congruent, then they do not have the same measure.

Study Tip

Contrapositive

The relationship of the truth values of a conditional and its contrapositive is known as the Law of Contrapositive.

If a given conditional is true, the converse and inverse are not necessarily true. However, the contrapositive of a true conditional is always true, and the contrapositive of a false conditional is always false. Likewise, the converse and inverse of a conditional are either both true or both false.

Statements with the same truth values are said to be **logically equivalent**. So, a conditional and its contrapositive are logically equivalent as are the converse and inverse of a conditional. These relationships are summarized in the table below.

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

EXAMPLE 4 Related Conditionals

Write the converse, inverse, and contrapositive of the following statement. Determine whether each statement is *true* or *false*. If a statement is false, give a counterexample.
Linear pairs of angles are supplementary.

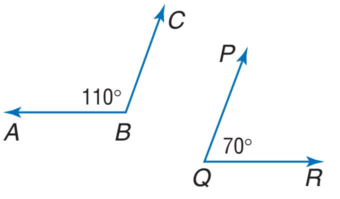
First, write the conditional in if-then form.

Conditional: If two angles form a linear pair, then they are supplementary. The conditional statement is true.

Write the converse by switching the hypothesis and conclusion.

Converse: If two angles are supplementary, then they form a linear pair. The converse is false. $\angle ABC$ and $\angle PQR$ are supplementary, but are not a linear pair.

Inverse: If two angles do not form a linear pair, then they are not supplementary. The inverse is false. $\angle ABC$ and $\angle PQR$ do not form a linear pair, but they are supplementary. The inverse is formed by negating the hypothesis and conclusion of the conditional.



The contrapositive is formed by negating the hypothesis and conclusion of the converse.

Contrapositive: If two angles are not supplementary, then they do not form a linear pair. The contrapositive is true.

CHECK Your Progress

4. *Vertical angles are congruent.*

CHECK Your Understanding

Example 1 (p. 91)

Identify the hypothesis and conclusion of each statement.

1. If it rains on Monday, then I will stay home.

2. If $x - 3 = 7$, then $x = 10$.

Example 2 (p. 92)

Write each statement in if-then form.

3. A 32-ounce pitcher holds a quart of liquid.

4. The sum of the measures of supplementary angles is 180.

5. **FORESTRY** In different regions of the country, different variations of trees dominate the landscape. Write the three conditionals in if-then form.

- In Colorado, aspen trees cover high areas of the mountains.
- In Florida, cypress trees rise from swamps.
- In Vermont, maple trees are prevalent.

Example 3 (p. 92)

Determine the truth value of the following statement for each set of conditions.
If you drive faster than 65 miles per hour, then you will receive a speeding ticket.

6. You drive 70 miles per hour, and you receive a speeding ticket.

7. You drive 62 miles per hour, and you do not receive a speeding ticket.

8. You drive 68 miles per hour, and you do not receive a speeding ticket.

Example 4 (p. 94)

Write the converse, inverse, and contrapositive of each conditional statement. Determine whether each related conditional is true or false. If a statement is false, find a counterexample.

9. If plants have water, then they will grow.

10. Flying in an airplane is safer than riding in a car.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–18	1
19–26	2
27–36	3
37–42	4

Identify the hypothesis and conclusion of each statement.

11. If you are a teenager, then you are at least 13 years old.

12. If you have a driver's license, then you are at least 16 years old.

13. If $2x + 6 = 10$, then $x = 2$.

14. If three points lie on a line, then they are collinear.

15. "If there is no struggle, there is no progress." (Frederick Douglass)

16. If the measure of an angle is between 0 and 90, then the angle is acute.

17. If a quadrilateral has four congruent sides, then it is a square.

18. If a convex polygon has five sides, then it is a pentagon.

Write each statement in if-then form.

19. Get a free water bottle with a one-year gym membership.

20. Math teachers love to solve problems.

21. "I think, therefore I am." (Descartes)

22. Adjacent angles have a common side.

23. Vertical angles are congruent.

24. Equiangular triangles are equilateral.

25. **MUSIC** Different instruments are emphasized in different types of music.

- Jazz music often incorporates trumpet or saxophone.
- Rock music emphasizes guitar and drums.
- In hip-hop music the bass is featured.

26. **ART** Several artists have their own museums dedicated to exhibiting their work. At the Andy Warhol Museum in Pittsburgh, Pennsylvania, most of the collection is Andy Warhol's artwork.

Real-World Link

The Andy Warhol Museum has over 4000 works of art including prints, paintings, films, photographs, and sculpture. About 500 of these pieces are on display at a time. The museum also has works by colleagues of Andy Warhol.

Source: warhol.org

Determine the truth value of the following statement for each set of conditions.

If you are over 18 years old, then you vote in all elections.

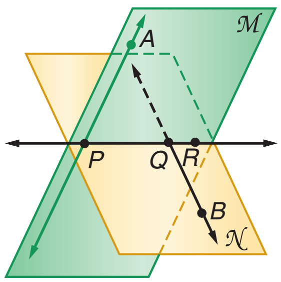
27. You are 19 years old and you vote.

28. You are 21 years old and do not vote.

29. You are 17 years old and do not vote.

30. Your dad is 45 years old and does not vote.

In the figure, P, Q, and R are collinear, P and A lie in plane $\mathcal{M}$, and Q and B lie in plane $\mathcal{N}$. Determine the truth value of each statement.



31. P, Q, and R lie in plane $\mathcal{M}$.

32. $\overleftrightarrow{QB}$ lies in plane $\mathcal{N}$.

33. Q lies in plane $\mathcal{M}$.

34. P, Q, A, and B are coplanar.

35. $\overleftrightarrow{AP}$ contains Q.

36. Planes $\mathcal{M}$ and $\mathcal{N}$ intersect at $\overleftrightarrow{RQ}$.

"Self Portrait 9", 1986. Andy Warhol. Synthentic polymer paint and screenprint on canvas. National Gallery of Victoria, Melbourne, Australia.

Write the converse, inverse, and contrapositive of each conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

37. If you live in Dallas, then you live in Texas.

38. If you exercise regularly, then you are in good shape.

39. The sum of two complementary angles is 90.

40. All rectangles are quadrilaterals.

41. All right angles measure 90.

42. Acute angles have measures less than 90.

SEASONS **For Exercises 43 and 44, use the following information.**
Due to the movement of Earth around the Sun, summer days in Alaska have more hours of daylight than darkness, and winter days have more hours of darkness than daylight.

43. Write two true conditional statements in if-then form for summer days and winter days in Alaska.

44. Write the converse of the two true conditional statements. State whether each is *true* or *false*. If a statement is false, find a counterexample.

EXTRA PRACTICE
See pages 803, 829.
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H.O.T. Problems

45. OPEN ENDED Write an example of a conditional statement.

46. REASONING Compare and contrast the inverse and contrapositive of a conditional.

47. CHALLENGE Write a false conditional. Is it possible to insert the word *not* into your conditional to make it true? If so, write the true conditional.

48. *Writing in Math* Refer to page 91. Describe how conditional statements are used in advertisements. Include an example of a conditional statement in if-then form that could be used in an advertisement.

STANDARDIZED TEST PRACTICE

49. *"If the sum of the measures of two angles is 90, then the angles are complementary angles."*
Which of the following is the converse of the conditional above?

A If the angles are complementary angles, then the sum of the measures of two angles is 90.

B If the angles are not complementary angles, then the sum of the measures of two angles is 90.

C If the angles are complementary angles, then the sum of the measures of two angles is not 90.

D If the angles are not complementary angles, then the sum of the measures of two angles is not 90.

50. REVIEW What is $\frac{10a^2 - 15ab}{4a^2 - 9b^2}$ reduced to lowest terms?

F $\frac{5a}{2a - 3b}$

G $\frac{5a}{2a + 3b}$

H $\frac{a}{2a + 3b}$

J $\frac{a}{2a - 3b}$

Graphing Calculator

For Exercises 51–53, refer to the following information.

The program at the right assigns random single digit integers to A and B. Then the program evaluates A and B and assigns a value to C.

```
PROGRAM: BOOLEAN
:randInt (0, 9)→A
:randInt (0, 9)→B
:if A≥2 and B=3
:Then:4→C
:Else:5→C
:End
```

51. Copy the program into your graphing calculator. Execute the program five times.

52. Write the conditional statement used in the program that assigns the value 4 to C.

53. Write the conditional statement that assigns the value 5 to C.

Spiral Review

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value. (Lesson 2-2)

p: George Washington was the first president of the United States.
q: A hexagon has five sides.
r: $60 \times 3 = 18$

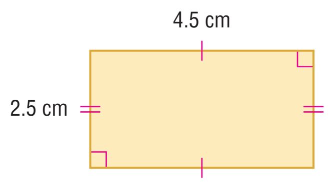
54. $p \wedge q$ **55.** $q \vee r$ **56.** $p \vee q$

57. $\sim q \vee r$ **58.** $p \wedge \sim q$ **59.** $\sim p \wedge \sim r$

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. (Lesson 2-1)

60. $ABCD$ is a rectangle. **61.** $J(-3, 2)$, $K(1, 8)$, $L(5, 2)$ **62.** In $\triangle PQR$, $m\angle PQR = 90$.

For Exercises 63–66, use the rectangle at the right. (Lesson 1-6)



63. Find the perimeter of the rectangle.

64. Find the area of the rectangle.

65. Suppose the length and width of the rectangle are each doubled. What effect does this have on the perimeter?

66. Describe the effect on the area.

Use the Distance Formula to find the distance between each pair of points. (Lesson 1-3)

67. $C(-2, -1)$, $D(0, 3)$ **68.** $J(-3, 5)$, $K(1, 0)$ **69.** $P(-3, -1)$, $Q(2, -3)$

For Exercises 70–72, draw and label a figure for each relationship. (Lesson 1-1)

70. $\overleftrightarrow{FG}$ lies in plane M and contains point H.

71. Lines r and s intersect at point W.

72. Line ℓ contains P and Q, but does not contain R.

GET READY for the Next Lesson

PREREQUISITE SKILL Identify the operation used to change Equation (1) to Equation (2). (Pages 781–782)

73. (1) $3x + 4 = 5x - 8$
(2) $3x = 5x - 12$

74. (1) $\frac{1}{2}(a - 5) = 12$
(2) $a - 5 = 24$

75. (1) $8p = 24$
(2) $p = 3$

READING MATH

Biconditional Statements

Ashley began a new summer job, earning \$10 an hour. If she works over 40 hours a week, she earns time and a half, or \$15 an hour. If she earns \$15 an hour, she has worked over 40 hours a week.

p: Ashley earns \$15 an hour
q: Ashley works over 40 hours a week

$p \rightarrow q$: If Ashley earns \$15 an hour, she has worked over 40 hours a week.
$q \rightarrow p$: If Ashley works over 40 hours a week, she earns \$15 an hour.

In this case, both the conditional and its converse are true. The conjunction of the two statements is called a **biconditional**.

KEY CONCEPT — *Biconditional Statement*

Words A biconditional statement is the conjunction of a conditional and its converse.

Symbols $(p \rightarrow q) \wedge (q \rightarrow p)$ is written $(p \leftrightarrow q)$ and read p *if and only if* q.

If and only if can be abbreviated *iff*.

So, the biconditional statement is as follows.

$p \leftrightarrow q$: Ashley earns \$15 an hour *if and only if* she works over 40 hours a week.

Examples

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

a. Two angle measures are complements if and only if their sum is 90.
Conditional: If two angle measures are complements, then their sum is 90.
Converse: If the sum of two angle measures is 90, then they are complements.
Both the conditional and the converse are true, so the biconditional is true.

b. $x > 9$ iff $x > 0$
Conditional: If $x > 9$, then $x > 0$. **Converse:** If $x > 0$, then $x > 9$.
The conditional is true, but the converse is not. Let $x = 2$. Then $2 > 0$ but $2 \not> 9$. So, the biconditional is false.

Reading to Learn

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

1. A calculator will run if and only if it has batteries.
2. Two lines intersect if and only if they are not vertical.
3. Two angles are congruent if and only if they have the same measure.
4. $3x - 4 = 20$ iff $x = 7$.

2-4 Deductive Reasoning

Main Ideas

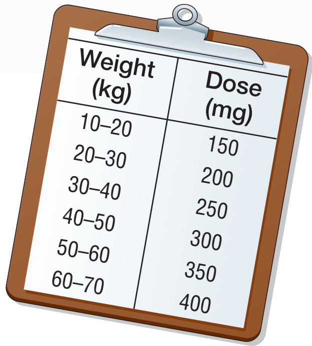
- Use the Law of Detachment.
- Use the Law of Syllogism.

New Vocabulary

deductive reasoning
Law of Detachment
Law of Syllogism

GET READY for the Lesson

When you are ill, your doctor may prescribe an antibiotic to help you get better. Doctors may use a dose chart like the one shown to determine the correct amount of medicine you should take.



Law Of Detachment The process that doctors use to determine the amount of medicine a patient should take is called **deductive reasoning**. Unlike inductive reasoning, which uses examples to make a conjecture, deductive reasoning uses facts, rules, definitions, or properties to reach logical conclusions. Inductive reasoning by itself does not prove anything, but deductive reasoning can be used to prove statements.

One form of deductive reasoning that is used to draw conclusions from true conditional statements is called the **Law of Detachment**.

Study Tip

Validity

When you apply the Law of Detachment, make sure that the conditional is true before you test the validity of the conclusion.

KEY CONCEPT *Law of Detachment*

Words	If $p \rightarrow q$ is true and p is true, then q is also true.
Symbols	$[(p \rightarrow q) \wedge p] \rightarrow q$

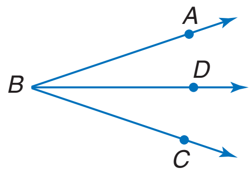
EXAMPLE Determine Valid Conclusions

1 **The statement below is a true conditional. Determine whether each conclusion is valid based on the given information. Explain your reasoning.**

If a ray is an angle bisector, then it divides the angle into two congruent angles.

Given: $\overrightarrow{BD}$ bisects $\angle ABC$.

Conclusion: $\angle ABD \cong \angle CBD$



The hypothesis states that $\overrightarrow{BD}$ is the bisector of $\angle ABC$. Since the conditional is true and the hypothesis is true, the conclusion is valid.

CHECK Your Progress

1. *If segments are parallel, then they do not intersect.*
   Given: $\overline{AB}$ and $\overline{CD}$ do not intersect.
   Conclusion: $\overline{AB} \parallel \overline{CD}$

Law of Syllogism Another law of logic is the **Law of Syllogism**. It is similar to the Transitive Property of Equality.

KEY CONCEPT *Law of Syllogism*

Words If $p \rightarrow q$ and $q \rightarrow r$ are true, then $p \rightarrow r$ is also true.

Symbols $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Example If $2x = 14$, then $x = 7$ and if $x = 7$, then $\frac{1}{x} = \frac{1}{7}$. Therefore, if $2x = 14$ then $\frac{1}{x} = \frac{1}{7}$.

Study Tip

Conditional Statements

Label the hypotheses and conclusions of a series of statements before applying the Law of Syllogism.

Real-World EXAMPLE Determine Valid Conclusions From Two Conditionals

2 **CHEMISTRY** **Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements.**

a. (1) If the symbol of a substance is Pb, then it is lead.
(2) If a substance is lead, then its atomic number is 82.

Let p, q, and r represent the parts of the statements.

p: the symbol of a substance is Pb

q: it is lead

r: the atomic number is 82

Statement (1): $p \rightarrow q$

Statement (2): $q \rightarrow r$

Since the given statements are true, use the Law of Syllogism to conclude $p \rightarrow r$. That is, *If the symbol of a substance is Pb, then its atomic number is 82.*

b. (1) Water can be represented by H_2O.
(2) Hydrogen (H) and oxygen (O) are in the atmosphere.

There is no valid conclusion. While both statements are true, the conclusion of each statement is not used as the hypothesis of the other.

CHECK Your Progress

2A. (1) If you stand in line, then you will get to ride the new roller coaster.
(2) If you are at least 48 inches tall, you will get to ride the new roller coaster.

2B. (1) If a polygon has six congruent sides, then it is a regular hexagon.
(2) If a regular hexagon has a side length of 3 units, then the perimeter is 3(6) or 18 units.

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EXAMPLE 3 Analyze Conclusions

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

(1) Vertical angles are congruent.

(2) If two angles are congruent, then their measures are equal.

(3) If two angles are vertical, then their measures are equal.

p: two angles are vertical

q: they are congruent

r: their measures are equal

Statement (3) is a valid conclusion by the Law of Syllogism.

Concepts in Motion
Interactive Lab
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CHECK Your Progress

3. (1) The length of a side of square A is the same as the length of a side of square B.
(2) If the lengths of the sides of two squares are the same, then the squares have the same perimeter.
(3) Square A and square B have the same perimeter.

CHECK Your Understanding

Example 1 (p. 99)

Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

If two angles are vertical angles, then they are congruent.

1. Given: $\angle A$ and $\angle B$ are vertical angles.
Conclusion: $\angle A \cong \angle B$

2. Given: $\angle C \cong \angle D$
Conclusion: $\angle C$ and $\angle D$ are vertical angles.

Example 2 (p. 100)

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it. If not, write *no conclusion*.

3. If you are 18 years old, you can vote.
You can vote.

4. The midpoint divides a segment into two congruent segments.
If two segments are congruent, then their measures are equal.

Example 3 (p. 101)

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

5. (1) If Molly arrives at school at 7:30 A.M., she will get help in math.
(2) If Molly gets help in math, then she will pass her math test.
(3) If Molly arrives at school at 7:30 A.M., then she will pass her math test.

6. (1) Right angles are congruent.
(2) $\angle X \cong \angle Y$
(3) $\angle X$ and $\angle Y$ are right angles.

INSURANCE For Exercises 7 and 8, use the following information.
An insurance company advertised the following monthly rates for life insurance.

If you are a:	Premium for $30,000 Coverage	Premium for $50,000 Coverage
Female, age 35	$14.35	$19.00
Male, age 35	$16.50	$21.63
Female, age 45	$21.63	$25.85
Male, age 45	$23.75	$28.90

7. If Marisol is 35 years old and she wants to purchase $30,000 of insurance from this company, then what is her premium?

8. Terry paid $21.63 for life insurance. Can you conclude that Terry is 35? Explain.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–16	1
17–20	2
21–26	3

For Exercises 9–16, determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

If two numbers are odd, then their sum is even.

9. **Given:** The sum of two numbers is 22.
Conclusion: The two numbers are odd.

10. **Given:** The numbers are 5 and 7.
Conclusion: The sum is even.

11. **Given:** 11 and 23 are added together.
Conclusion: The sum of 11 and 23 is even.

12. **Given:** The numbers are 2 and 6.
Conclusion: The sum is odd.

If three points are noncollinear, then they determine a plane.

13. **Given:** A, B, and C are noncollinear.
Conclusion: A, B, and C determine a plane.

14. **Given:** E, F, and G lie in plane M.
Conclusion: E, F, and G are noncollinear.

15. **Given:** P and Q lie on a line.
Conclusion: P and Q determine a plane.

16. **Given:** $\triangle XYZ$
Conclusion: X, Y, and Z determine a plane.

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it. If not, write *no conclusion*.

17. If you interview for a job, then you wear a suit.
If you interview for a job, then you will be offered that job.

18. If the measure of an angle is less than 90, then it is acute.
If an angle is acute, then it is not obtuse.

19. If X is the midpoint of segment YZ, then $YX = XZ$.
If the measures of two segments are equal, then they are congruent.

20. If two lines intersect to form a right angle, then they are perpendicular.
Lines ℓ and m are perpendicular.

Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

21. (1) Ballet dancers like classical music.
(2) If you like classical music, then you enjoy the opera.
(3) If you are a ballet dancer, then you enjoy the opera.

22. (1) If the measure of an angle is greater than 90, then it is obtuse.
(2) $m\angle ABC > 90$
(3) $\angle ABC$ is obtuse.

23. (1) Vertical angles are congruent.
(2) $\angle 3 \cong \angle 4$
(3) $\angle 3$ and $\angle 4$ are vertical angles.

24. (1) If an angle is obtuse, then it cannot be acute.
(2) $\angle A$ is obtuse.
(3) $\angle A$ cannot be acute.

25. (1) If you drive safely, then you can avoid accidents.
(2) Tika drives safely.
(3) Tika can avoid accidents.

26. (1) If you are athletic, then you enjoy sports.
(2) If you are competitive, then you enjoy sports.
(3) If you are competitive, then you are athletic.

Real-World Link

Carly Patterson was the first American gymnast to win the all-around competition since Mary Lou Retton 20 years before.

Source: www.dallas.about.com

27. LITERATURE John Steinbeck, a Pulitzer Prize-winning author, lived in Monterey, California, for part of his life. In 1945, he published the book, *Cannery Row*, about many of his working-class heroes from Monterey. If you visited Cannery Row in Monterey during the 1940s, then you could hear the grating noise of the fish canneries. Write a valid conclusion to the hypothesis *If John Steinbeck lived in Monterey in 1941, ...*

28. SPORTS In the 2004 Summer Olympics, gymnast Carly Patterson won the gold medal in the women's individual all-around competition. Use the two true conditional statements to reach a valid conclusion about the 2004 competition.

(1) If the sum of Carly Patterson's individual scores is greater than the rest of the competitors, then she wins the competition.

(2) If a gymnast wins the competition, then she earns a gold medal.

EXTRA PRACTICE

See pages 804, 829.

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H.O.T. Problems

29. OPEN ENDED Write an example to illustrate the correct use of the Law of Detachment.

30. REASONING Explain how the Transitive Property of Equality is similar to the Law of Syllogism.

31. FIND THE ERROR An article in a magazine states that if you get seasick, then you will get dizzy. It also says that if you get seasick, you will get an upset stomach. Suzanne says that this means that if you get dizzy, then you will get an upset stomach. Lakeisha says that she is wrong. Who is correct? Explain.

32. CHALLENGE Suppose all triangles that satisfy Property B satisfy the Pythagorean Theorem. Is the following statement *true* or *false*? Justify your answer using what you have learned in Lessons 2-3 and 2-4.
A triangle that is not a right triangle does not satisfy Property B.

33. *Writing in Math* Refer to page 99. Explain how a doctor uses deductive reasoning to diagnose an illness, such as strep throat or chickenpox.

STANDARDIZED TEST PRACTICE

34. Determine which statement follows logically from the given statements.

If you order two burritos, you also get nachos.

Michael ordered two burritos.

A Michael ordered one burrito.

B Michael will order two burritos.

C Michael ordered nachos and burritos.

D Michael got nachos.

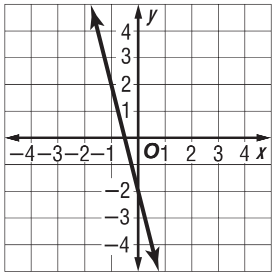
35. **REVIEW** What is the slope of this line?

F $\frac{1}{4}$

G $-\frac{1}{4}$

H 4

J -4



Spiral Review

MARKETING **For Exercises 36–38, use the following information.** (Lesson 2-3)

Marketing executives use if-then statements in advertisements to promote products or services. An ad for a car repair shop reads, *If you're looking for fast and reliable car repair, visit AutoCare.*

36. Write the converse of the conditional.

37. What do you think the advertiser wants people to conclude about AutoCare?

38. Does the advertisement say that AutoCare is fast and reliable?

Construct a truth table for each compound statement. (Lesson 2-2)

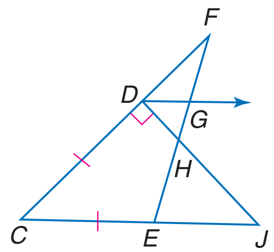
39. $q \wedge r$

40. $\sim p \vee r$

41. $p \wedge (q \vee r)$

42. $p \vee (\sim q \wedge r)$

For Exercises 43–47, refer to the figure at the right. (Lesson 1-5)



43. Which angle is complementary to $\angle FDG$?

44. Name a pair of vertical angles.

45. Name a pair of angles that are noncongruent and supplementary.

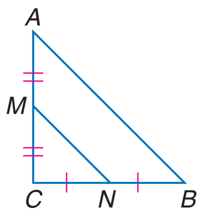
46. Identify $\angle FDH$ and $\angle CDH$ as congruent, adjacent, vertical, complementary, supplementary, and/or a linear pair.

47. Can you assume that $\overline{DC} \cong \overline{CE}$? Explain.

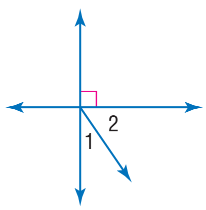
GET READY for the Next Lesson

PREREQUISITE SKILL **Write what you can assume about the segments or angles listed for each figure.** (Lesson 1-5)

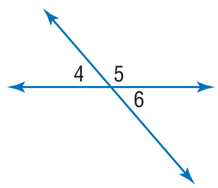
48. $\overline{AM}, \overline{CM}, \overline{CN}, \overline{BN}$



49. $\angle 1, \angle 2$



50. $\angle 4, \angle 5, \angle 6$



2-5 Postulates and Paragraph Proofs

Main Ideas

- Identify and use basic postulates about points, lines, and planes.
- Write paragraph proofs.

New Vocabulary

postulate
axiom
theorem
proof
paragraph proof
informal proof

GET READY for the Lesson

U.S. Supreme Court Justice William Douglas stated, "The First Amendment makes confidence in the common sense of our people and in the maturity of their judgment the great postulate of our democracy." The writers of the constitution assumed that citizens would act and speak with common sense and maturity. Some statements in geometry also must be assumed or accepted as true.

Points, Lines, and Planes A **postulate** or **axiom** is a statement that is accepted as true. In Chapter 1, you studied basic ideas about points, lines, and planes. These ideas can be stated as postulates.

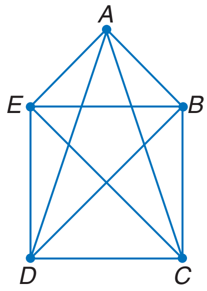
POSTULATES	
2.1	Through any two points, there is exactly one line.
2.2	Through any three points not on the same line, there is exactly one plane.

Real-World EXAMPLE Points and Lines

1 **COMPUTERS** **Each of five computers needs to be connected to every other computer. How many connections need to be made?**

Explore There are five computers, and each is connected to four others.

Plan Draw a diagram to illustrate the solution.



Solve Let noncollinear points A, B, C, D, and E represent the five computers. Connect each point with every other point.

Between every two points there is exactly one segment. So, the connection between computer A and computer B is the same as between computer B and computer A. For the five points, ten segments can be drawn.

Check $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{BC}$, $\overline{BD}$, $\overline{BE}$, $\overline{CD}$, $\overline{CE}$, and $\overline{DE}$ each represent a connection. So, there will be ten connections in all.

CHECK Your Progress

1. Determine the number of segments that can connect 4 points.

Study Tip

Drawing Diagrams

When listing segments, start with one vertex and draw all of the segments from that vertex. Then move on to the other vertices until all possible segments have been drawn.

Other postulates are based on relationships among points, lines, and planes.

POSTULATES

2.3 A line contains at least two points.

2.4 A plane contains at least three points not on the same line.

2.5 If two points lie in a plane, then the entire line containing those points lies in that plane.

2.6 If two lines intersect, then their intersection is exactly one point.

2.7 If two planes intersect, then their intersection is a line.

EXAMPLE 2 Use Postulates

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

a. If points *A*, *B*, and *C* lie in plane *M*, then they are collinear.

Sometimes; *A*, *B*, and *C* do not have to be collinear to lie in plane *M*.

b. There is exactly one plane that contains noncollinear points *P*, *Q*, and *R*.

Always; Postulate 2.2 states that through any three noncollinear points, there is exactly one plane.

c. There are at least two lines through points *M* and *N*.

Never; Postulate 2.1 states that through any two points, there is exactly one line.

CHECK Your Progress

2. If two coplanar lines intersect, then the point of intersection lies in the same plane as the two lines.

Paragraph Proofs Undefined terms, definitions, postulates, and algebraic properties of equality are used to prove that other statements or conjectures are true. Once a statement or conjecture has been shown to be true, it is called a **theorem**, and it can be used to justify that other statements are true.

You will study and use various methods to verify or prove statements and conjectures in geometry. A **proof** is a logical argument in which each statement you make is supported by a statement that is accepted as true. One type of proof is called a **paragraph proof** or **informal proof**. In this type of proof, you write a paragraph to explain why a conjecture for a given situation is true.

Study Tip

Axiomatic System

An **axiomatic system** is a set of axioms, from which some or all axioms can be used together to logically derive theorems.

KEY CONCEPT

Five essential parts of a good proof:

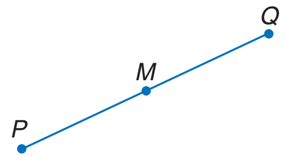
- State the theorem or conjecture to be proven.
- List the given information.
- If possible, draw a diagram to illustrate the given information.
- State what is to be proved.
- Develop a system of deductive reasoning.

EXAMPLE 3 Write a Paragraph Proof

Given that M is the midpoint of $\overline{PQ}$, write a paragraph proof to show that $\overline{PM} \cong \overline{MQ}$.

Given: M is the midpoint of $\overline{PQ}$.

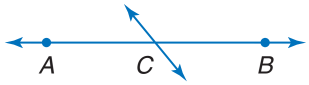
Prove: $\overline{PM} \cong \overline{MQ}$



From the definition of midpoint of a segment, $PM = MQ$. This means that $\overline{PM}$ and $\overline{MQ}$ have the same measure. By the definition of congruence, if two segments have the same measure, then they are congruent. Thus, $\overline{PM} \cong \overline{MQ}$.

CHECK Your Progress

3. Given that $\overline{AC} \cong \overline{CB}$, and C is between A and B, write a paragraph proof to show that C is the midpoint of $\overline{AB}$.



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> **Study Tip**
>
> **Proofs**
>
> Before writing a proof, you should have a plan. One strategy is to *work backward*. Start with what you want to prove, and work backward step by step until you reach the given information.

Once a conjecture has been proven true, it can be stated as a theorem and used in other proofs. The conjecture in Example 3 is the Midpoint Theorem.

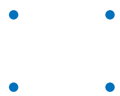
THEOREM 2.1 ***Midpoint Theorem***

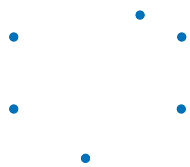
If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

CHECK Your Understanding

Example 1 (p. 105)

Determine the number of segments that can be drawn connecting each set of points.

1. 

2. 

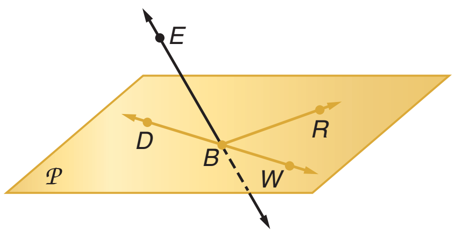
3. **DANCING** Six students will dance at the opening of a new community center. The students, each connected to each of the other students with wide colored ribbons, will move in a circular motion. How many ribbons are needed?

Example 2 (p. 106)

4. Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain. *The intersection of three planes is two lines.*

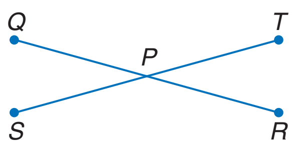
Example 3 (p. 107)

In the figure at the right, $\overleftrightarrow{BD}$ and $\overleftrightarrow{BR}$ are in plane $\mathcal{P}$, and W is on $\overleftrightarrow{BD}$. State the postulate that can be used to show each statement is true.



5. B, D, and W are collinear.
6. E, B, and R are coplanar.

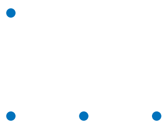
7. **PROOF** In the figure at the right, P is the midpoint of $\overline{QR}$ and $\overline{ST}$, and $\overline{QR} \cong \overline{ST}$. Write a paragraph proof to show that $PQ = PT$.

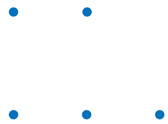


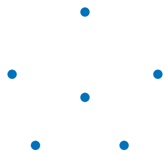
Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–10	1
11–14	2
15, 16	3

Determine the number of segments that can be drawn connecting each set of points.

8. 

9. 

10. 

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

11. Three points determine a plane.

12. Points G and H are in plane X. Any point collinear with G and H is in plane X.

13. The intersection of two planes can be a point.

14. Points S, T, and U determine three lines.

15. **PROOF** Point C is the midpoint of $\overline{AB}$ and B is the midpoint of $\overline{CD}$. Prove that $\overline{AC} \cong \overline{BD}$.

16. **PROOF** Point L is the midpoint of $\overline{JK}$. $\overline{JK}$ intersects $\overline{MK}$ at K. If $\overline{MK} \cong \overline{JL}$, prove that $\overline{LK} \cong \overline{MK}$.

Real-Life Career

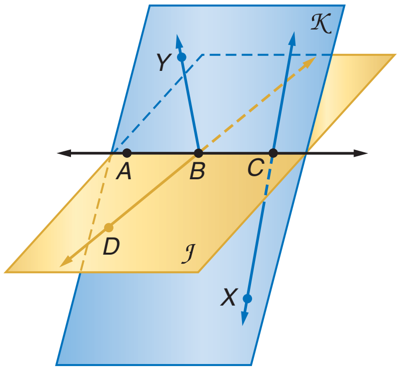
Detective

A police detective gathers facts and collects evidence for use in criminal cases. The facts and evidence are used together to prove a suspect's guilt in court.

Math Online

For more information, go to geometryonline.com.

In the figure at the right, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ lie in plane $\mathcal{J}$, and $\overleftrightarrow{BY}$ and $\overleftrightarrow{CX}$ lie in plane $\mathcal{K}$. State the postulate that can be used to show each statement is true.



17. C and D are collinear.

18. $\overleftrightarrow{XB}$ lies in plane $\mathcal{K}$.

19. Points A, C, and X are coplanar.

20. $\overleftrightarrow{AD}$ lies in plane $\mathcal{J}$.

21. **CAREERS** Many professions use deductive reasoning and paragraph proofs. For example, a police officer uses deductive reasoning investigating a traffic accident and then writes the findings in a report. List a profession, and describe how it can use paragraph proofs.

22. **MODELING** Isabel's teacher asked her to make a model showing the number of lines and planes formed from four points that are noncollinear and noncoplanar. Isabel decided to make a mobile of straws, pipe cleaners, and colored sheets of tissue paper. She plans to glue the paper to the straws and connect the straws together to form a group of connected planes. How many planes and lines will she have?

H.O.T. Problems

23. **REASONING** Explain how deductive reasoning is used in a proof. List the types of reasons that can be used for justification in a proof.

24. **OPEN ENDED** Draw figures to illustrate Postulates 2.6 and 2.7.

EXTRA PRACTICE
See pages 803, 829.
Math Online
Self-Check Quiz at
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25. Which One Doesn't Belong? Identify the term that does not belong with the other three. Explain your reasoning.

postulate	conjecture	theorem	axiom

26. CHALLENGE Three noncollinear points lie in a single plane. In Exercise 22, you found the number of planes defined by four noncollinear points. What are the least and greatest number of planes defined by five noncollinear points?

27. *Writing in Math* Refer to page 105. Describe how postulates are used in literature. Include an example of a postulate in historic United States' documents.

STANDARDIZED TEST PRACTICE

28. Which statement *cannot* be true?

A Three noncollinear points determine a plane.

B Two lines intersect at exactly one point.

C At least two lines can contain the same two points.

D A midpoint divides a segment into two congruent segments.

29. REVIEW Which is one of the solutions to the equation $3x^2 - 5x + 1 = 0$?

F $\frac{5 + \sqrt{13}}{6}$

G $\frac{-5 - \sqrt{13}}{6}$

H $\frac{5}{6} - \sqrt{13}$

J $-\frac{5}{6} + \sqrt{13}$

Spiral Review

30. Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*. (Lesson 2-4)
(1) Part-time jobs require 20 hours of work per week.
(2) Diego has a part-time job.
(3) Diego works 20 hours per week.

31. Write the converse, inverse, and contrapositive of the conditional statement. Determine whether the related conditional is *true* or *false*. If a statement is false, find a counterexample. *If you have access to the Internet at your house, then you have a computer.* (Lesson 2-3)

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** (Pages 781–782)

32. $m - 17 = 8$ **33.** $3y = 57$ **34.** $\frac{y}{6} + 12 = 14$ **35.** $-t + 3 = 27$

CHAPTER 2 Mid-Chapter Quiz

Lessons 2-1 through 2-5

Determine whether each conjecture is *true* or *false*. Give a counterexample for any *false* conjecture. (Lesson 2-1)

1. **Given:** $WX = XY$
   Conjecture: W, X, Y are collinear.
2. **Given:** $\angle 1$ and $\angle 2$ are not complementary. $\angle 2$ and $\angle 3$ are complementary.
   Conjecture: $m\angle 1 = m\angle 3$

3. **PETS** Michaela took a survey of six friends and created the table shown below.

Name	Number of Cats	Number of Dogs	Number of Fish
Kristen	2	0	0
Jorge	0	0	5
Mark	0	2	2
Carissa	1	1	0
Alex	2	1	10
Akilah	1	1	1

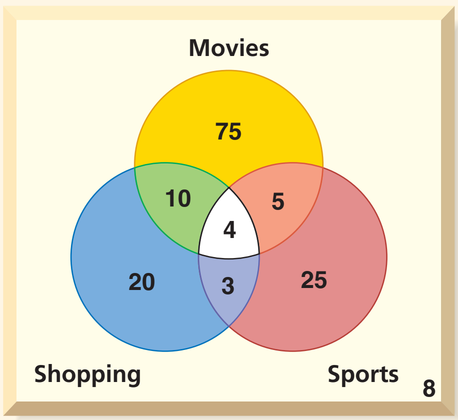
Michaela reached the following conclusion. *If a person has 3 or more pets, then they have a dog*. Is this conclusion valid? If not, find a counterexample. (Lesson 2-1)

Construct a truth table for each compound statement. (Lesson 2-2)

4. $\sim p \wedge q$

5. $p \vee (q \wedge r)$

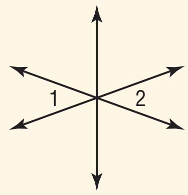
6. **RECREATION** A group of 150 students were asked what they like to do during their free time. How many students like going to the movies or shopping? (Lesson 2-2)



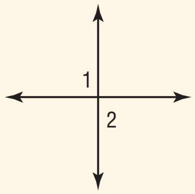
7. **MULTIPLE CHOICE** Which figure can serve as a *counterexample* to the conjecture below?

   If $\angle 1$ and $\angle 2$ share exactly one point, then they are vertical angles.

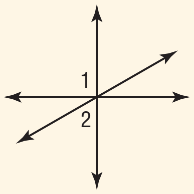
A



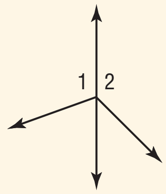
C



B



D



8. Write the converse, inverse, and contrapositive of the following conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample. (Lesson 2-3)

   If two angles are adjacent, then the angles have a common vertex.

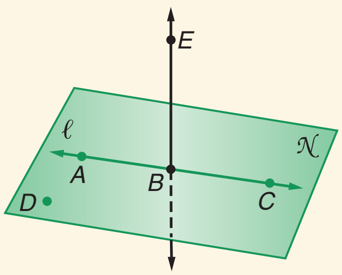
9. Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*. (Lesson 2-4)

   (1) If n is an integer, then n is a real number.

   (2) n is a real number.

   (3) n is an integer.

In the figure below, A, B, and C are collinear. Points A, B, C, and D lie in plane $\mathcal{N}$. State the postulate or theorem that can be used to show each statement is true. (Lesson 2-5)



10. A, B, and D determine plane $\mathcal{N}$.
11. $\overleftrightarrow{BE}$ intersects $\overleftrightarrow{AC}$ at B.
12. ℓ lies in plane $\mathcal{N}$.

2-6 Algebraic Proof

Main Ideas

- Use algebra to write two-column proofs.
- Use properties of equality in geometry proofs.

New Vocabulary

deductive argument
two-column proof
formal proof

GET READY for the Lesson

Lawyers develop their cases using logical arguments based on evidence to lead a jury to a conclusion favorable to their case. At the end of a trial, a lawyer makes closing remarks summarizing the evidence and testimony that they feel proves their case. These closing arguments are similar to a proof in mathematics.

Algebraic Proof Algebra is a system with sets of numbers, operations, and properties that allow you to perform algebraic operations. The following table summarizes several properties of real numbers that you studied in algebra.

Study Tip

Commutative and Associative Properties

Throughout this text, we shall assume the Commutative and Associative Properties for addition and multiplication.

CONCEPT SUMMARY — *Properties of Real Numbers*

The following properties are true for any numbers a, b, and c.

Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Addition and Subtraction Properties	If $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties	If $a = b$, then $a \cdot c = b \cdot c$ and if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

The properties of equality can be used to justify each step when solving an equation. A group of algebraic steps used to solve problems form a **deductive argument**.

EXAMPLE 1 Verify Algebraic Relationships

Solve $3(x - 2) = 42$. Justify each step.

Algebraic Steps	Properties
$3(x - 2) = 42$	Original equation
$3x - 6 = 42$	Distributive Property
$3x - 6 + 6 = 42 + 6$	Addition Property
$3x = 48$	Substitution Property
$\frac{3x}{3} = \frac{48}{3}$	Division Property
$x = 16$	Substitution Property

CHECK Your Progress

1. Solve $2x + 3 = 5$. Justify each step.

Study Tip

Mental Math

If your teacher permits you to do so, some steps may be eliminated by performing mental calculations. For example, in Example 2, statements 4 and 6 could be omitted. Then the reason for statements 5 and 7 would be Addition Property and Division Property, respectively.

Example 1 is a proof of the conditional statement *If* $3(x - 2) = 42$, *then* $x = 16$. Notice that the column on the left is a step-by-step process that leads to a solution. The column on the right contains the reason for each statement. In geometry, a similar format is used to prove conjectures and theorems. A **two-column proof**, or **formal proof**, contains *statements* and *reasons* organized in two columns.

EXAMPLE 2 Write a Two-Column Proof

Write a two-column proof to show that if $3\left(x - \frac{5}{3}\right) = 1$, then $x = 2$.

Statements	Reasons
1. $3\left(x - \frac{5}{3}\right) = 1$	**1.** Given
2. $3x - 3\left(\frac{5}{3}\right) = 1$	**2.** Distributive Property
3. $3x - 5 = 1$	**3.** Substitution
4. $3x - 5 + 5 = 1 + 5$	**4.** Addition Property
5. $3x = 6$	**5.** Substitution
6. $\frac{3x}{3} = \frac{6}{3}$	**6.** Division Property
7. $x = 2$	**7.** Substitution

CHECK Your Progress

2. The Pythagorean Theorem states that in a right triangle ABC, $c^2 = a^2 + b^2$. Write a two-column proof to show that $a = \sqrt{c^2 - b^2}$.

Geometric Proof Segment measures and angle measures are real numbers, so properties of real numbers can be used to discuss their relationships.

Property	Segments	Angles
Reflexive	$AB = AB$	$m\angle 1 = m\angle 1$
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

STANDARDIZED TEST EXAMPLE Justify Geometric Relationships

3 If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then which of the following is a valid conclusion?

I $AB = CD$ and $CD = EF$

II $\overline{AB} \cong \overline{EF}$

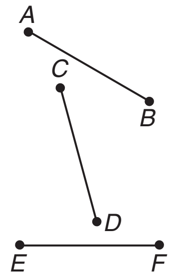
III $AB = EF$

A I only

B I and II

C I and III

D I, II, and III



Test-Taking Tip

Analyzing Statements
More than one statement may be correct. Work through each problem completely before indicating your answer.

Read the Test Item

Determine whether the statements are true based on the given information.

Solve the Test Item

Statement I: Examine the given information, $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$. From the definition of congruent segments, $AB = CD$ and $CD = EF$. Thus, Statement I is true.

Statement II: By the definition of congruent segments, if $AB = EF$, then $\overline{AB} \cong \overline{EF}$. Statement II is true also.

Statement III: If $AB = CD$ and $CD = EF$, then $AB = EF$ by the Transitive Property. Thus, Statement III is true.

Because Statements I, II, and III are true, choice D is correct.

CHECK Your Progress

3. If $m\angle 1 = m\angle 2$ and $m\angle 2 = 90$, then which of the following is a valid conclusion?

F $m\angle 1 = 45$

G $m\angle 1 = 90$

H $m\angle 1 + m\angle 2 = 180$

J $m\angle 1 + m\angle 2 = 90$

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In Example 3, each conclusion was justified using a definition or property. This process is used in geometry to verify and prove statements.

EXAMPLE Geometric Proof

4 **TIME** **On a clock, the angle formed by the hands at 2:00 is a 60° angle. If the angle formed at 2:00 is congruent to the angle formed at 10:00, prove that the angle at 10:00 is a 60° angle.**

Given: $m\angle 2 = 60$
$\angle 2 \cong \angle 10$

Prove: $m\angle 10 = 60$

(continued on the next page)

Extra Examples at geometryonline.com

Aaron Haupt

Proof:

Statements	Reasons
1. $m\angle 2 = 60$; $\angle 2 \cong \angle 10$	**1.** Given
2. $m\angle 2 = m\angle 10$	**2.** Definition of congruent angles
3. $60 = m\angle 10$	**3.** Substitution
4. $m\angle 10 = 60$	**4.** Symmetric Property

CHECK Your Progress

4. It is given that $\angle A$ and $\angle B$ are congruent. The measure of $\angle A$ is 110. Write a two-column proof to show that the measure of $\angle B$ is 110.

CHECK Your Understanding

Example 1 (p. 112)

State the property that justifies each statement.

1. If $\frac{x}{2} = 7$, then $x = 14$.

2. If $x = 5$ and $b = 5$, then $x = b$.

3. If $XY - AB = WZ - AB$, then $XY = WZ$.

Example 2 (p. 112)

4. Complete the following proof.

Given: $5 - \frac{2}{3}x = 1$

Prove: $x = 6$

Proof:

Statements	Reasons
a. ___?___	**a.** Given
b. $3\left(5 - \frac{2}{3}x\right) = 3(1)$	**b.** ___?___
c. $15 - 2x = 3$	**c.** ___?___
d. ___?___	**d.** Subtraction Property
e. $x = 6$	**e.** ___?___

Example 3 (p. 113)

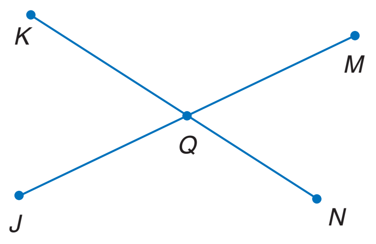
5. MULTIPLE CHOICE If $\overline{JM}$ and $\overline{KN}$ intersect at Q to form $\angle JQK$ and $\angle MQN$, which of the following is *not* a valid conclusion?

A $\angle JQK$ and $\angle MQN$ are vertical angles.

B $\angle JQK$ and $\angle MQN$ are supplementary.

C $\angle JQK \cong \angle MQN$

D $m\angle JQK = m\angle MQN$



Examples 2 and 4 (pp. 112–114)

PROOF **Write a two-column proof for each conditional.**

6. If $25 = -7(y - 3) + 5y$, then $-2 = y$.

7. If rectangle $ABCD$ has side lengths $AD = 3$ and $AB = 10$, then $AC = BD$.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–13	1
14–17	3
18–23	2
24, 25	4

State the property that justifies each statement.

8. If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, $m\angle A = m\angle C$.

9. If $HJ + 5 = 20$, then $HJ = 15$.

10. If $XY + 20 = YW$ and $XY + 20 = DT$, then $YW = DT$.

11. If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 90$.

12. If $\frac{1}{2}AB = \frac{1}{2}EF$, then $AB = EF$.

13. $AB = AB$

14. If $2\left(x - \frac{3}{2}\right) = 5$, then $2x - 3 = 5$.

15. If $m\angle 4 = 35$ and $m\angle 5 = 35$, then $m\angle 4 = m\angle 5$.

16. If $\frac{1}{2}AB = \frac{1}{2}CD$, then $AB = CD$.

17. If $EF = GH$ and $GH = JK$, then $EF = JK$.

Complete each proof.

18. Given: $\frac{3x + 5}{2} = 7$

Prove: $x = 3$

Proof:

Statements	Reasons
a. $\frac{3x + 5}{2} = 7$	**a.** ___?___
b. ___?___	**b.** Multiplication Property
c. $3x + 5 = 14$	**c.** ___?___
d. $3x = 9$	**d.** ___?___
e. ___?___	**e.** Division Property

19. Given: $2x - 7 = \frac{1}{3}x - 2$

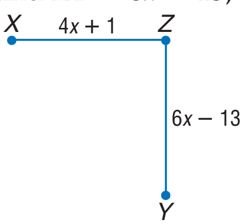
Prove: $x = 3$

Proof:

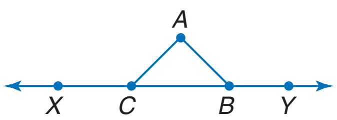
Statements	Reasons
a. ___?___	**a.** Given
b. ___?___	**b.** Multiplication Property
c. $6x - 21 = x - 6$	**c.** ___?___
d. ___?___	**d.** Subtraction Property
e. $5x = 15$	**e.** ___?___
f. ___?___	**f.** Division Property

PROOF **Write a two-column proof.**

20. If $XZ = ZY$, $XZ = 4x + 1$, and $ZY = 6x - 13$, then $x = 7$.



21. If $m\angle ACB = m\angle ABC$, then $\angle XCA \cong \angle YBA$.



EXTRA PRACTICE
See pages 804, 829.

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Real-World Link

The Formula Society of Automotive Engineers at University of California, Berkeley, holds a competition each year for the design and construction of a race car. The cars are judged on many factors, including acceleration.

Source: www.dailycal.org

PROOF Write a two-column proof.

22. If $-\frac{1}{2}m = 9$, then $m = -18$.

23. If $5 - \frac{2}{3}z = 1$, then $z = 6$.

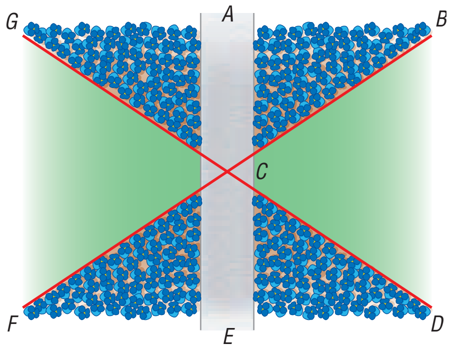
24. If $4 - \frac{1}{2}a = \frac{7}{2} - a$, then $a = -1$.

25. If $-2y + \frac{3}{2} = 8$, then $y = -\frac{13}{4}$.

26. **PHYSICS** Acceleration, distance traveled, velocity, and time are related in the formula $d = vt + \frac{1}{2}at^2$. Solve for a and justify each step.

27. **CHEMISTRY** The Ideal Gas law is given by the formula $PV = nRT$, where P = pressure, V = volume, n = the amount of a substance, R is a constant value, and T is the temperature. Solve the formula for T and justify each step.

28. **GARDENING** In the arrangement of pansies shown, the walkway divides the two sections of pansies into four beds of the same size. If $m\angle ACB = m\angle DCE$, what could you conclude about the relationship among $\angle ACB$, $\angle DCE$, $\angle ECF$, and $\angle ACG$?

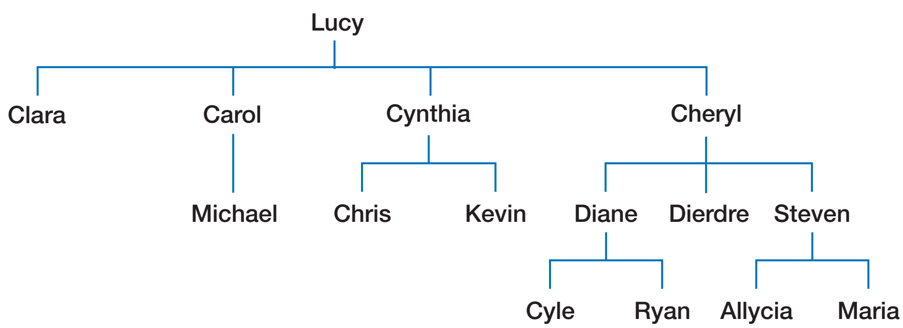


H.O.T. Problems

29. **OPEN ENDED** Write a statement that illustrates the Substitution Property of Equality.

30. **REASONING** Compare one part of a conditional to the *Given* statement of a proof. What part is related to the *Prove* statement?

31. **CHALLENGE** Below is a family tree of the Gibbs family. Clara, Carol, Cynthia, and Cheryl are all daughters of Lucy. Because they are sisters, they have a transitive and symmetric relationship. That is, Clara is a sister of Carol, Carol is a sister of Cynthia, so Clara is a sister of Cynthia.



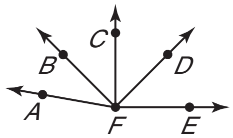
What other relationships in a family have reflexive, symmetric, or transitive relationships? Explain why. Remember that the child or children of each person are listed beneath that person's name. Consider relationships such as first cousin, ancestor or descendent, aunt or uncle, sibling, or any other relationship.

32. ***Writing in Math*** Compare proving a theorem of mathematics to proving a case in a court of law. Include a description of how evidence is used to influence jurors' conclusions in court and a description of the evidence used to make conclusions in mathematics.

Berkeley Formula SAE

STANDARDIZED TEST PRACTICE

33. In the diagram below, $m\angle CFE = 90°$ and $\angle AFB \cong \angle CFD$.



Which of the following conclusions does not have to be true?

A $m\angle BFD = m\angle BFD$

B $\overline{BF}$ bisects $\angle BFD$.

C $m\angle CFD = m\angle AFB$

D $\angle CFE$ is a right angle.

34. REVIEW Which expression can be used to find the values of $s(n)$ in the table?

n	−8	−4	−1	0	1
$s(n)$	1.00	2.00	2.75	3.00	3.25

F $-n + 7$

G $-2n + 3$

H $\frac{1}{2}n + 5$

J $\frac{1}{4}n + 3$

Spiral Review

35. CONSTRUCTION There are four buildings on the Medfield High School Campus, no three of which stand in a straight line. How many sidewalks need to be built so that each building is directly connected to every other building? (Lesson 2-5)

Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.
A number is divisible by 3 if it is divisible by 6. (Lesson 2-4)

36. Given: 24 is divisible by 6. Conclusion: 24 is divisible by 3.

37. Given: 27 is divisible by 3. Conclusion: 27 is divisible by 6.

38. Given: 85 is not divisible by 3. Conclusion: 85 is not divisible by 6.

Write each statement in if-then form. (Lesson 2-3)

39. "He that can have patience can have what he will." (Benjamin Franklin)

40. "To be without some of the things you want is an indispensable part of happiness." (Bertrand Russell)

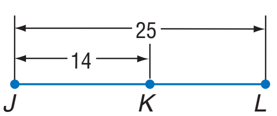
41. "Respect yourself and others will respect you." (Confucius)

42. "A fanatic is one who can't change his mind and won't change the subject." (Sir Winston Churchill)

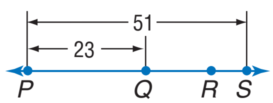
GET READY for the Next Lesson

PREREQUISITE SKILL Find the measure of each segment. (Lesson 1-2)

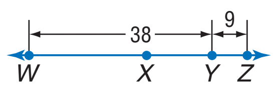
43. $\overline{KL}$



44. $\overline{QS}$



45. $\overline{WZ}$



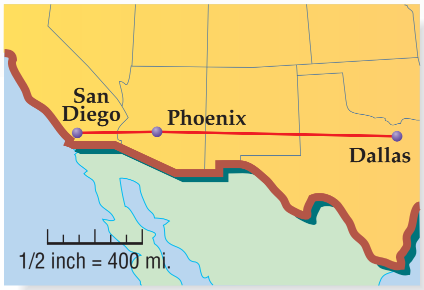
2-7 Proving Segment Relationships

Main Ideas

- Write proofs involving segment addition.
- Write proofs involving segment congruence.

GET READY for the Lesson

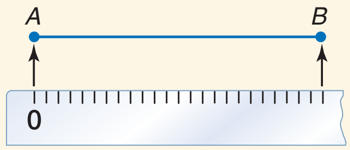
When leaving San Diego, the pilot said that the flight would be about 360 miles to Phoenix. When the plane left Phoenix, the pilot said that the flight would be about 1070 miles to Dallas. Distances in a straight line on a map are sometimes measured with a ruler.



Segment Addition In Lesson 1-2, you measured segments with a ruler by placing the mark for zero on one endpoint, then finding the distance to the other endpoint. This illustrates the **Ruler Postulate**.

POSTULATE 2.8 *Ruler Postulate*

The points on any line or line segment can be paired with real numbers so that, given any two points *A* and *B* on a line, *A* corresponds to zero, and *B* corresponds to a positive real number.



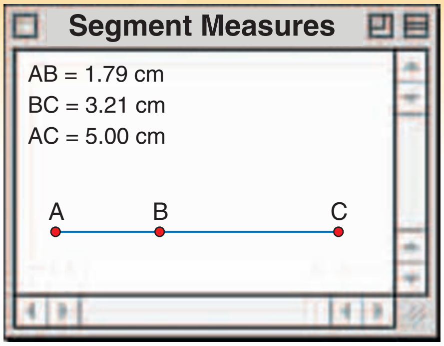
The Ruler Postulate can be used to further investigate line segments.

GEOMETRY LAB

Adding Segment Measures

CONSTRUCT A FIGURE

- Use The Geometer's Sketchpad to construct $\overline{AC}$.
- Place point *B* on $\overline{AC}$.
- Find *AB*, *BC*, and *AC*.



ANALYZE THE MODEL

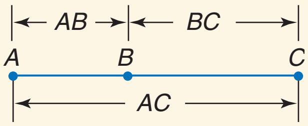
1. What is the sum $AB + BC$?
2. Move *B*. Find *AB*, *BC*, and *AC*. What is the sum of $AB + BC$?
3. Repeat step 2 three times. Record your results.
4. What is true about the relationship of *AB*, *BC*, and *AC*?
5. Is it possible to place *B* on $\overline{AC}$ so that this relationship is not true?

The Geometry Lab suggests the following postulate.

POSTULATE 2.9 — ***Segment Addition Postulate***

If A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C.



EXAMPLE 1 Proof With Segment Addition

Prove the following.

Given: $PQ = RS$

Prove: $PR = QS$

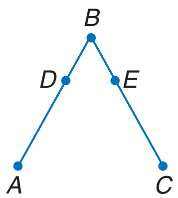
Proof:

Statements	Reasons
1. $PQ = RS$	**1.** Given
2. $PQ + QR = QR + RS$	**2.** Addition Property
3. $PQ + QR = PR$ $QR + RS = QS$	**3.** Segment Addition Postulate
4. $PR = QS$	**4.** Substitution

CHECK Your Progress

1. Given: $\overline{AD} \cong \overline{CE}$, $\overline{DB} \cong \overline{EB}$

Prove: $\overline{AB} \cong \overline{CB}$



Segment Congruence In algebra, you learned about the properties of equality. The Reflexive Property of Equality states that a quantity is equal to itself. The Symmetric Property of Equality states that if $a = b$, then $b = a$. And the Transitive Property of Equality states that for any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$. These properties of equality are similar to the following properties of congruence.

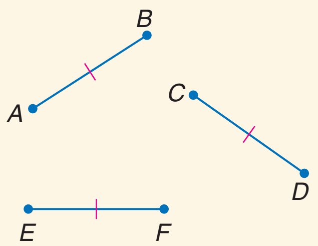
THEOREM 2.2 — ***Segment Congruence***

Congruence of segments is reflexive, symmetric, and transitive.

Reflexive Property $\overline{AB} \cong \overline{AB}$

Symmetric Property If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive Property If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.



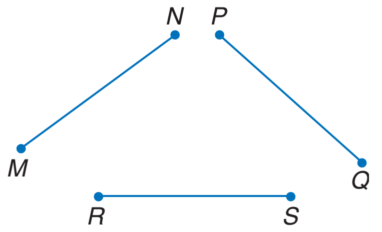
You will prove the first two properties in Exercises 4 and 5.

Vocabulary Link
Symmetric
Everyday Use balanced or proportional
Math Use if $a = b$, then $b = a$

PROOF Transitive Property of Congruence

Given: $\overline{MN} \cong \overline{PQ}$
$\overline{PQ} \cong \overline{RS}$

Prove: $\overline{MN} \cong \overline{RS}$



Proof:

Method 1 Paragraph Proof

Since $\overline{MN} \cong \overline{PQ}$ and $\overline{PQ} \cong \overline{RS}$, $MN = PQ$ and $PQ = RS$ by the definition of congruent segments. By the Transitive Property of Equality, $MN = RS$. Thus, $\overline{MN} \cong \overline{RS}$ by the definition of congruent segments.

Method 2 Two-Column Proof

Statements	Reasons
1. $\overline{MN} \cong \overline{PQ}$, $\overline{PQ} \cong \overline{RS}$	**1.** Given
2. $MN = PQ$, $PQ = RS$	**2.** Definition of congruent segments
3. $MN = RS$	**3.** Transitive Property
4. $\overline{MN} \cong \overline{RS}$	**4.** Definition of congruent segments

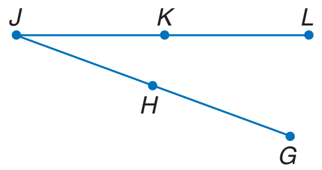
Theorems about congruence can be used to prove segment relationships.

EXAMPLE Proof With Segment Congruence

2 **Prove the following.**

Given: $\overline{JK} \cong \overline{KL}$, $\overline{HJ} \cong \overline{GH}$, $\overline{KL} \cong \overline{HJ}$

Prove: $\overline{GH} \cong \overline{JK}$



Proof:

Method 1 Paragraph Proof

It is given that $\overline{JK} \cong \overline{KL}$ and $\overline{KL} \cong \overline{HJ}$. Thus, $\overline{JK} \cong \overline{HJ}$ by the Transitive Property. It is also given that $\overline{HJ} \cong \overline{GH}$. By the Transitive Property, $\overline{JK} \cong \overline{GH}$. Therefore, $\overline{GH} \cong \overline{JK}$ by the Symmetric Property.

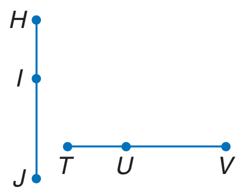
Method 2 Two-Column Proof

Statements	Reasons
1. $\overline{JK} \cong \overline{KL}$, $\overline{KL} \cong \overline{HJ}$	**1.** Given
2. $\overline{JK} \cong \overline{HJ}$	**2.** Transitive Property
3. $\overline{HJ} \cong \overline{GH}$	**3.** Given
4. $\overline{JK} \cong \overline{GH}$	**4.** Transitive Property
5. $\overline{GH} \cong \overline{JK}$	**5.** Symmetric Property

CHECK Your Progress

2. Given: $\overline{HI} \cong \overline{TU}$
$\overline{HJ} \cong \overline{TV}$

Prove: $\overline{IJ} \cong \overline{UV}$



Online **Personal Tutor at** geometryonline.com

CHECK Your Understanding

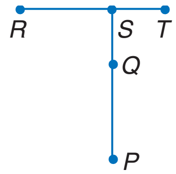
Example 1 (p. 119)

1. Copy and complete the proof.

Given: $\overline{PQ} \cong \overline{RS}$, $\overline{QS} \cong \overline{ST}$

Prove: $\overline{PS} \cong \overline{RT}$

Proof:



Statements	Reasons
a. ____?____, ____?____	**a.** Given
b. $PQ = RS$, $QS = ST$	**b.** ____?____
c. $PS = PQ + QS$, $RT = RS + ST$	**c.** ____?____
d. ____?____	**d.** Substitution Property
e. ____?____	**e.** Substitution Property
f. $\overline{PS} \cong \overline{RT}$	**f.** ____?____

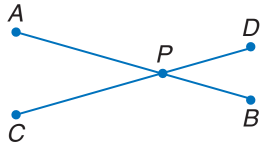
Example 2 (p. 120)

2. PROOF Prove the following.

Given: $\overline{AP} \cong \overline{CP}$

$\overline{BP} \cong \overline{DP}$

Prove: $\overline{AB} \cong \overline{CD}$



Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
3, 6	1
4, 5, 7, 8	2

3. Copy and complete the proof.

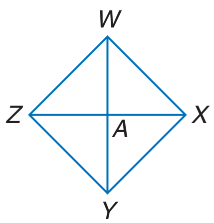
Given: $\overline{WY} \cong \overline{ZX}$

A is the midpoint of $\overline{WY}$.

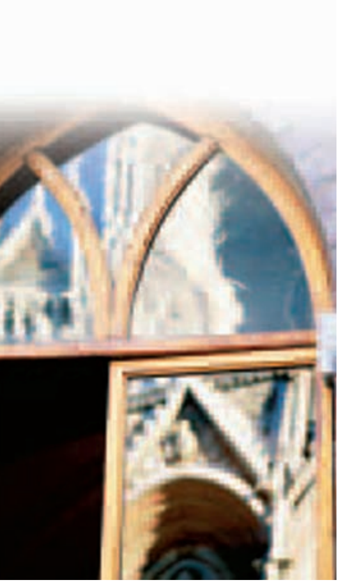
A is the midpoint of $\overline{ZX}$.

Prove: $\overline{WA} \cong \overline{ZA}$

Proof:



Statements	Reasons
a. $\overline{WY} \cong \overline{ZX}$ A is the midpoint of $\overline{WY}$. A is the midpoint of $\overline{ZX}$.	**a.** ____?____
b. $WY = ZX$	**b.** ____?____
c. ____?____	**c.** Def. of midpoint
d. $WY = WA + AY$, $ZX = ZA + AX$	**d.** ____?____
e. $WA + AY = ZA + AX$	**e.** ____?____
f. $WA + WA = ZA + ZA$	**f.** ____?____
g. $2WA = 2ZA$	**g.** ____?____
h. ____?____	**h.** Division Property
i. $\overline{WA} \cong \overline{ZA}$	**i.** ____?____



Real-World Link

Windows come in many different shapes and sizes. Some commonly used shapes for windows are circles, rectangles, squares, triangles, pentagons, and octagons.

Source: www.pella.com

EXTRA PRACTICE

See pages 804, 829.

Math Online

Self-Check Quiz at geometryonline.com

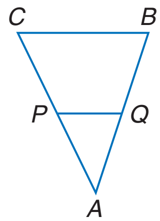
Prove the following.

4. Reflexive Property of Congruence (Theorem 2.2)
5. Symmetric Property of Congruence (Theorem 2.2)

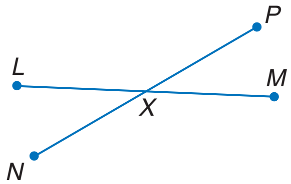
PROOF **Prove the following.**

6. If $AB = BC$, then $AC = 2BC$.

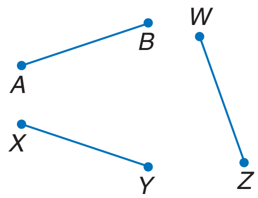
7. If $\overline{AB} \cong \overline{BC}$ and $\overline{PC} \cong \overline{QB}$, then $\overline{AB} \cong \overline{AC}$.



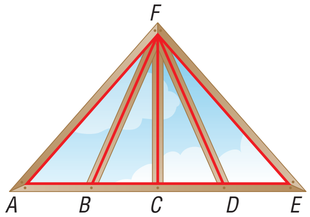
8. If $\overline{LM} \cong \overline{PN}$ and $\overline{XM} \cong \overline{XN}$, then $\overline{LX} \cong \overline{PX}$.



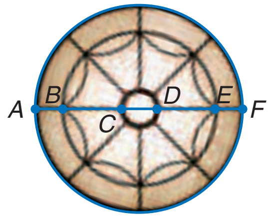
9. If $\overline{XY} \cong \overline{WZ}$ and $\overline{WZ} \cong \overline{AB}$, then $\overline{XY} \cong \overline{AB}$.



10. **DESIGN** The front of a building has a triangular window. If $\overline{AB} \cong \overline{DE}$ and C is the midpoint of $\overline{BD}$, prove that $\overline{AC} \cong \overline{CE}$.



11. **LIGHTING** In the light fixture, $\overline{AB} \cong \overline{EF}$ and $\overline{BC} \cong \overline{DE}$. Prove that $\overline{AC} \cong \overline{DF}$.

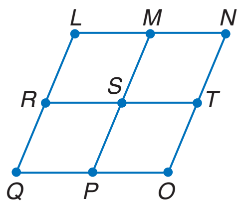


H.O.T. Problems

12. **OPEN ENDED** Draw three congruent segments, and illustrate the Transitive Property using these segments.

13. **REASONING** Choose two cities from a United States road map. Describe the distance between the cities using the Reflexive Property.

14. **CHALLENGE** Given that $\overline{LN} \cong \overline{RT}$, $\overline{RT} \cong \overline{QO}$, $\overline{LQ} \cong \overline{NO}$, $\overline{MP} \cong \overline{NO}$, S is the midpoint of $\overline{RT}$, M is the midpoint of $\overline{LN}$, and P is the midpoint of $\overline{QO}$, list three statements that you could prove using the postulates, theorems, and definitions that you have learned.



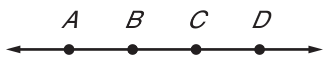
15. ***Writing in Math*** How can segment relationships be used for travel? Include an explanation of how a passenger can use the distances the pilot announced to find the total distance from San Diego to Dallas and an explanation of why the Segment Addition Postulate may or may not be useful when traveling.

STANDARDIZED TEST PRACTICE

16. Which reason can be used to justify Statement 5 in the proof below?

Given: $\overline{AB} \cong \overline{BC}, \overline{BC} \cong \overline{CD}$

Prove: $3AB = AD$



Statements	**Reason**
1. $\overline{AB} \cong \overline{BC}, \overline{BC} \cong \overline{CD}$	**1.** Given
2. $\overline{AB} \cong \overline{CD}$	**2.** ?
3. $AB = BC, BC = CD$	**3.** ?
4. $CD = BC$	**4.** ?
5. $AB + BC + CD = AD$	**5.** ?
6. $AB + AB + AB = AD$	**6.** Subst.
7. $3AB = AD$	**7.** Def. of Mult.

A Angle Addition Postulate

B Segment Congruence

C Segment Addition Postulate

D Midpoint Theorem

17. REVIEW Haru made a scale model of the park near his house. Every inch represents 5 feet. If the main sidewalk in his model is 45 inches long, how long is the actual sidewalk in the park?

F 225 ft

G 125 ft

H 15 ft

J 5 ft

18. REVIEW Which expression is equivalent to $\frac{12x^{-4}}{4x^{-8}}$?

A $\frac{1}{3x^4}$

B $3x^4$

C $8x^2$

D $\frac{x^4}{3}$

Spiral Review

State the property that justifies each statement. (Lesson 2-6)

19. If $m\angle P + m\angle Q = 110$ and $m\angle R = 110$, then $m\angle P + m\angle Q = m\angle R$.

20. If $x(y + z) = a$, then $xy + xz = a$.

21. If $n - 17 = 39$, then $n = 56$.

22. If $cv = md$ and $md = 15$, then $cv = 15$.

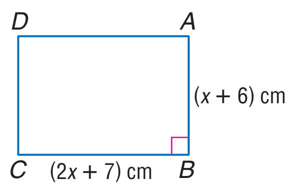
Determine whether each statement is *always*, *sometimes*, or *never* true. Explain. (Lesson 2-5)

23. A midpoint divides a segment into two noncongruent segments.

24. Three lines intersect at a single point.

25. The intersection of two planes forms a line.

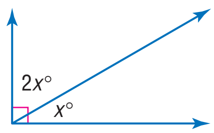
26. If the perimeter of rectangle $ABCD$ is 44 centimeters, find x and the dimensions of the rectangle. (Lesson 1-6)



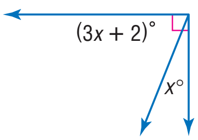
GET READY for the Next Lesson

PREREQUISITE SKILL **Find x.** (Lesson 1-5)

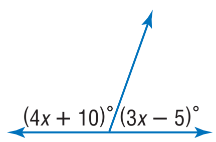
27.



28.



29.



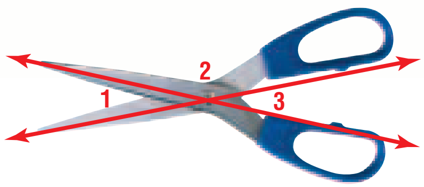
2-8 Proving Angle Relationships

Main Ideas

- Write proofs involving supplementary and complementary angles.
- Write proofs involving congruent and right angles.

GET READY for the Lesson

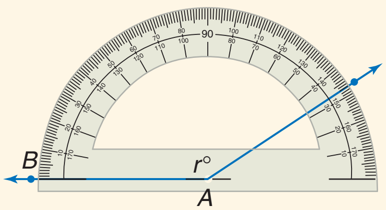
Notice that when a pair of scissors is opened, the angle formed by the two blades, $\angle 1$, and the angle formed by a blade and a handle, $\angle 2$, are a linear pair. Likewise, the angle formed by a blade and a handle, $\angle 2$, and the angle formed by the two handles, $\angle 3$, also forms a linear pair.



Supplementary and Complementary Angles Recall that when you measure angles with a protractor, you position the protractor so that one of the rays aligns with zero degrees and then determine the position of the second ray. To draw an angle of a given measure, align a ray with the zero degree mark and use the desired angle measure to position the second ray. The Protractor Postulate ensures that there is one ray you could draw with a given ray to create an angle with a given measure.

POSTULATE 2.10 ***Protractor Postulate***

Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.

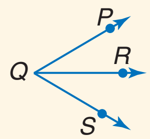


In Lesson 2-7, you learned about the Segment Addition Postulate. A similar relationship exists between the measures of angles.

POSTULATE 2.11 ***Angle Addition Postulate***

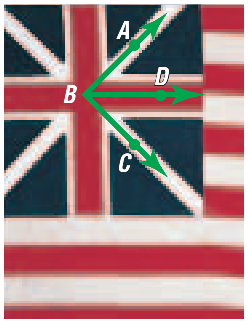
If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$.

If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$.



You can use the Angle Addition Postulate to solve problems involving angle measures.

C Squared Studios/Getty Images



Real-World Link

The Grand Union Flag was the first flag used by the colonial United States that resembles the current flag. The square in the corner resembles the flag of Great Britain.

Source: www.usflag.org

Review Vocabulary

Supplementary Angles two angles with measures that add to 180 (Lesson 1-5)

Complementary Angles two angles with measures that add to 90 (Lesson 1-5)

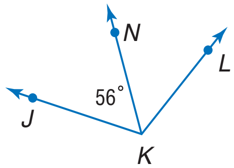
EXAMPLE Angle Addition

1 HISTORY The Grand Union Flag at the left contains several angles. If $m\angle ABD = 44$ and $m\angle ABC = 88$, find $m\angle DBC$.

$m\angle ABD + m\angle DBC = m\angle ABC$	Angle Addition Postulate
$44 + m\angle DBC = 88$	$m\angle ABD = 44$, $m\angle ABC = 88$
$m\angle DBC = 44$	Subtraction Property

CHECK Your Progress

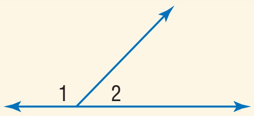
1. Find $m\angle NKL$ if $m\angle JKL = 2m\angle JKN$.



The Angle Addition Postulate can be used with other angle relationships to provide additional theorems relating to angles.

THEOREMS

2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.



$m\angle 1 + m\angle 2 = 180$

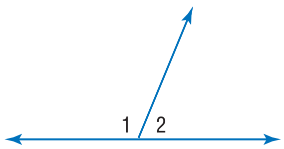
2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

$m\angle 1 + m\angle 2 = 90$

You will prove Theorems 2.3 and 2.4 in Exercises 16 and 17, respectively.

EXAMPLE Supplementary Angles

2 If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 67$, find $m\angle 1$.

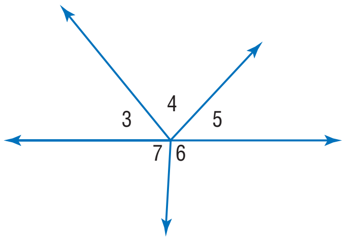


$m\angle 1 + m\angle 2 = 180$	Supplement Theorem
$m\angle 1 + 67 = 180$	$m\angle 2 = 67$
$m\angle 1 = 113$	Subtraction Property

CHECK Your Progress

2A. Find the measures of $\angle 3$, $\angle 4$, and $\angle 5$ if $m\angle 3 = x + 20$, $m\angle 4 = x + 40$, and $m\angle 5 = x + 30$.

2B. If $\angle 6$ and $\angle 7$ form a linear pair and $m\angle 6 = 3x + 32$ and $m\angle 7 = 5x + 12$, find x, $m\angle 6$, and $m\angle 7$.



Extra Examples at geometryonline.com

file photo

Congruent and Right Angles The properties of algebra that applied to the congruence of segments and the equality of their measures also hold true for the congruence of angles and the equality of their measures.

THEOREM 2.5

Congruence of angles is reflexive, symmetric, and transitive.

Reflexive Property $\angle 1 \cong \angle 1$

Symmetric Property If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

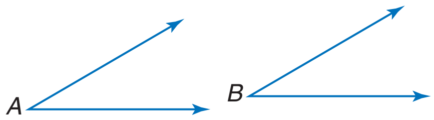
Transitive Property If $\angle 1 \cong \angle 2$, and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

You will prove the Reflexive and Transitive Properties of Angle Congruence in Exercises 18 and 19.

PROOF Symmetric Property of Congruence

Given: $\angle A \cong \angle B$

Prove: $\angle B \cong \angle A$



Method 1

Paragraph Proof:

We are given $\angle A \cong \angle B$. By the definition of congruent angles, $m\angle A = m\angle B$. Using the Symmetric Property, $m\angle B = m\angle A$. Thus, $\angle B \cong \angle A$ by the definition of congruent angles.

Method 2

Two-Column Proof:

Statements	Reasons
1. $\angle A \cong \angle B$	**1.** Given
2. $m\angle A = m\angle B$	**2.** Definition of Congruent Angles
3. $m\angle B = m\angle A$	**3.** Symmetric Property
4. $\angle B \cong \angle A$	**4.** Definition of Congruent Angles

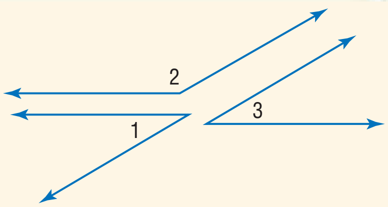
Algebraic properties can be applied to prove theorems for congruence relationships involving supplementary and complementary angles.

THEOREMS

2.6 Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation: $\measuredangle$ suppl. to same $\angle$ or $\cong$ $\measuredangle$ are $\cong$.

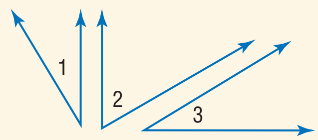
Example: If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.



2.7 Angles complementary to the same angle or to congruent angles are congruent.

Abbreviation: $\measuredangle$ compl. to same $\angle$ or $\cong$ $\measuredangle$ are $\cong$.

Example: If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 + m\angle 3 = 90$, then $\angle 1 \cong \angle 3$.

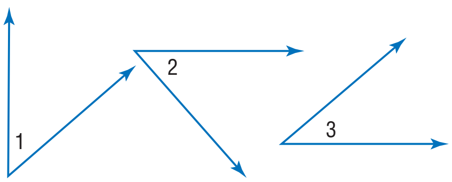


You will prove Theorem 2.6 in Exercise 3.

PROOF Theorem 2.7

Given: $\angle 1$ and $\angle 3$ are complementary.
$\angle 2$ and $\angle 3$ are complementary.

Prove: $\angle 1 \cong \angle 2$

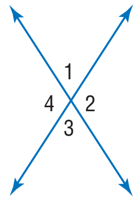


Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 3$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	**1.** Given
2. $m\angle 1 + m\angle 3 = 90$; $m\angle 2 + m\angle 3 = 90$	**2.** Definition of complementary angles
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**3.** Substitution
4. $m\angle 3 = m\angle 3$	**4.** Reflexive Property
5. $m\angle 1 = m\angle 2$	**5.** Subtraction Property
6. $\angle 1 \cong \angle 2$	**6.** Definition of congruent angles

EXAMPLE 3 Use Supplementary Angles

In the figure, $\angle 1$ and $\angle 2$ form a linear pair and $\angle 2$ and $\angle 3$ form a linear pair. Prove that $\angle 1$ and $\angle 3$ are congruent.



Given: $\angle 1$ and $\angle 2$ form a linear pair.
$\angle 2$ and $\angle 3$ form a linear pair.

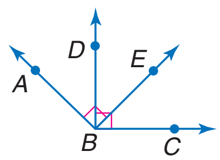
Prove: $\angle 1 \cong \angle 3$

Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ form a linear pair. $\angle 2$ and $\angle 3$ form a linear pair.	**1.** Given
2. $\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	**2.** Supplement Theorem
3. $\angle 1 \cong \angle 3$	**3.** $\angle$s suppl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.

CHECK Your Progress

3. In the figure, $\angle ABE$ and $\angle DBC$ are right angles. Prove that $\angle ABD \cong \angle EBC$.



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Note that in Example 3, $\angle 1$ and $\angle 3$ are vertical angles. The conclusion in the example is a proof for the following theorem.

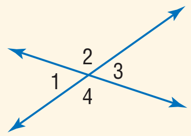
Review Vocabulary

Vertical Angles two nonadjacent angles formed by intersecting lines (Lesson 1-5)

THEOREM 2.8 *Vertical Angles Theorem*

If two angles are vertical angles, then they are congruent.

Abbreviation: Vert. $\angle$s are $\cong$.



$\angle 1 \cong \angle 3$ and
$\angle 2 \cong \angle 4$

EXAMPLE 4 Vertical Angles

If $\angle 1$ and $\angle 2$ are vertical angles and $m\angle 1 = x$ and $m\angle 2 = 228 - 3x$, find $m\angle 1$ and $m\angle 2$.

$\angle 1 \cong \angle 2$	Vertical Angles Theorem	Substitute to find the angle measures.
$m\angle 1 = m\angle 2$	Definition of congruent angles	$m\angle 1 = x$
$x = 228 - 3x$	Substitution	$= 57$
$4x = 228$	Add $3x$ to each side.	$m\angle 2 = m\angle 1$
$x = 57$	Divide each side by 4.	$= 57$

CHECK Your Progress

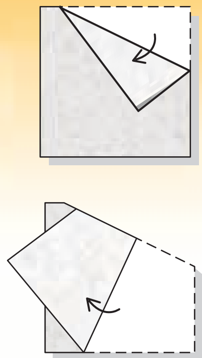
4. If $\angle 3$ and $\angle 4$ are vertical angles, $m\angle 3 = 6x + 2$, and $m\angle 4 = 8x - 14$, find $m\angle 3$ and $m\angle 4$.

You can create right angles and investigate congruent angles by paper folding.

GEOMETRY LAB

Right Angles

MAKE A MODEL



- Fold the paper so that one corner is folded downward.
- Fold along the crease so that the top edge meets the side edge.
- Unfold the paper and measure each of the angles.
- Repeat the activity three more times.

ANALYZE THE MODEL

1. What do you notice about the lines formed?

2. What do you notice about each pair of adjacent angles?

3. What are the measures of the angles formed?

4. **MAKE A CONJECTURE** What is true about perpendicular lines?

The following theorems support the conjectures you made in the Geometry Lab.

THEOREMS *Right Angle Theorems*

2.9	Perpendicular lines intersect to form four right angles.
2.10	All right angles are congruent.
2.11	Perpendicular lines form congruent adjacent angles.
2.12	If two angles are congruent and supplementary, then each angle is a right angle.
2.13	If two congruent angles form a linear pair, then they are right angles.

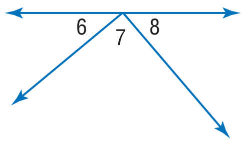
You will prove these theorems in Exercises 20–24.

Check Your Understanding

Find the measure of each numbered angle.

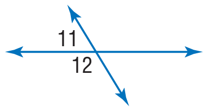
Example 1 (p. 125)

1. $\angle 6$ and $\angle 8$ are complementary, $m\angle 8 = 47$



Example 2 (p. 125)

2. $m\angle 11 = x - 4$, $m\angle 12 = 2x - 5$



Example 3 (p. 127)

3. PROOF Copy and complete the proof of Theorem 2.6.

Given: $\angle 1$ and $\angle 2$ are supplementary.
$\angle 3$ and $\angle 4$ are supplementary.
$\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$

Proof:

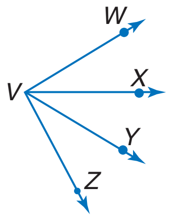
Statements	Reasons
a. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary. $\angle 1 \cong \angle 4$	**a.** ?
b. $m\angle 1 + m\angle 2 = 180$ $m\angle 3 + m\angle 4 = 180$	**b.** ?
c. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	**c.** ?
d. $m\angle 1 = m\angle 4$	**d.** ?
e. $m\angle 2 = m\angle 3$	**e.** ?
f. $\angle 2 \cong \angle 3$	**f.** ?

Example 4 (p. 128)

4. PROOF Write a two-column proof.

Given: $\overrightarrow{VX}$ bisects $\angle WVY$.
$\overrightarrow{VY}$ bisects $\angle XVZ$.

Prove: $\angle WVX \cong \angle YVZ$



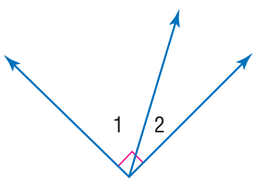
Exercises

HOMEWORK HELP

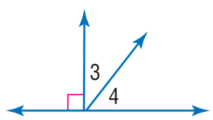
For Exercises	See Examples
5–7	1
8–10	2
14–19	3
11–13	4

Find the measure of each numbered angle.

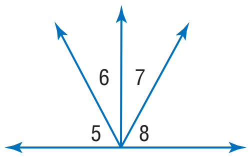
5. $m\angle 1 = 64$



6. $m\angle 3 = 38$

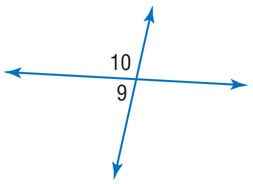


7. $\angle 7$ and $\angle 8$ are complementary. $\angle 5 \cong \angle 8$ and $m\angle 6 = 29$.

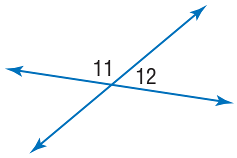


Find the measure of each numbered angle.

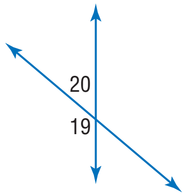
8. $m\angle 9 = 2x - 4$, $m\angle 10 = 2x + 4$



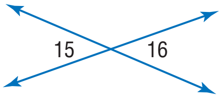
9. $m\angle 11 = 4x$, $m\angle 12 = 2x - 6$



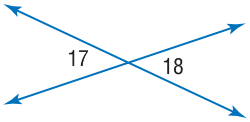
10. $m\angle 19 = 100 + 20x$, $m\angle 20 = 20x$



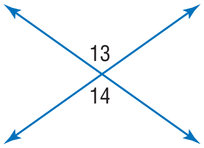
11. $m\angle 15 = x$, $m\angle 16 = 6x - 290$



12. $m\angle 17 = 2x + 7$, $m\angle 18 = x + 30$

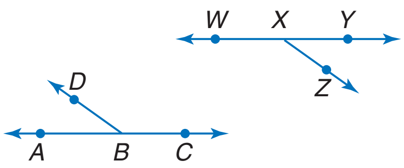


13. $m\angle 13 = 2x + 94$, $m\angle 14 = 7x + 49$

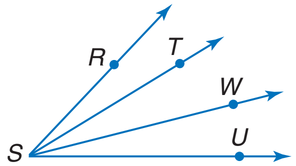


PROOF Write a two-column proof.

14. **Given:** $\angle ABD \cong \angle YXZ$
Prove: $\angle CBD \cong \angle WXZ$



15. **Given:** $m\angle RSW = m\angle TSU$
Prove: $m\angle RST = m\angle WSU$



Real-World Link

Interstate highways that run from north to south are odd-numbered with the lowest numbers in the west. East-west interstates are even-numbered and begin in the south.

Source: www.infoplease.com

Write a proof for each theorem.

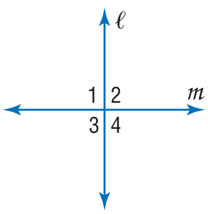
16. Supplement Theorem

17. Complement Theorem

18. Reflexive Property of Angle Congruence

19. Transitive Property of Angle Congruence

PROOF Use the figure to write a proof of each theorem.



20. Theorem 2.9

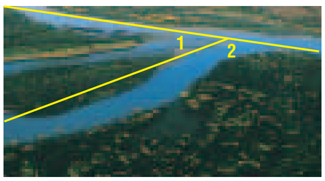
21. Theorem 2.10

22. Theorem 2.11

23. Theorem 2.12

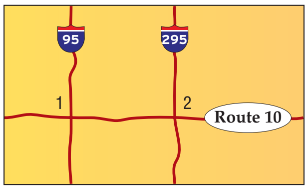
24. Theorem 2.13

25. **RIVERS** Tributaries of rivers sometimes form a linear pair of angles when they meet the main river. The Yellowstone River forms the linear pair $\angle 1$ and $\angle 2$ with the Missouri River. If $m\angle 1$ is 28, find $m\angle 2$.



EXTRA PRACTICE
See pages 804, 829.
Math Online
Self-Check Quiz at geometryonline.com

26. **HIGHWAYS** Near the city of Hopewell, Virginia, Route 10 runs perpendicular to Interstate 95 and Interstate 295. Show that the angles at the intersections of Route 10 with Interstate 95 and Interstate 295 are congruent.



H.O.T. Problems

27. **OPEN ENDED** Draw three congruent angles. Use these angles to illustrate the Transitive Property for angle congruence.

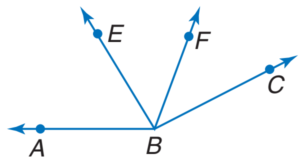
28. FIND THE ERROR Tomas and Jacob wrote equations involving the angle measures shown. Who is correct? Explain your reasoning.

Tomas

$m\angle ABE + m\angle EBC = m\angle ABC$

Jacob

$m\angle ABE + m\angle FBC = m\angle ABC$

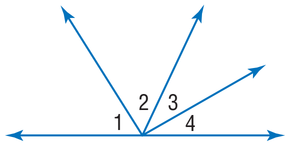


REASONING Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

29. Two angles that are nonadjacent are vertical.

30. Two acute angles that are congruent are complementary to the same angle.

31. CHALLENGE What conclusion can you make about the sum of $m\angle 1$ and $m\angle 4$ if $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$? Explain.



32. *Writing in Math* Refer to page 124. Describe how scissors illustrate supplementary angles. Is the relationship the same for two angles complementary to the same angle?

STANDARDIZED TEST PRACTICE

33. The measures of two complementary angles are in the ratio 4:1. What is the measure of the smaller angle?

A 15

B 18

C 24

D 36

34. REVIEW Simplify $4(3x - 2)(2x + 4) + 3x^2 + 5x - 6$.

F $9x^2 + 3x - 14$

G $9x^2 + 13x - 14$

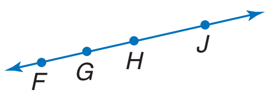
H $27x^2 + 37x - 38$

J $27x^2 + 27x - 26$

Spiral Review

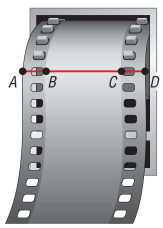
PROOF Write a two-column proof. (Lesson 2-7)

35. Given: G is between F and H.
H is between G and J.
Prove: $FG + GJ = FH + HJ$



36. Given: X is the midpoint of $\overline{WY}$.
Prove: $WX + YZ = XZ$

37. PHOTOGRAPHY Film is fed through a camera by gears that catch the perforation in the film. The distance from the left edge of the film, A, to the right edge of the image, C, is the same as the distance from the left edge of the image, B, to the right edge of the film, D. Show that the two perforated strips are the same width. (Lesson 2-6)



CHAPTER 2

Study Guide and Review

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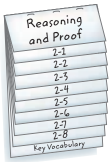
FOLDABLES Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Inductive Reasoning and Logic (Lessons 2-1 and 2-2)

- If a statement is represented by p, then *not* p is the negation of the statement.
- A conjunction is a compound statement formed by joining two or more statements with the word *and*.
- A disjunction is a compound statement formed by joining two or more statements with the word *or*.

Conditional Statements (Lesson 2-3)

- An if-then statement is written in the form *if* p, *then* q in which p is the hypothesis, and q is the conclusion.
- The converse is formed by exchanging the hypothesis and conclusion of the conditional.
- The inverse is formed by negating both the hypothesis and conclusion of the conditional.
- The contrapositive is formed by negating both the hypothesis and conclusion of the converse statement.

Deductive Reasoning (Lesson 2-4)

- Law of Detachment: If $p \to q$ is true and p is true, then q is also true.
- Law of Syllogism: If $p \to q$ and $q \to r$ are true, then $p \to r$ is also true.

Proof (Lessons 2-5 through 2-8)

- State what is to be proven.
- List the given information.
- If possible, draw a diagram.
- State what is to be proved.
- Develop of system of deductive reasoning.

Key Vocabulary

conclusion (p. 91)
conditional statement (p. 91)
conjecture (p. 78)
conjunction (p. 84)
contrapositive (p. 93)
converse (p. 93)
counterexample (p. 79)
deductive argument (p. 111)
deductive reasoning (p. 99)
disjunction (p. 84)
hypothesis (p. 91)
if-then statement (p. 91)
inductive reasoning (p. 78)
inverse (p. 93)
negation (p. 83)
paragraph proof (p. 106)
postulate (p. 105)
proof (p. 106)
theorem (p. 106)
truth value (p. 83)
two-column proof (p. 112)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. Theorems are accepted as true.
2. A disjunction is true only when both statements in it are true.
3. In a two-column proof, the properties that justify each step are called reasons.
4. Inductive reasoning uses facts, rules, definitions, or properties to reach logical conclusions.
5. The Reflexive Property of Equality states that for every number a, $a = a$.
6. A negation is another term for axiom.
7. To show that a conjecture is false you would give a counterexample.
8. An if-then statement consists of a conjecture and a conclusion.
9. The contrapositive of a conditional is formed by exchanging the hypothesis and conclusion of the conditional statement.
10. A disjunction is formed by joining two or more sentences with the word *or*.

Vocabulary Review at geometryonline.com

Lesson-by-Lesson Review

2-1 Inductive Reasoning and Conjecture (pp. 78–82)

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

11. $\angle A$ and $\angle B$ are supplementary.

12. X, Y, and Z are collinear and $XY = YZ$.

13. TRAFFIC While driving on the freeway, Tonya noticed many cars ahead of her had stopped. So she immediately took the next exit. Make a conjecture about why Tonya chose to exit the freeway.

Example 1 **Given that points P, Q, and R are collinear, determine whether the conjecture that Q is between P and R is *true* or *false*. If the conjecture is false, give a counterexample.**

The figure below can be used to disprove the conjecture. In this case, R is between P and Q. Since we can find a counterexample, the conjecture is false.

2-2 Logic (pp. 83–90)

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

p: $-1 > 0$

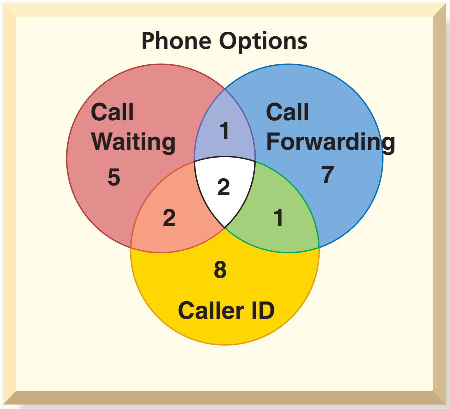
q: In a right triangle with right angle C, $a^2 + b^2 = c^2$.

r: The sum of the measures of two supplementary angles is 180°.

14. p and $\sim q$

15. $\sim p \vee \sim r$

16. PHONES The results of a survey about phone service options are shown below. How many customers had both call waiting and caller ID?



Example 2 **Use the following statements to write a compound statement for the conjunction and disjunction below. Then find each truth value.**

p: $\sqrt{15} = 5$

q: The measure of a right angle equals 90.

a. **p and q**

$\sqrt{15} = 5$, and the measure of a right angle equals 90.

p and q is false because p is false and q is true.

b. **$p \vee q$**

$\sqrt{15} = 5$, or the measure of a right angle equals 90.

$p \vee q$ is true because q is true. It does not matter that p is false.

CHAPTER 2

Study Guide and Review

2-3 Conditional Statements (pp. 91–97)

Write the converse, inverse, and contrapositive of each conditional. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

17. March has 31 days.

18. If an ordered pair for a point has 0 for its x-coordinate, then the point lies on the y-axis.

TEMPERATURE **Find the truth value of the following statement for each set of conditions.**

Water freezes when the temperature is at most 0°C.

19. Water freezes at −10°C.

20. Water freezes at 15°C.

21. Water does not freeze at −2°C.

22. Water does not freeze at 30°C.

Example 3 **Identify the hypothesis and conclusion of the statement *The intersection of two planes is a line*. Then write the statement in if-then form.**

Hypothesis: two planes intersect

Conclusion: their intersection is a line

If two planes intersect, then their intersection is a line.

Example 4 **Write the converse of the statement *All fish live under water*. Determine whether the converse is true or false. If it is is false, find a counterexample.**

Converse: If it lives under water, it is a fish. False; dolphins live under water, but are not fish.

2-4 Deductive Reasoning (pp. 99–104)

Determine whether statement (3) follows from statements (1) and (2) by the Laws of Detachment or Syllogism. If so, state which law was used. If not, write *invalid*.

23. (1) If a student attends North High School, then he or she has an ID number.
(2) Josh attends North High School.
(3) Josh has an ID number.

24. (1) If a rectangle has four congruent sides, then it is a square.
(2) A square has diagonals that are perpendicular.
(3) A rectangle has diagonals that are perpendicular.

Example 5 **Use the Law of Syllogism to determine whether a valid conclusion can be reached from the following statements.**

(1) If a body in our solar system is the Sun, then it is a star.

(2) Stars are in constant motion.

p: A body in our solar system is the Sun.

q: It is a star.

r: Stars are in constant motion.

Statement (1): $p \rightarrow q$

Statement (2): $q \rightarrow r$

Since the given statements are true, use the Law of Syllogism to conclude $p \rightarrow r$. That is, *If a body in our solar system is the Sun, then it is in constant motion.*

Mixed Problem Solving
For mixed problem-solving practice, see page 829.

2-5 Postulates and Paragraph Proofs (pp. 105–109)

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

25. The intersection of two different lines is a line.

26. If P is the midpoint of $\overline{XY}$, then $XP = PY$.

27. Four points determine six lines.

28. If $MX = MY$, then M is the midpoint of $\overline{XY}$.

29. HAMMOCKS Maurice has six trees in a regular hexagonal pattern in his backyard. How many different possibilities are there for tying his hammock to any two of those trees?

Example 6 **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.**

Two points determine a line.

According to a postulate relating to points and lines, two points determine a line. Thus, the statement is *always* true.

If two angles are right angles, they are adjacent.

If two right angles form a linear pair, then they would be adjacent. This statement is *sometimes* true.

2-6 Algebraic Proof (pp. 111–117)

State the property that justifies each statement.

30. If $3(x + 2) = 6$, then $3x + 6 = 6$.

31. If $10x = 20$, then $x = 2$.

32. If $AB + 20 = 45$, then $AB = 25$.

33. If $3 = CD$ and $CD = XY$, then $3 = XY$.

Write a two-column proof.

34. If $5 = 2 - \frac{1}{2}x$, then $x = -6$.

35. If $x - 1 = \frac{x - 10}{-2}$, then $x = 4$.

36. If $AC = AB$, $AC = 4x + 1$, and $AB = 6x - 13$, then $x = 7$.

37. If $MN = PQ$ and $PQ = RS$, then $MN = RS$.

38. BIRTHDAYS Mark has the same birthday as Cami. Cami has the same birthday as Briana. Which property would show that Mark has the same birthday as Briana?

Example 7

Given: $2x + 6 = 3 + \frac{5}{3}x$

Prove: $x = -9$

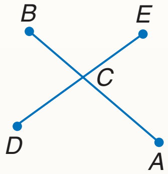
Proof:

Statements	Reasons
1. $2x + 6 = 3 + \frac{5}{3}x$	**1.** Given
2. $3(2x + 6) = 3\left(3 + \frac{5}{3}x\right)$	**2.** Multiplication Property
3. $6x + 18 = 9 + 5x$	**3.** Distributive Property
4. $6x + 18 - 5x = 9 + 5x - 5x$	**4.** Subtraction Property
5. $x + 18 = 9$	**5.** Substitution
6. $x + 18 - 18 = 9 - 18$	**6.** Subtraction Property
7. $x = -9$	**7.** Substitution

2-7 Proving Segment Relationships (pp. 118–123)

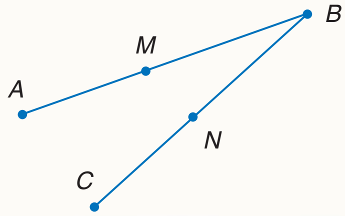
PROOF Write a two-column proof.

39. Given: $BC = EC$, $CA = CD$
Prove: $BA = DE$



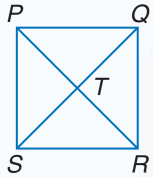
40. Given: $AB = CD$
Prove: $AC = BD$

41. KANSAS The distance from Salina to Kansas City is represented by AB, and the distance from Wichita to Kansas City is represented by CB. If $AB = CB$, M is the midpoint of AB, and N is the midpoint of CD, prove $AM = CN$.



Example 8 Write a two-column proof.

Given: $QT = RT$, $TS = TP$
Prove: $QS = RP$
Proof:



Statements	Reasons
1. $QT = RT$, $TS = TP$	**1.** Given
2. $QT + TS = RT + TS$	**2.** Addition Prop.
3. $QT + TS = RT + TP$	**3.** Substitution
4. $QT + TS = QS$, $RT + TP = RP$	**4.** Seg. Add. Post.
5. $QS = RP$	**5.** Substitution

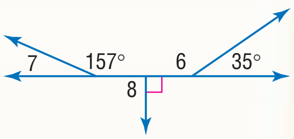
2-8 Proving Angle Relationships (pp. 124–131)

Find the measure of each angle.

42. $\angle 6$

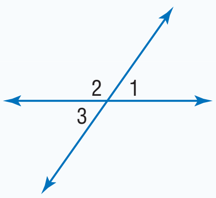
43. $\angle 7$

44. $\angle 8$



45. PROOF Write a two-column proof.
Given: $\angle 1$ and $\angle 2$ form a linear pair
$m\angle 2 = 2(m\angle 1)$
Prove: $m\angle 1 = 60$

Example 9 Find the measure of each numbered angle if $m\angle 3 = 55$.



$m\angle 1 = 55$, since $\angle 1$ and $\angle 3$ are vertical angles.

$\angle 2$ and $\angle 3$ form a linear pair.

$55 + m\angle 2 = 180$	Def. of suppl. ∡
$m\angle 2 = 180 - 55$	Subtract.
$m\angle 2 = 125$	Simplify.

Chapter 2 Practice Test

Determine whether each conjecture is *true* or *false*. Explain your answer and give a counterexample for any false conjecture.

1. **Given:** $\angle A \cong \angle B$
   Conjecture: $\angle B \cong \angle A$
2. **Given:** y is a real number.
   Conjecture: $-y > 0$
3. **Given:** $3a^2 = 48$
   Conjecture: $a = 4$

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value.

p: $-3 > 2$
q: $3x = 12$ when $x = 4$.
r: An equilateral triangle is also equiangular.

4. p and q
5. p or q
6. $p \vee (q \wedge r)$

7. **ADVERTISING** Identify the hypothesis and conclusion of the following statement. Then write it in if-then form.
   Hard-working people deserve a great vacation.

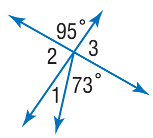
Determine whether statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

8. (1) Perpendicular lines intersect.
   (2) Lines m and n are perpendicular.
   (3) Lines m and n intersect.

9. (1) If n is an integer, then n is a real number.
   (2) n is a real number.
   (3) n is an integer.

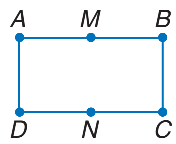
Find the measure of each numbered angle.

10. $\angle 1$
11. $\angle 2$
12. $\angle 3$



PROOF Write the indicated type of proof.

13. two-column
    If $y = 4x + 9$ and $x = 2$, then $y = 17$.
14. two-column
    If $2(n - 3) + 5 = 3(n - 1)$, prove that $n = 2$.
15. paragraph proof
    Given: $AM = CN$, $MB = ND$
    Prove: $AB = CD$



16. paragraph proof
    If M is the midpoint of $\overline{AB}$, and Q is the midpoint of $\overline{AM}$, then $AQ = \frac{1}{4}AB$.

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.

17. Two angles that form a right angle are complementary.
18. Two angles that form a linear pair are congruent.

Identify the hypothesis and conclusion of each statement and write each statement in if-then form. Then write the converse, inverse, and contrapositive of each conditional.

19. An apple a day keeps the doctor away.
20. A rolling stone gathers no moss.

21. **MULTIPLE CHOICE** Refer to the following statements.

    p: There are 52 states in the United States.
    q: $12 + 8 = 20$
    r: A week has 8 days.

    Which compound statement is true?

    A p and q
    B p or q
    C p or r
    D q and r

Standardized Test Practice

CHAPTER 2

Cumulative, Chapters 1–2

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. *"Two lines that do not intersect are always parallel."*

Which of the following best describes a *counterexample* to the assertion above?

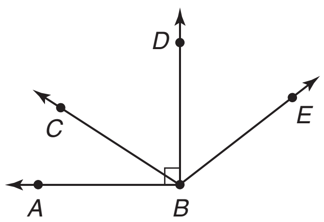
A coplanar lines

B parallel lines

C perpendicular lines

D skew lines

2. Consider the following statements about the figure shown below.



p: $\angle ABC$ is an acute angle.
q: $\angle ABC$ and $\angle CBD$ are supplementary angles.
r: $m\angle ABE$ is greater than 90°.

Which of the following compound statements is *not* true?

F $p \vee q$

G $\sim q \wedge r$

H $\sim r \wedge \sim q$

J $\sim p \vee \sim q$

3. Which of the following *best* describes an axiom?

A a conjecture made using examples

B a conjecture made using facts, rules, definition, or properties

C a statement that is accepted as true

D a statement or conjecture that has been shown to be true

4. Determine which statement follows logically from the given statements.

If it rains today, the game will be cancelled.
Cancelled games are made up on Saturdays.

F If a game is cancelled, it was because of rain.

G If it rains today, the game will be made up on Saturday.

H Some cancelled games are not made up on Saturdays.

J If it does not rain today, the game will not be made up on Saturday.

TEST-TAKING TIP

Question 4 When answering a multiple-choice question, always read every answer choice and eliminate those you decide are definitely wrong. This way, you may deduce the correct answer.

5. Which of the following statements is the contrapositive of the conditional statement: If the sum of the measures of the angles of a polygon is 180°, then the polygon is a triangle?

A If a polygon is not a triangle, then the sum of the measures of the angles of the polygon is not 180°.

B If the sum of the measures of the angles of polygon is not 180°, then the polygon is not a triangle.

C If a polygon is a triangle, then the sum of the measures of the angles of the polygon is 180°.

D If a polygon is not a triangle, then the sum of the measures of the angles of the polygon is 180°.

6. GRIDDABLE Samantha has 3 more trophies than Martha. Melinda has triple the number of trophies that Samantha has. Altogether the girls have 22 trophies. How many trophies does Melinda have?

For test-taking strategies and more practice, see pages 841–856.

7. Use the proof to answer the question below.

Given: $\angle A$ is the complement of $\angle B$;
$m\angle B = 46$

Prove: $m\angle A = 44$

Statement	Reason
1. A is the complement of $\angle B$; $m\angle B = 46$	**1.** Given
2. $m\angle A + m\angle B = 90$	**2.** Def. of comp. angles
3. $m\angle A + 46 = 90$	**3.** Substitution Prop.
4. $m\angle A + 46 - 46 = 90 - 46$	**4.** ?
5. $m\angle A = 44$	**5.** Substitution Prop.

What reason can be given to justify Statement 4?

F Addition Property
G Substitution Property
H Subtraction Property
J Symmetric Property

8. **Given:** Points A, B, C, and D are collinear, with point B between points A and C and point C between points B and D. Which of the following does *not* have to be true?

A $AB + BD = AD$
B $\overline{AB} \cong \overline{CD}$
C $\overline{BC} \cong \overline{BC}$
D $BC + CD = BD$

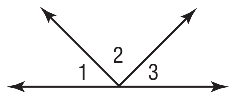
9. A farmer needs to make a 1000-square-foot rectangular enclosure for her cows. She wants to save money by purchasing the least amount of fencing possible to enclose the area. What whole-number dimensions will require the least amount of fencing?

F 8 ft by 125 ft
G 10 ft by 100 ft
H 20 ft by 50 ft
J 25 ft by 40 ft

10. **Given:** $\angle EFG$ and $\angle GFH$ are complementary. Which of the following *must* be true?

A $\overrightarrow{FE} \perp \overrightarrow{FG}$
B $\overrightarrow{FG}$ bisects $\angle EFH$.
C $m\angle EFG + m\angle GFH = 180$
D $\angle GFH$ is an acute angle.

11. In the diagram below, $\angle 1 \cong \angle 3$.



Which of the following conclusions does *not* have to be true?

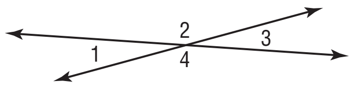
F $m\angle 1 - m\angle 2 + m\angle 3 = 90$
G $m\angle 1 + m\angle 2 + m\angle 3 = 180$
H $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$
J $m\angle 2 - m\angle 1 = m\angle 2 - m\angle 3$

Pre-AP

Record your answer on a sheet of paper. Show your work.

12. **Given:** $\angle 1$ and $\angle 3$ are vertical angles.
$m\angle 1 = 3x + 5$, $m\angle 3 = 2x + 8$

Prove: $m\angle 1 = 14$



13. From a single point in her yard, Marti measures and marks distances of 18 feet and 30 feet for two sides of her garden. What length should the third side of her garden be so that it will form a right angle with the 18-foot side? If Marti decided to use the same length of fencing in a square configuration, how long would each side of the fence be? Which configuration would provide the largest area for her garden?

NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	2-1	2-2	2-5	2-4	2-3	781	2-6	2-7	1-6	1-6	2-8	2-8	1-3