Parallel and Perpendicular Lines

BIG Ideas

- Identify angle relationships that occur with parallel lines and a transversal, and identify and prove lines parallel from given angle relationships.

- Use slope to analyze a line and to write its equation.

- Find the distance between a point and a line and between two parallel lines.

Key Vocabulary

parallel lines (p. 142)

transversal (p. 143)

🌐 Real-World Link

Architecture The East Building of the National Gallery of Art in Washington, D.C., designed by architect I.M. Pei, has an H-shaped façade. Viewed from above, the building appears to be made of interlocking diamonds. This design creates many parallel lines.

FOLDABLES Study Organizer

Parallel and Perpendicular Lines Make this Foldable to help you organize your notes. Begin with one sheet of $8\frac{1}{2}$" × 11" paper.
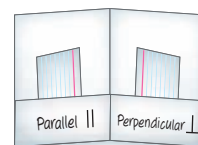
1 **Fold** in half matching the short sides.

2 **Unfold** and fold the long side up 2 inches to form a pocket.

3 **Staple** or glue the outer edges to complete the pocket.

4 **Label** each side as shown. Use index cards to record examples.

Parallel ‖   Perpendicular ⊥

Royalty-Free/SuperStock

GET READY for Chapter 3

Diagnose Readiness You have two options for checking Prerequisite Skills.
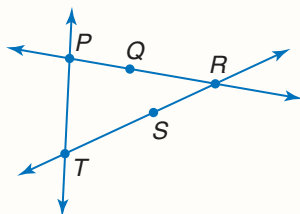
Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Name all of the lines that contain the given point. (Lesson 1-1)

1. Q

2. R

3. S

4. T

Name all angles congruent to the given angle. (Lessons 1-5)

5. $\angle 2$ **6.** $\angle 5$
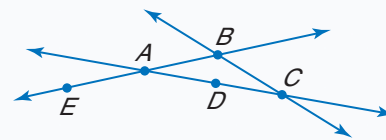
7. $\angle 3$ **8.** $\angle 8$
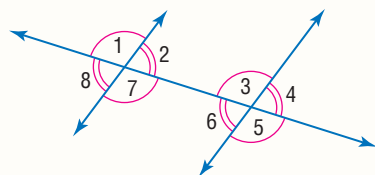
9. **MOVIES** A local movie theater is running a promotion in which a large popcorn costs $2 with the purchase of two adult tickets. If Mr. and Mrs. Elian spent $19 at the movie theater, write an equation to represent the cost and solve for the cost of one adult ticket. (Prerequisite Skill)
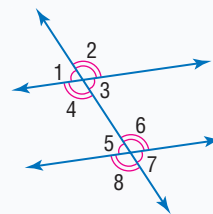
QUICK Review

EXAMPLE 1

Name all of the lines that contain the point C.

Point C is the intersection point of lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$.

EXAMPLE 2

Name all angles congruent to $\angle 5$.

Look at the congruence marks. From the figure, $\angle 1$, $\angle 3$, and $\angle 7$ are each congruent to $\angle 5$.

EXAMPLE 3

Find the value of y in $2x - y = 4$ if $x = -4$.

$2x - y = 4$	Write the equation.
$-y = -2x + 4$	Subtract 2x from each side.
$y = 2x - 4$	Divide each side by −1.
$y = 2(-4) - 4$	Substitute −4 for x.
$y = -8 - 4$	Multiply.
$y = -12$	Simplify.

Parallel Lines and Transversals

Main Ideas

- Identify the relationships between two lines or two planes.
- Name angles formed by a pair of lines and a transversal.

New Vocabulary

parallel lines
parallel planes
skew lines
transversal
consecutive interior angles
alternate exterior angles
alternate interior angles
corresponding angles

▶ GET READY for the Lesson

The Dana-Thomas House in Springfield, Illinois, is perhaps architect Frank Lloyd Wright's best preserved and most complete early "prairie" house. There are several examples of parallel lines, parallel planes, and skew lines in the design.

Relationships Between Lines and Planes Lines ℓ and m are coplanar because they lie in the same plane. If the lines were extended indefinitely, they would not intersect. Coplanar lines that do not intersect are called **parallel lines**. Segments and rays contained within parallel lines are also parallel.

The symbol ‖ means *is parallel to*. Arrows are used in diagrams to indicate that lines are parallel. In the figure, the arrows indicate that $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{RS}$.
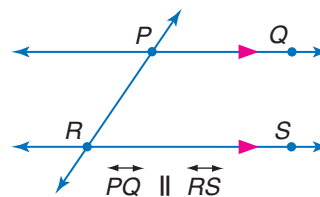
Similarly, two planes can intersect or be parallel. In the photograph above, the front faces of the building are contained in **parallel planes**. The walls and the floor of each level lie in intersecting planes.

$$\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$$

The symbol ∦ means *is not parallel to.*
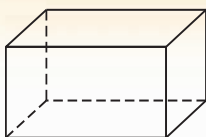

GEOMETRY LAB

Draw a Rectangular Prism

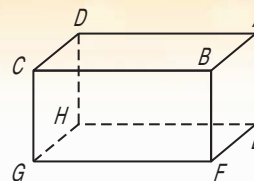**A rectangular prism can be drawn using parallel lines and parallel planes.**

Step 1 Draw two parallel planes to represent the top and bottom.

Step 2 Draw the edges. Make any hidden edges dashed.

Step 3 Label the vertices.

ANALYZE

1. Identify the parallel planes in the figure.
2. Name the planes that intersect plane *ABC* and name their intersections.
3. Identify all segments parallel to $\overline{BF}$.

Notice that in the Geometry Lab, $\overline{AE}$ and $\overline{GF}$ do not intersect. These segments are not parallel since they do not lie in the same plane. Lines that do not intersect and are not coplanar are called **skew lines**. Segments and rays contained in skew lines are also skew.

EXAMPLE Identify Relationships

1
a. **Name all planes that are parallel to plane ABG.**
   plane CDE

b. **Name all segments that intersect $\overline{CH}$.**
   $\overline{BC}, \overline{CD}, \overline{CE}, \overline{EH}$, and $\overline{GH}$

c. **Name all segments that are skew to $\overline{BG}$.**
   $\overline{AD}, \overline{CD}, \overline{CE}, \overline{EF}$, and $\overline{EH}$
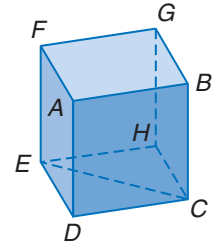
CHECK Your Progress
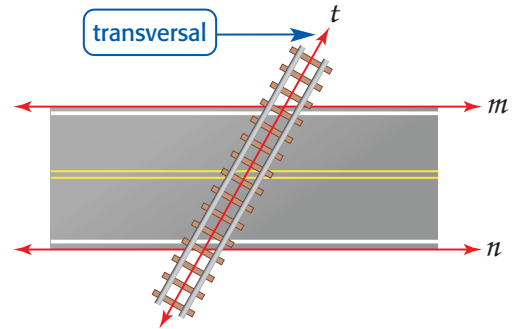
1. Name all segments that are parallel to $\overline{EF}$.

Angle Relationships In the drawing of the railroad crossing, notice that the tracks, represented by line t, intersect the sides of the road, represented by lines m and n. A line that intersects two or more lines in a plane at different points is called a **transversal**.
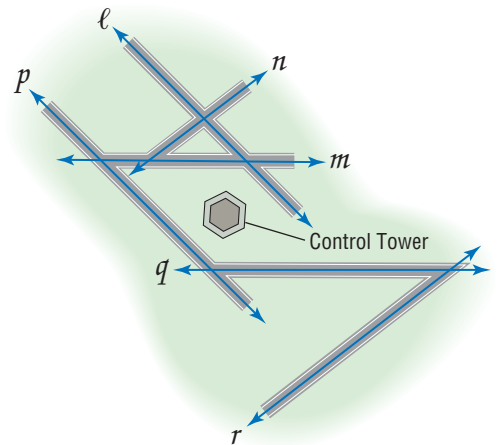
transversal

Real-World EXAMPLE Identify Transversals

2
AIRPORTS Some of the runways at O'Hare International Airport are shown below. Identify the sets of lines to which each given line is a transversal.

a. **line q**
   If the lines are extended, line q intersects lines ℓ, n, p, and r.

b. **line m**
   lines ℓ, n, p, and r

c. **line n**
   lines ℓ, m, p, and q

Control Tower

CHECK Your Progress

2. line r

In the drawing of the railroad crossing above, notice that line t forms eight angles with lines m and n. These angles are given special names, as are specific pairings of these angles.
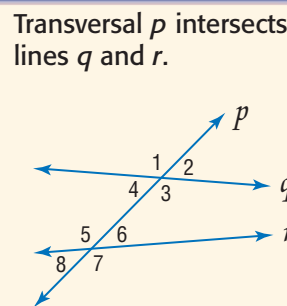
Same Side Interior Angles

Consecutive interior angles are also called *same side interior angles*.

KEY CONCEPT — Transversals and Angles

Name	Angles
exterior angles	∠1, ∠2, ∠7, ∠8
interior angles	∠3, ∠4, ∠5, ∠6
consecutive interior angles	∠3 and ∠6, ∠4 and ∠5
alternate exterior angles	∠1 and ∠7, ∠2 and ∠8
alternate interior angles	∠3 and ∠5, ∠4 and ∠6
corresponding angles	∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8

Transversal *p* intersects lines *q* and *r*.

EXAMPLE — Identify Angle Relationships

3 Refer to the figure below. Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

a. ∠1 and ∠7
alternate exterior

b. ∠2 and ∠10
corresponding

c. ∠8 and ∠9
consecutive interior

d. ∠3 and ∠12
corresponding

e. ∠4 and ∠10
alternate interior

f. ∠6 and ∠11
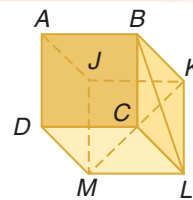alternate exterior
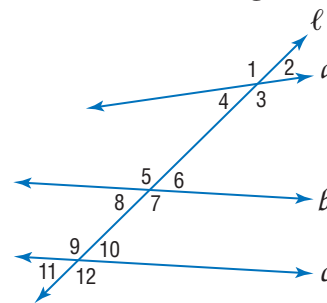
CHECK Your Progress

3A. ∠4 and ∠11

3B. ∠2 and ∠8

Personal Tutor at geometryonline.com

CHECK Your Understanding

Example 1 (p. 143)
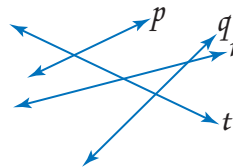
For Exercises 1–3, refer to the figure at the right.
1. Name all planes that intersect plane *ADM*.
2. Name all segments that are parallel to $\overline{CD}$.
3. Name all segments that intersect $\overline{KL}$.

Example 2 (p. 143)

Identify the pairs of lines to which each given line is a transversal.
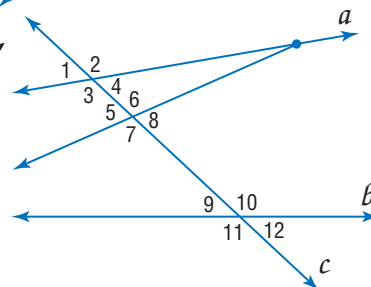4. *p*
5. *r*
6. *q*
7. *t*

Example 3 (p. 144)

Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior angles*.
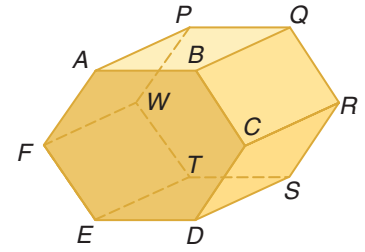8. ∠7 and ∠10
9. ∠1 and ∠5
10. ∠4 and ∠6
11. ∠8 and ∠1

For Exercises 12–19, refer to the figure at the right.

12. Name all segments parallel to $\overline{AB}$.

13. Name all planes intersecting plane BCR.

14. Name all segments parallel to $\overline{TW}$.

15. Name all segments skew to $\overline{DE}$.

16. Name all planes intersecting plane EDS.

17. Name all segments skew to $\overline{AP}$.

18. Name all segments parallel to $\overline{DC}$.

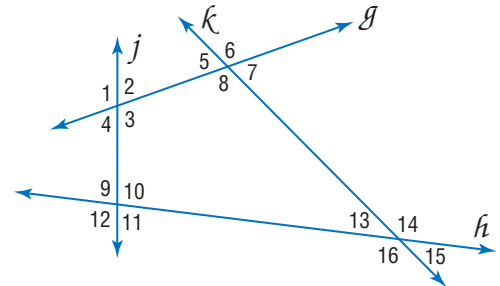**19.** Name all segments parallel to $\overline{DS}$.

Identify the pairs of lines to which each given line is a transversal.

20. a **21.** b

22. c **23.** r
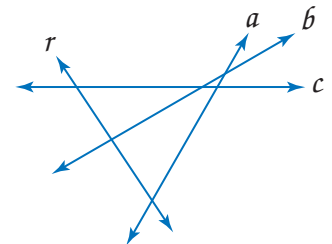
Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.

24. $\angle2$ and $\angle10$ **25.** $\angle1$ and $\angle11$

26. $\angle5$ and $\angle3$ **27.** $\angle6$ and $\angle14$

28. $\angle5$ and $\angle15$ **29.** $\angle11$ and $\angle13$

30. $\angle8$ and $\angle3$ **31.** $\angle9$ and $\angle4$

32. $\angle6$ and $\angle16$ **33.** $\angle7$ and $\angle3$

34. $\angle10$ and $\angle13$ **35.** $\angle12$ and $\angle14$

36. AVIATION Airplanes heading east are assigned an altitude level that is an odd number of thousands of feet. Airplanes heading west are assigned an altitude level that is an even number of thousands of feet. If one airplane is flying northwest at 34,000 feet and another airplane is flying east at 25,000 feet, describe the type of lines formed by the paths of the airplanes. Explain your reasoning.

Real-World Career

Air Traffic Controller

The air traffic controller monitors and coordinates air traffic either at an airport or between airports. They monitor the position of airplanes ensuring that planes are a safe distance apart.

Math Online

For more information, go to geometryonline.com.

MONUMENTS For Exercises 37–40, refer to the photograph of the Lincoln Memorial.

37. Describe a pair of parallel lines found on the Lincoln Memorial.

38. Find an example of parallel planes.

39. Locate a pair of skew lines.

40. Identify a transversal passing through a pair of lines.

Name the transversal that forms each pair of angles. Then identify the special name for the angle pair.

41. ∠3 and ∠10

42. ∠2 and ∠12

43. ∠8 and ∠14

44. ∠9 and ∠16
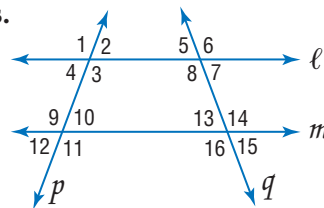
STRUCTURES For Exercises 45-47, refer to the drawing of the gazebo at the right.

45. Name all labeled segments parallel to $\overline{BF}$.

46. Name all labeled segments skew to $\overline{AC}$.

47. Are any of the planes on the gazebo parallel to plane *ADE*? Explain.
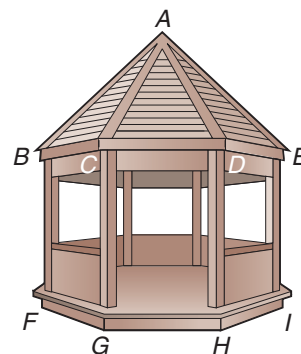
48. RESEARCH The word *parallel* describes computer processes that occur simultaneously, or devices, such as printers, that receive more than one bit of data at a time. Find two other examples for uses of the word *parallel* in other subject areas such as history, music, or sports.

H.O.T. Problems

49. OPEN ENDED Draw a solid figure with parallel planes. Describe which parts of the figure are parallel.

50. FIND THE ERROR Juanita and Eric are naming alternate interior angles in the figure at the right. One of the angles must be ∠4. Who is correct? Explain your reasoning.
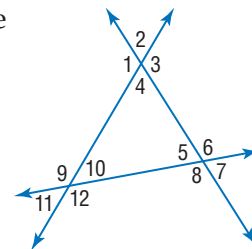
> **Juanita**
>
> ∠4 and ∠9
>
> ∠4 and ∠6

> **Eric**
>
> ∠4 and ∠10
>
> ∠4 and ∠5

CHALLENGE Suppose there is a line ℓ and a point *P* not on the line.

51. In space, how many lines can be drawn through *P* that do not intersect ℓ?

52. In space, how many lines can be drawn through *P* that are parallel to ℓ?

53. *Writing in Math* Use the information about architecture on page 142 to explain how parallel lines and planes are used in architecture. Include a description of where you might find examples of parallel lines and parallel planes, and skew lines and nonparallel planes.

54. Which of the following angle pairs are alternate exterior angles?
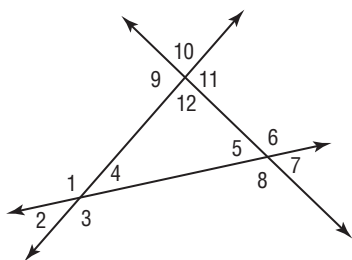
  A $\angle 1$ and $\angle 5$

  B $\angle 2$ and $\angle 10$

  C $\angle 2$ and $\angle 6$

  D $\angle 5$ and $\angle 9$

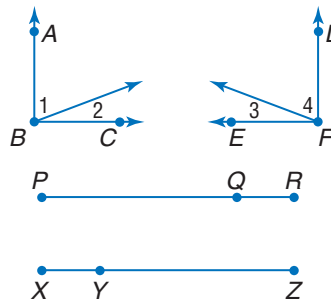**55. REVIEW** Which coordinate points represent the x- and y-intercepts of the graph shown below?
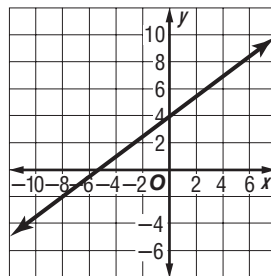
  F $(-5.6, 0), (0, 4)$

  G $(5.6, 0), (4, 0)$

  H $(6, 0), (0, 4)$

  J $(0, 4), (0, 6)$

Spiral Review

56. PROOF Write a two-column proof. (Lesson 2-8)

  Given: $m\angle ABC = m\angle DFE, m\angle 1 = m\angle 4$

  Prove: $m\angle 2 = m\angle 3$

57. PROOF Write a paragraph proof. (Lesson 2-7)

  Given: $\overline{PQ} \cong \overline{ZY}, \overline{QR} \cong \overline{XY}$

  Prove: $\overline{PR} \cong \overline{XZ}$

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment or the Law of Syllogism. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write *no conclusion.* (Lesson 2-4)

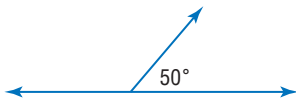**58.** (1) If two angles are vertical, then they do not form a linear pair.

  (2) If two angles form a linear pair, then they are not congruent.

59. (1) If an angle is acute, then its measure is less than 90.

  (2) $\angle EFG$ is acute.

GET READY for the Next Lesson

PREREQUISITE SKILL State the measures of linear pairs of angles in each figure. (Lesson 2-6)
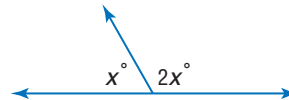
60.

61.

**62.**

Geometry Software Lab
Angles and Parallel Lines

You can use The Geometer's Sketchpad® to investigate the measures of angles formed by two parallel lines and a transversal.

ACTIVITY

Step 1   Draw parallel lines.

- Construct points A and B.
- Construct a line through the points.
- Place point C so that it does not lie on $\overleftrightarrow{AB}$.
- Construct a line through C parallel to $\overleftrightarrow{AB}$.
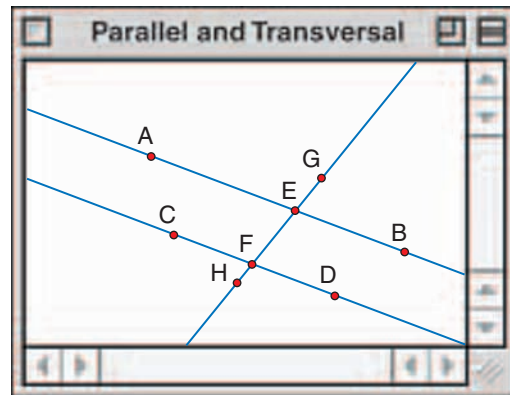- Place point D on this line.
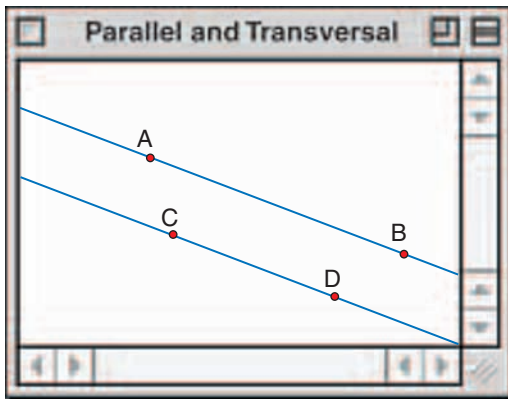
Step 2   Construct a transversal.

- Place point E on $\overleftrightarrow{AB}$ and point F on $\overleftrightarrow{CD}$.
- Construct a line through points E and F.
- Place points G and H on $\overleftrightarrow{EF}$.

Step 3   Measure angles.

- Measure each angle.

Parallel and Transversal

Parallel and Transversal

ANALYZE THE RESULTS

1. List pairs of angles by the special names you learned in Lesson 3-1. Which pairs have the same measure?

2. What is the relationship between consecutive interior angles?

3. **Make a conjecture** about the following pairs of angles formed by two parallel lines and a transversal. Write your conjecture in if-then form.
   - **a.** corresponding angles
   - **b.** alternate interior angles
   - **c.** alternate exterior angles
   - **d.** consecutive interior angles

4. Rotate the transversal. Are the angles with equal measures in the same relative location as the angles with equal measures in your original drawing?

5. Test your conjectures by rotating the transversal and analyzing the angles.

6. Rotate the transversal so that the measure of at least one angle is 90.
   - **a.** What do you notice about the measures of the other angles?
   - **b.** Make a conjecture about a transversal that is perpendicular to one of two parallel lines.

Angles and Parallel Lines

Main Ideas

- Use the properties of parallel lines to determine congruent angles.
- Use algebra to find angle measures.

▶ GET READY for the Lesson

In the painting, the artist uses lines and transversals to create patterns. The figure on the painting shows two parallel lines with a transversal passing through them. There is a special relationship between the angle pairs formed by these lines.
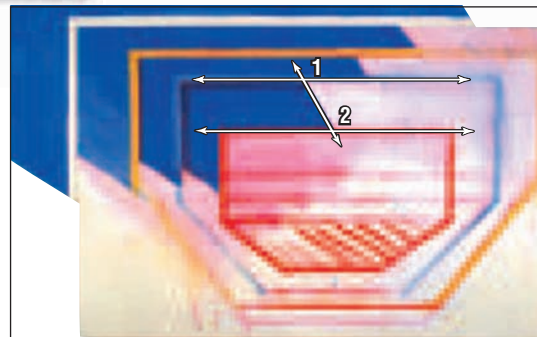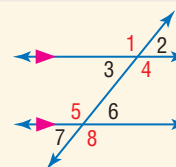
Source: *Vista* by Richard Smith

Parallel Lines and Angle Pairs In the figure above, ∠1 and ∠2 are corresponding angles. When the two lines are parallel, there is a special relationship between these pairs of angles.

POSTULATE 3.1   *Corresponding Angles Postulate*

If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

Examples: ∠1 ≅ ∠5, ∠2 ≅ ∠6, ∠3 ≅ ∠7, ∠4 ≅ ∠8

Review Vocabulary

Vertical Angles two nonadjacent angles formed by two intersecting lines (Lesson 1-6)

EXAMPLE   Determine Angle Measures

① **In the figure, $m\angle 3 = 133$. Find $m\angle 5$.**

∠3 ≅ ∠7	Corresponding Angles Postulate
∠7 ≅ ∠5	Vertical Angles Theorem
∠3 ≅ ∠5	Transitive Property
$m\angle 3 = m\angle 5$	Definition of congruent angles
$133 = m\angle 5$	Substitution
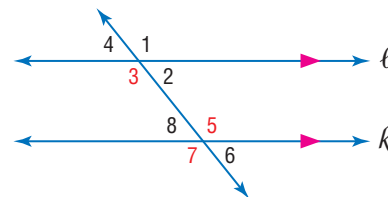
✓ CHECK Your Progress

1. In the figure, $m\angle 8 = 47$. Find $m\angle 4$.

In Example 1, alternate interior angles 3 and 5 are congruent. This suggests another special relationship between angles formed by two parallel lines and a transversal. Other relationships are summarized in Theorems 3.1, 3.2, and 3.3.

THEOREM   Parallel Lines and Angle Pairs

Theorems	Examples	Model
3.1 Alternate Interior Angles If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.	$\angle 4 \cong \angle 5$ $\angle 3 \cong \angle 6$	
3.2 Consecutive Interior Angles If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.	$\angle 4$ and $\angle 6$ are supplementary. $\angle 3$ and $\angle 5$ are supplementary.	
3.3 Alternate Exterior Angles If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.	$\angle 1 \cong \angle 8$ $\angle 2 \cong \angle 7$	

You will prove Theorems 3.2 and 3.3 in Exercises 26 and 23, respectively.

Proof Theorem 3.1

Given: $a \parallel b$; p is a transversal of a and b.

Prove: $\angle 2 \cong \angle 7, \angle 3 \cong \angle 6$

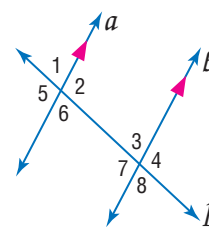**Paragraph Proof:**   We are given that $a \parallel b$ with a transversal p. By the Corresponding Angles Postulate, $\angle 2 \cong \angle 4$ and $\angle 8 \cong \angle 6$. Also, $\angle 4 \cong \angle 7$ and $\angle 3 \cong \angle 8$ because vertical angles are congruent. Therefore, $\angle 2 \cong \angle 7$ and $\angle 3 \cong \angle 6$ since congruence of angles is transitive.

A special relationship occurs when the transversal is a perpendicular line.

THEOREM 3.4   Perpendicular Transversal Theorem

In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

Proof Theorem 3.4

Given: $p \parallel q, t \perp p$

Prove: $t \perp q$
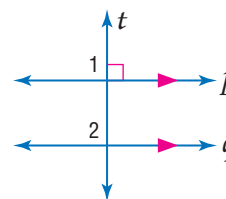
Proof:

Statements	Reasons
1. $p \parallel q, t \perp p$	**1.** Given
2. $\angle 1$ is a right angle.	**2.** Definition of $\perp$ lines
3. $m\angle 1 = 90$	**3.** Definition of right angle
4. $\angle 1 \cong \angle 2$	**4.** Corresponding Angles Postulate
5. $m\angle 1 = m\angle 2$	**5.** Definition of congruent angles
6. $m\angle 2 = 90$	**6.** Substitution Property
7. $\angle 2$ is a right angle.	**7.** Definition of right angles
8. $t \perp q$	**8.** Definition of $\perp$ lines

2 What is $m\angle GHI$?

A 50° **C** 130°

B 110° **D** 140°
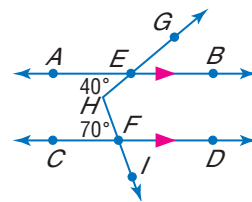
Read the Test Item

You need to find $m\angle GHI$.

Solve the Test Item

Draw $\overleftrightarrow{JK}$ through H parallel to $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

$\angle EHK \cong \angle AEH$ Alternate Interior Angles Theorem

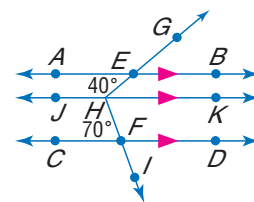$m\angle EHK = m\angle AEH$ Definition of congruent angles

$m\angle EHK = 40$ Substitution

$\angle FHK \cong \angle CFH$ Alternate Interior Angles Theorem

$m\angle FHK = m\angle CFH$ Definition of congruent angles

$m\angle FHK = 70$ Substitution

$m\angle GHI = m\angle EHK + m\angle FHK$ Angle Addition Postulate

$\qquad = 40 + 70$ or 110 $\quad m\angle EHK = 40, m\angle FHK = 70$

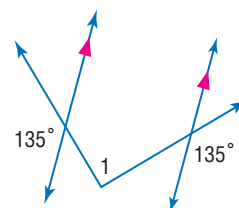Thus, the answer is choice B.

CHECK Your Progress

2. What is $m\angle 1$?

F 45° **H** 90°

G 65° **J** 135°

Online **Personal Tutor at** geometryonline.com

Algebra and Angle Measures Angles formed by two parallel lines and a transversal can be used to find unknown values.

EXAMPLE **Find Values of Variables**

3 **ALGEBRA** If $m\angle 1 = 3x + 40$ and $m\angle 3 = 2x + 70$, find x.

Since $\overleftrightarrow{FG} \parallel \overleftrightarrow{EH}$, $\angle 1 \cong \angle 3$ by the Corresponding Angles Postulate.

$m\angle 1 = m\angle 3$ Definition of congruent angles

$3x + 40 = 2x + 70$ Substitution

$x = 30$ Subtract $2x$ and 40 from each side.
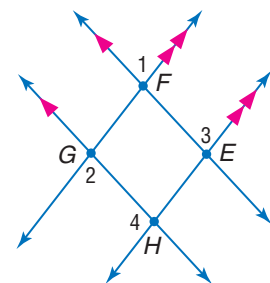
CHECK Your Progress

3. Refer to the figure. If $m\angle 2 = 4x + 7$ and $m\angle 3 = 5x - 13$, find $m\angle 3$.

Example 1
(p. 149)

In the figure, $m\angle 3 = 110$ and $m\angle 12 = 55$. Find the measure of each angle.

1. $\angle 1$ **2.** $\angle 6$ **3.** $\angle 2$

Example 2
(p. 151)

4. STANDARDIZED TEST PRACTICE What is $m\angle 1$?

A $5°$ **C** $36°$

B $31°$ **D** $67°$

Example 3
(p. 151)

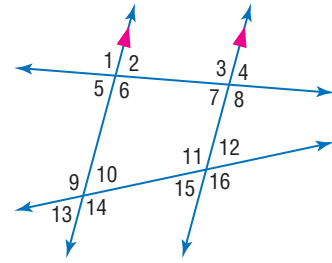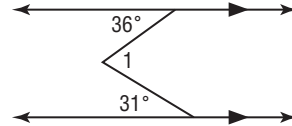Find x and y in each figure.

5. $(8y + 2)°$ $(25y - 20)°$ $10x°$

6. $(3y + 1)°$ $(4x - 5)°$ $(3x + 11)°$
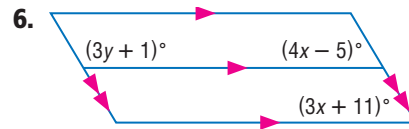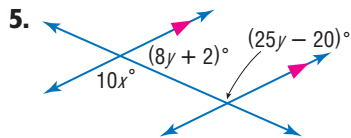
Exercises

In the figure, $m\angle 3 = 43$. Find the measure of each angle.

7. $\angle 2$ **8.** $\angle 7$

9. $\angle 10$ **10.** $\angle 11$

11. $\angle 13$ **12.** $\angle 16$
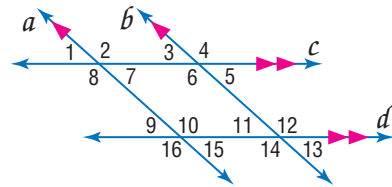
In the figure, $m\angle 1 = 50$ and $m\angle 3 = 60$. Find the measure of each angle.

13. $\angle 4$ **14.** $\angle 5$

15. $\angle 2$ **16.** $\angle 6$

17. $\angle 7$ **18.** $\angle 8$

Find x and y in each figure.

19. $4x°$ $56°$ $(3y - 11)°$

20. $(3x - 15)°$ $68°$ $2x°$ $(y^2)°$
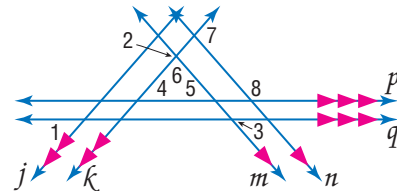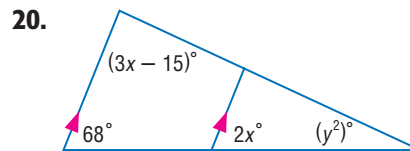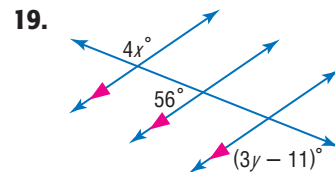
Find $m\angle 1$ in each figure.

21. $110°$ 1 $37°$

22. 1 $90°$ $157°$
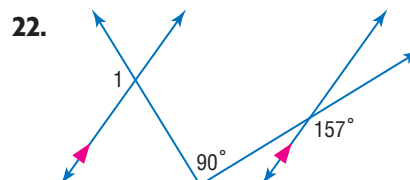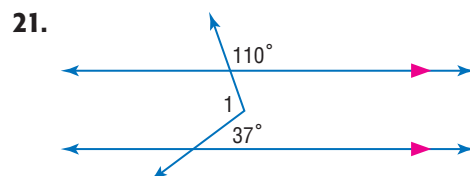
23. PROOF Copy and complete the proof of Theorem 3.3.
Given: $\ell \parallel m$
Prove: $\angle 1 \cong \angle 8$
$\angle 2 \cong \angle 7$
Proof:

Statements	Reasons
1. $\ell \parallel m$	**1.** ___?___
2. $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$	**2.** ___?___
3. $\angle 5 \cong \angle 8$, $\angle 6 \cong \angle 7$	**3.** ___?___
4. $\angle 1 \cong \angle 8$, $\angle 2 \cong \angle 7$	**4.** ___?___

24. CARPENTRY Anthony is building a picnic table for his patio. He cut one of the legs at an angle of 40°. At what angle should he cut the other end to ensure that the top of the table is parallel to the ground? Explain.
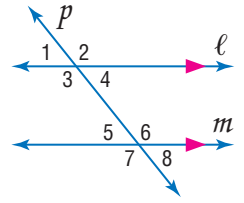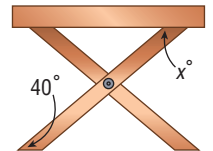
25. CONSTRUCTION Parallel drainage pipes are connected with a third pipe. The connector pipe makes a 65° angle with a pipe as shown. What is the measure of the angle it makes with the pipe on the other side of the road? Explain.

connector pipe

pipe

65°

?°
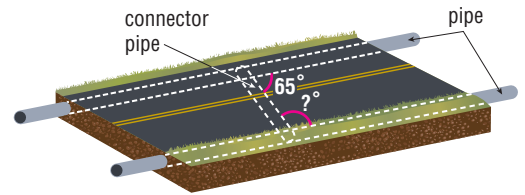
26. PROOF Write a two-column proof of Theorem 3.2.

Refer to the figure for Exercises 27 and 28.
27. Determine whether $\angle 1$ is *always, sometimes,* or *never* congruent to $\angle 2$. Explain.
28. Determine the minimum number of angle measures you would have to know to find the measures of all of the angles in the figure.
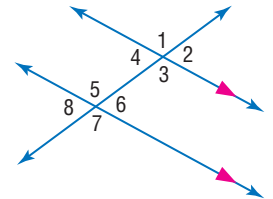
H.O.T. Problems

29. OPEN ENDED Use a straightedge and protractor to draw a pair of parallel lines cut by a transversal so that one pair of corresponding angles measures 35°.

30. REASONING Make a conjecture about two exterior angles on the same side of a transversal. Prove your conjecture.

31. CHALLENGE Explain why you can conclude that $\angle 2$ and $\angle 6$ are supplementary, but you cannot state that $\angle 4$ and $\angle 6$ are necessarily supplementary.
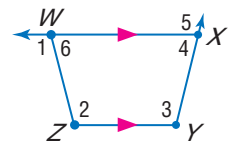
32. *Writing in Math* Use the information about art from page 149 to explain how angles and lines can be used in art. Include a description of how angles and lines are used to create patterns and examples from two different artists that use lines and angles.

33. An architect wants to design a shopping district between Huntington Avenue, Payton Drive, and Mesopotamia Boulevard.
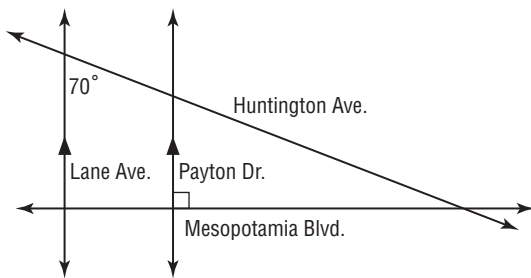
What are the measures of the three angles of the shopping district?

A 90°, 70°, 20°

B 90°, 62°, 38°

C 90°, 60°, 30°

D 100°, 30°, 20°

34. REVIEW Emma has been hiring more workers for her donut shop. The table shows the number of additional workers compared to the number of donuts the shop can make in an hour.

Additional Workers	Donuts Made
0	45
1	70
2	95
3	120

Which equation best describes the relationship between w, the number of additional workers, and d, the number of donuts the shop can make in an hour?

F $45w + 25 = d$

G $d - 45 = 25w$

H $d + 45 = 25w$

J $45w - 25 = d$

Spiral Review

For Exercises 35–37, refer to the figure at the right. (Lesson 3-1)

35. Name all segments parallel to $\overline{AB}$.

36. Name all segments skew to $\overline{CH}$.

37. Name all planes parallel to AEF.
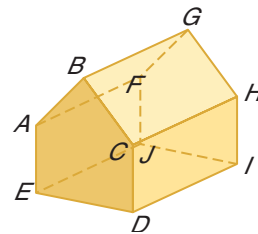
Find the measure of each numbered angle. (Lesson 2-8)

38.

39.
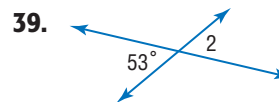
Identify the hypothesis and conclusion of each statement. (Lesson 2-3)

40. If it rains this evening, then I will mow the lawn tomorrow.

41. A balanced diet will keep you healthy.

GET READY for the Next Lesson

PREREQUISITE SKILL Simplify each expression.

42. $\dfrac{7-9}{8-5}$

43. $\dfrac{-3-6}{2-8}$

44. $\dfrac{14-11}{23-15}$

45. $\dfrac{15-23}{14-11}$

46. $\dfrac{2}{9} \cdot \left(-\dfrac{18}{5}\right)$

Graphing Calculator Lab
Investigating Slope

The rate of change of the steepness of a line is called the *slope*. Slope can be used to investigate the relationship between real-world quantities.

▶ SET UP the Lab

- Connect the data collection device to the graphing calculator. Place on a desk or table so that the data collection device can read the motion of a walker.
- Mark the floor at a distance of 1 meter and 6 meters from the device.

ACTIVITY

Step 1   Have one group member stand at the 1-meter mark. When another group member presses the button to begin collecting data, the walker begins to walk away from the device. Walk at a slow, steady pace.

Step 2   Stop collecting data when the walker passes the 6-meter mark. Save the data as Trial 1.

Step 3   Repeat the experiment, walking more quickly. Save the data as Trial 2.

Step 4   For Trial 3, repeat the experiment by walking toward the data collection device slowly.

Step 5   Repeat the experiment, walking quickly toward the device. Save the data as Trial 4.

ANALYZE THE RESULTS

1. Compare and contrast the graphs for Trials 1 and 2.

2. Use the TRACE feature of the calculator to find the coordinates of two points on each graph. Record the coordinates in a table like the one shown. Then use the points to find the slope of the line.

Trial	Point A (x_1, y_1)	Point B (x_2, y_2)	Slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$
1			
2			

3. Compare and contrast the slopes for Trials 1 and 2.

4. The slope of a line describes the rate of change of the quantities represented by the *x*- and *y*-values. What is represented by the rate of change in this experiment?

5. **MAKE A CONJECTURE** What would the graph look like if you were to collect data while the walker was standing still? Use the data collection device to test your conjecture.

Main Ideas

- Find slopes of lines.
- Use slope to identify parallel and perpendicular lines.

New Vocabulary

slope
rate of change

▶ GET READY for the Lesson

Traffic signs are often used to alert drivers to road conditions. The sign at the right indicates a hill with a 6% *grade*. This means that the road will rise or fall 6 feet vertically for every 100 horizontal feet traveled.

6%

Slope of a Line   The **slope** of a line is the ratio of its vertical rise to its horizontal run.

$$\text{slope} = \frac{\text{vertical rise}}{\text{horizontal run}}$$

You can use the coordinates of points on a line to derive a formula for slope. In a coordinate plane, the slope of a line is the ratio of the change along the y-axis to the change along the x-axis. The vertical rise is computed by finding the difference in y-values of the coordinates of two points on the line. Likewise, the horizontal run is defined by the difference in x-values of the coordinates of two points on the line.
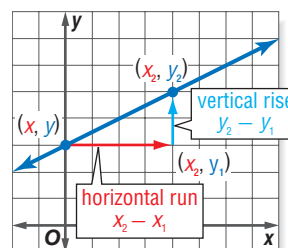
KEY CONCEPT   *Slope Formula*

The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

Study Tip

Slope

Lines with positive slope *rise* as you move from left to right, while lines with negative slope *fall* as you move from left to right.

The slope of a line indicates whether the line rises to the right, falls to the right, or is horizontal. The slope of a vertical line, where $x_1 = x_2$, is undefined.

Study Tip

Common Misconception

A line with a slope of 0 is a horizontal line. The slope of a vertical line is undefined.

1 Find the slope of each line.

a.

From $(-3, -2)$ to $(-1, 2)$, go up 4 units and right 2 units.

$$\frac{\text{rise}}{\text{run}} = \frac{4}{2} \text{ or } 2$$
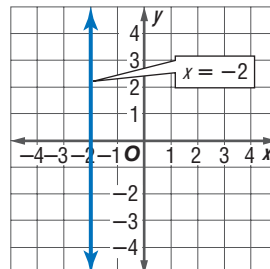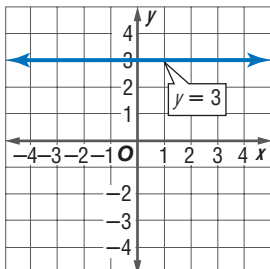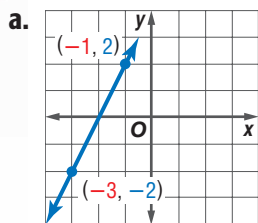
b.
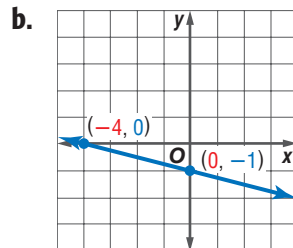
Let $(-4, 0)$ be (x_1, y_1) and $(0, -1)$ be (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}$$

$$= \frac{-1 - 0}{0 - (-4)} \text{ or } -\frac{1}{4}$$

c.
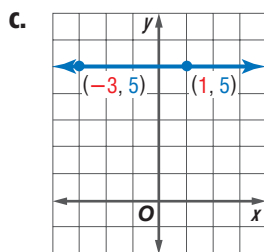
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 5}{-3 - 1}$$

$$= \frac{0}{-4} \text{ or } 0$$
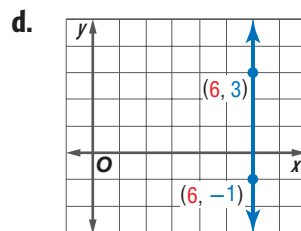
d.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 - (-1)}{6 - 6}$$

$$= \frac{4}{0}, \text{ which is undefined}$$

CHECK Your Progress

1A. the line containing $(-6, -2)$ and $(3, -5)$

1B. the line containing $(8, -3)$ and $(-6, -2)$

The slope of a line can be used to identify the coordinates of any point on the line. It can also be used to describe a rate of change. The **rate of change** describes how a quantity is changing over time.

Real-World Link

Between 2000 and 2003, annual sales of exercise equipment increased by an average rate of $314.3 million per year. In 2003, the total sales were $4553 million.

Source: *Statistical Abstract of the United States*

Real-World EXAMPLE **Use Rate of Change to Solve a Problem**

2 **FITNESS** Refer to the information at the left. If sales of fitness equipment increase at the same rate, what will the total sales be in 2010?

Let $(x_1, y_1) = (2003, 4553)$ and $m = 314.3$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$314.3 = \frac{y_2 - 4553}{2010 - 2003} \quad m = 314.3, y_1 = 4553, x_1 = 2003, \text{ and } x_2 = 2010$$

$$314.3 = \frac{y_2 - 4553}{7} \quad \text{Simplify.}$$

(continued on the next page)

Marco Garcia/Stringer/Getty Images

$2200.1 = y_2 - 4553$ Multiply each side by 7.

$6753.1 = y_2$ Add 4553 to each side.

The coordinates of the point representing the sales for 2010 are $(2010, 6753.1)$. Thus, the total sales in 2010 will be about $6753.1 million.

CHECK Your Progress

2. DOWNLOADS In 2004, 200 million songs were legally downloaded from the Internet. In 2003, 20 million songs were legally downloaded. If this increases at the same rate, how many songs will be legally downloaded in 2008?

Personal Tutor at geometryonline.com

Parallel and Perpendicular Lines In the Geometry Lab, you will explore the slopes of parallel and perpendicular lines.

GEOMETRY LAB

Parallel and Perpendicular Lines

MAKE A MODEL

Materials: dry spaghetti, grid paper

1. Place a piece of spaghetti on the grid paper through points $(-3, 0)$ and $(2, -3)$. Label this line ℓ.

2. Place a second piece of spaghetti on the grid such that it is parallel to line ℓ. Label this line m.

3. Place a third piece of spaghetti so it is perpendicular to lines ℓ and m. Label this line n.
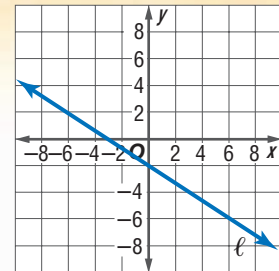
ANALYZE THE MODEL

1. What is the slope of line ℓ?

2. List two points that line m contains. What is the slope of line m?

3. List two points that line n contains. Determine the slope of line n.

4. Compare the slopes of line n and line m.

MAKE A CONJECTURE

5. Make a conjecture about the slopes of two parallel lines.

6. Make a conjecture about the slopes of two perpendicular lines.

7. Test your conjectures with different points.

The Geometry Lab suggests two important algebraic properties of parallel and perpendicular lines.

Study Tip

Look Back

To review **if and only if statements**, see Reading Math, page 98.

POSTULATES   *Parallel and Perpendicular Lines*

3.2 Two nonvertical lines have the same slope if and only if they are parallel.

3.3 Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Determine Line Relationships

3 Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*.

a. $A(-2, -5), B(4, 7), C(0, 2), D(8, -2)$

Find the slopes of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

slope of $\overleftrightarrow{AB} = \dfrac{7 - (-5)}{4 - (-2)}$ slope of $\overleftrightarrow{CD} = \dfrac{-2 - 2}{8 - 0}$

$\qquad\qquad = \dfrac{12}{6}$ or 2 $= -\dfrac{4}{8}$ or $-\dfrac{1}{2}$

The product of the slopes is $2\left(-\dfrac{1}{2}\right)$ or -1. So, $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.

b. $A(-8, -7), B(4, -4), C(-2, -5), D(1, 7)$

slope of $\overleftrightarrow{AB} = \dfrac{-4 - (-7)}{4 - (-8)}$ slope of $\overleftrightarrow{CD} = \dfrac{7 - (-5)}{1 - (-2)}$

$\qquad\qquad = \dfrac{3}{12}$ or $\dfrac{1}{4}$ $= \dfrac{12}{3}$ or 4

The slopes are not the same, so $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are not parallel. The product of the slopes is $4\left(\dfrac{1}{4}\right)$ or 1. So, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are neither parallel nor perpendicular.

Concepts in Motion

Interactive Lab
geometryonline.com

✓ CHECK Your Progress

3A. $A(14, 13), B(-11, 0), C(-3, 7), D(-4, -5)$

3B. $A(3, 6), B(-9, 2), C(-12, -6), D(15, 3)$

The relationships of the slopes of lines can be used to graph a line parallel or perpendicular to a given line.

EXAMPLE **Use Slope to Graph a Line**

4 Graph the line that contains $P(-2, 1)$ and is perpendicular to $\overleftrightarrow{JK}$ with $J(-5, -4)$ and $K(0, -2)$.

First, find the slope of $\overleftrightarrow{JK}$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$\quad = \dfrac{-2 - (-4)}{0 - (-5)}$ Substitution

$\quad = \dfrac{2}{5}$ Simplify.
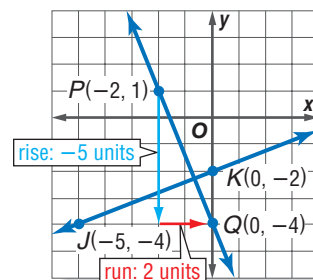
The product of the slopes of two perpendicular lines is -1.

Since $\dfrac{2}{5}\left(-\dfrac{5}{2}\right) = -1$, the slope of the line perpendicular to $\overleftrightarrow{JK}$ through $P(-2, 1)$ is $-\dfrac{5}{2}$.

Graph the line. Start at $(-2, 1)$. Move down 5 units and then move right 2 units.

Label the point Q. Draw $\overleftrightarrow{PQ}$.

Study Tip

Negative Slopes
To help determine direction with negative slopes, remember that
$-\dfrac{5}{2} = \dfrac{-5}{2} = \dfrac{5}{-2}$.

✓ CHECK Your Progress

4. Graph the line that contains $P(0, 1)$ and is perpendicular to $\overleftrightarrow{QR}$ with $Q(-6, -2)$ and $R(0, -6)$.

Example 1
(p. 157)

Find the slope of each line.

1. ℓ
2. m

Example 2
(pp. 157–158)

MOUNTAIN BIKING For Exercises 3–5, use the following information.
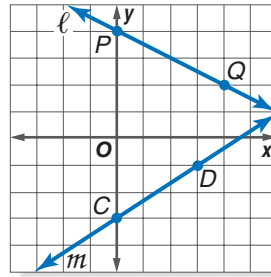A certain mountain bike trail has a section of trail with a grade of 8%.

3. What is the slope of the hill?

4. After riding on the trail, a biker is 120 meters below her original starting position. If her starting position is represented by the origin on a coordinate plane, what are possible coordinates of her current position?

5. How far has she traveled down the hill? Round to the nearest meter.

Example 3
(p. 159)

6. Determine whether $\overleftrightarrow{GH}$ and $\overrightarrow{RS}$ are *parallel*, *perpendicular*, or *neither* given $G(15, -9)$, $H(9, -9)$, $R(-4, -1)$, and $S(3, -1)$.

Example 4
(p. 159)

Graph the line that satisfies each condition.

7. slope $= 2$, contains $P(1, 2)$

8. contains $A(6, 4)$, perpendicular to $\overleftrightarrow{MN}$ with $M(5, 0)$ and $N(1, 2)$

Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
9–12, 17–20	1
21, 22	2
13–16, 23–28	3
29–36	4

Find the slope of each line.

9. $\overleftrightarrow{AB}$

10. $\overrightarrow{PQ}$

11. $\overleftrightarrow{LM}$

12. $\overleftrightarrow{EF}$

13. a line parallel to $\overleftrightarrow{LM}$

14. a line perpendicular to $\overrightarrow{PQ}$

15. a line perpendicular to $\overleftrightarrow{EF}$

16. a line parallel to $\overleftrightarrow{AB}$
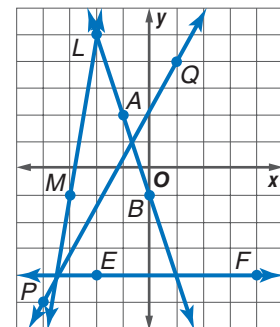
Determine the slope of the line that contains the given points.

17. $A(0, 2)$, $B(7, 3)$

18. $C(-2, -3)$, $D(-6, -5)$

19. $W(3, 2)$, $X(4, -3)$

20. $Y(1, 7)$, $Z(4, 3)$

21. **RECREATION** Paintball is one of the fastest growing sports. In 2002, 1,949,000 Americans from 12–17 years old participated in paintball. In 2005, 2,209,000 participated. If participation increases at the same rate, what will the participation be in 2012 to the nearest thousand?

22. TRAVEL On average, the rate of travel to Canada has been increasing by 486,500 visitors per year. In 2002, 16,161,000 Americans visited Canada. Approximately how many people will visit Canada in 2010?

Determine whether $\overleftrightarrow{PQ}$ and $\overleftrightarrow{UV}$ are *parallel*, *perpendicular*, or *neither*.

23. $P(-3, -2)$, $Q(9, 1)$, $U(3, 6)$, $V(5, -2)$ **24.** $P(-4, 0)$, $Q(0, 3)$, $U(-4, -3)$, $V(8, 6)$
25. $P(-10, 7)$, $Q(2, 1)$, $U(4, 0)$, $V(6, 1)$ **26.** $P(-9, 2)$, $Q(0, 1)$, $U(-1, 8)$, $V(-2, -1)$
27. $P(1, 1)$, $Q(9, 8)$, $U(-6, 1)$, $V(2, 8)$ **28.** $P(5, -4)$, $Q(10, 0)$, $U(9, -8)$, $V(5, -13)$

Graph the line that satisfies each condition.

29. slope $= -4$, passes through $P(-2, 1)$
30. contains $A(-1, -3)$, parallel to $\overleftrightarrow{CD}$ with $C(-1, 7)$ and $D(5, 1)$
31. contains $M(4, 1)$, perpendicular to $\overleftrightarrow{GH}$ with $G(0, 3)$ and $H(-3, 0)$
32. slope $= \frac{2}{5}$, contains $J(-7, -1)$
33. contains $Q(-2, -4)$, parallel to $\overleftrightarrow{KL}$ with $K(2, 7)$ and $L(2, -12)$
**34.** contains $W(6, 4)$, perpendicular to $\overleftrightarrow{DE}$ with $D(0, 2)$ and $E(5, 0)$.

35. Determine the value of x so that a line containing $(6, 2)$ and $(x, -1)$ has a slope of $-\frac{3}{7}$. Then graph the line.
36. Find the value of x so that the line containing $(4, 8)$ and $(2, -1)$ is perpendicular to the line containing $(x, 2)$ and $(-4, 5)$. Graph the lines.

POPULATION For Exercises 37–39, refer to the graph.

37. Estimate the annual rate of change of the median age from 1970 to 2000.
38. If the median age continues to increase at the same rate, what will be the median age in 2010?
39. Suppose that after 2000, the median age increases by $\frac{1}{3}$ of a year annually. In what year will the median age be 40.6?

Median Age

Age (years) vs. Year

16.7 ... 35.3
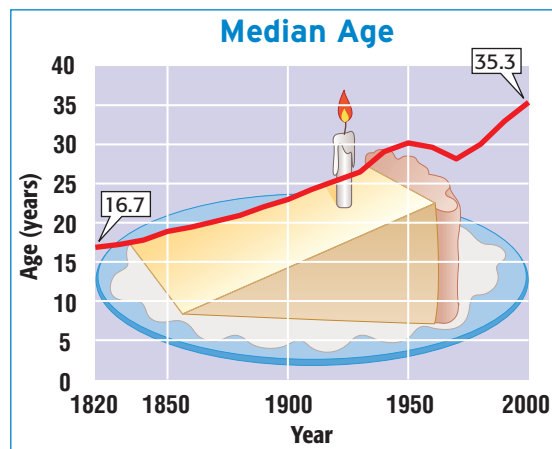
Source: Census Bureau

Math Online

STADIUMS For Exercises 40–42 use the following information.
Monster Park is home to the San Francisco 49ers. The attendance in 2000 was 541,960, and the attendance in 2004 was 518,271.

40. What is the approximate rate of change in attendance from 2000 to 2004?
41. If this rate of change continues, predict the attendance for 2012.
42. Will the attendance continue to decrease indefinitely? Explain.

H.O.T. Problems

43. FIND THE ERROR Curtis and Lori calculated the slope of the line containing $A(15, 4)$ and $B(-6, -13)$. Who is correct? Explain your reasoning.

Curtis
$$m = \frac{4 - (-13)}{15 - (-6)}$$
$$= \frac{17}{21}$$

Lori
$$m = \frac{4 - 13}{15 - 6}$$
$$= -1$$
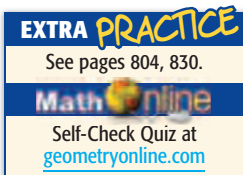
44. OPEN ENDED Give a real-world example of a line with a slope of 0 and a real-world example of a line with an undefined slope.

45. Which One Doesn't Belong? Identify the term that does not belong with the other three. Explain your reasoning.

| slope | rate of change | skew | steepness |

46. CHALLENGE The line containing the point $(5 + 2t, -3 + t)$ can be described by the equations $x = 5 + 2t$ and $y = -3 + t$. Write the slope-intercept form of the equation of this line.

47. *Writing in Math* Use the information about grade on page 156 to explain how slope is used in transportation. Include an explanation of why it is sometimes important to display the grade of a road and an example of slope used in transportation other than roads.

STANDARDIZED TEST PRACTICE

48. Which graph best represents the line passing through the point at $(-2, 5)$ and perpendicular to the graph of $y = \frac{2}{3}x$?

A

C

B

D
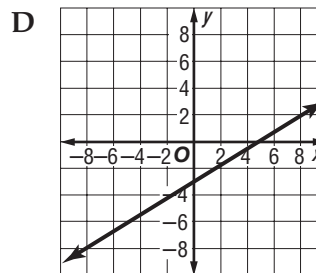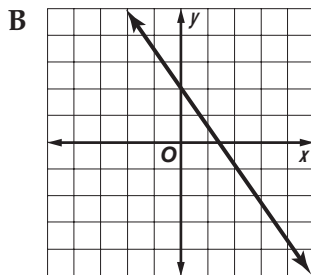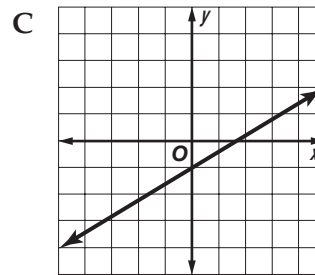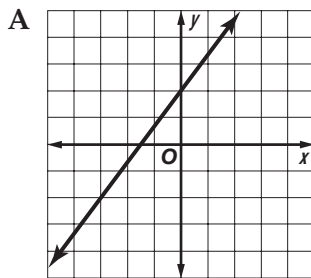
49. Which equation describes the line that passes through the point at $(-2, 1)$ and is perpendicular to the line $y = \frac{1}{3}x + 5$?

F $y = 3x + 7$

G $y = -3x - 5$

H $y = \frac{1}{3}x + 7$

J $y = -\frac{1}{3}x - 5$

50. REVIEW Which expression is equivalent to $4(x - 6) - \frac{1}{2}(x^2 + 8)$?
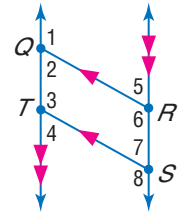
A $4x^2 + 4x - 28$

B $-\frac{1}{2}x^2 + 6x - 24$
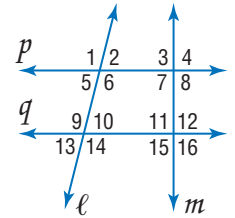
C $-\frac{1}{2}x^2 + 4x - 28$

D $3x - 20$

In the figure, $\overline{QR} \parallel \overline{TS}$, $\overleftrightarrow{QT} \parallel \overleftrightarrow{RS}$, and $m\angle 1 = 131$. Find the measure of each angle. (Lesson 3-2)
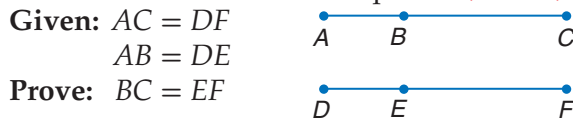
51. $\angle 6$ **52.** $\angle 7$ **53.** $\angle 4$

54. $\angle 2$ **55.** $\angle 5$ **56.** $\angle 8$

State the transversal that forms each pair of angles. Then identify the special name for each angle pair. (Lesson 3-1)

57. $\angle 1$ and $\angle 14$ **58.** $\angle 2$ and $\angle 10$

59. $\angle 3$ and $\angle 6$ **60.** $\angle 14$ and $\angle 15$

61. $\angle 7$ and $\angle 12$ **62.** $\angle 9$ and $\angle 11$

63. PROOF Write a two-column proof. (Lesson 2-6)
Given: $AC = DF$
$AB = DE$
Prove: $BC = EF$

Find the perimeter of $\triangle ABC$ to the nearest hundredth, given the coordinates of its vertices. (Lesson 1-6)

64. $A(10, -6)$, $B(-2, -8)$, $C(-5, -7)$ **65.** $A(-3, 2)$, $B(2, -9)$, $C(0, -10)$

DAYLIGHT SAVING TIME All of the states in the United States observe Daylight Saving Time except for Arizona and Hawaii. (Lesson 2-3)

66. Write a true conditional statement in if-then form for Daylight Saving Time.

67. Write the converse of the true conditional statement. State whether the statement is *true* or *false*. If false, find a counterexample.

Construct a truth table for each compound statement. (Lesson 2-2)

68. p and q

69. p or $\sim q$

70. $\sim p \wedge q$

71. $\sim p \wedge \sim q$

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. (Lesson 2-1)

72. Points H, I, and J are each located on different sides of a triangle.

73. Collinear points X, Y, and Z; Z is between X and Y.

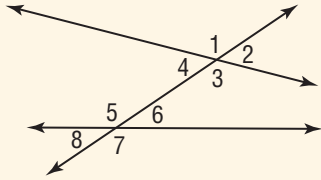**74.** $R(3, -4)$, $S(-2, -4)$, and $T(0, -4)$

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation for y. (Pages 781 and 782)

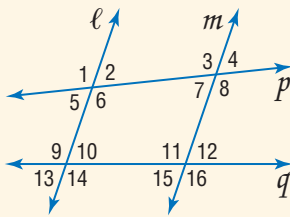**75.** $2x + y = 7$ **76.** $2x + 4y = 5$ **77.** $5x - 2y + 4 = 0$

1. MULTIPLE CHOICE $\angle 3$ and $\angle 5$ are _?_ angles. (Lesson 3-1)

A alternate exterior

B alternate interior

C consecutive interior

D corresponding

Name the transversal that forms each pair of angles. Then identify the special name for the angle pair. (Lesson 3-1)
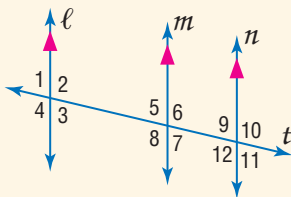

2. $\angle 1$ and $\angle 8$

3. $\angle 6$ and $\angle 10$

4. $\angle 11$ and $\angle 14$

Refer to the figure above. Find the measure of each angle if $\ell \parallel m$ and $m\angle 1 = 105$. (Lesson 3-2)

5. $\angle 6$ **6.** $\angle 4$

In the figure, $m\angle 9 = 75$. Find the measure of each angle. (Lesson 3-2)

7. $\angle 3$ **8.** $\angle 5$

9. $\angle 6$ **10.** $\angle 8$
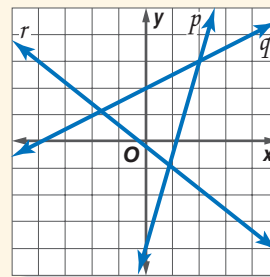
11. $\angle 11$ **12.** $\angle 12$

13. MULTIPLE CHOICE Find the slope of a line perpendicular to the line containing $(-5, 1)$ and $(-3, -2)$. (Lesson 3-3)

F $-\dfrac{2}{3}$

G $-\dfrac{3}{2}$

H $\dfrac{2}{3}$

J $\dfrac{3}{2}$

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*. (Lesson 3-3)

14. $A(3, -1), B(6, 1), C(-2, -2), D(2, 4)$

15. $A(-3, -11), B(3, 13), C(0, -6), D(8, -8)$

Find the slope of each line. (Lesson 3-3)

16. p

17. a line parallel to q

18. a line perpendicular to r

BASEBALL For Exercises 19 and 20 use the following information.

Minute Maid Ballpark in Houston is home to the Houston Astros. The average attendance per game in 2002 and 2004 are shown in the table. (Lesson 3-3)

Year	Average Attendance
2002	31,078
2004	38,122

19. What is the rate of change in average attendance per game from 2002 to 2004?

20. If this rate of change continues, predict the average attendance per game for the 2012 season.
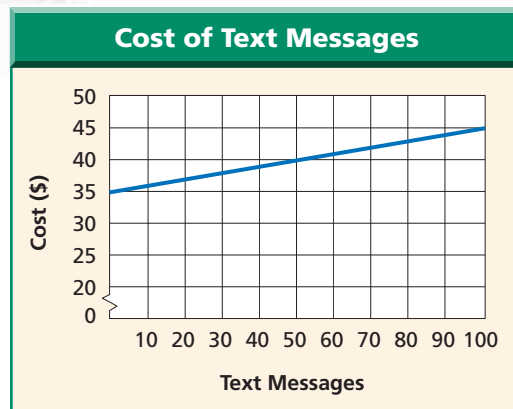
Equations of Lines

Main Ideas

- Write an equation of a line given information about its graph.
- Solve problems by writing equations.

New Vocabulary

slope-intercept form
point-slope form

GET READY for the Lesson

Julia's cell phone plan costs $35 per month for unlimited calls plus $0.10 per text message. The total charge C for a month can be represented by the equation $C = 0.1t + 35$.

Cost of Text Messages

(graph: Cost ($) on vertical axis from 0 to 50; Text Messages on horizontal axis from 10 to 100; a line rising from 35 at left to 45 at right)

Write Equations of Lines You may remember from algebra that an equation of a line can be written given any of the following:

- the slope and the y-intercept,
- the slope and the coordinates of a point on the line, or
- the coordinates of two points on the line.

The graph of $C = 0.1t + 35$ has a slope of 0.1, and it intersects the y-axis at 35. These two values can be used to write an equation of the line. The **slope-intercept form** of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y-intercept.

$$\overset{\text{slope}}{y = mx + \underset{y\text{-intercept}}{b}} \qquad C = \overset{\text{slope}}{0.1}t + \underset{y\text{-intercept}}{35}$$

Study Tip

Look Back

To review **writing an equation of a line**, see pages 786–787.

EXAMPLE Slope and y-Intercept

① Write an equation in slope-intercept form of the line with slope of -4 and y-intercept of 1.

$y = mx + b$ Slope-intercept form

$y = -4x + 1$ $m = -4, b = 1$

The slope-intercept form of the equation of the line is $y = -4x + 1$.

CHECK Your Progress

1. Write an equation in slope-intercept form of the line with slope of 3 and y-intercept of -8.

Another method used to write an equation of a line is the point-slope form of a linear equation. The **point-slope form** is $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of any point on the line and m is the slope of the line.

given point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

slope

EXAMPLE   Slope and a Point

2 Write an equation in point-slope form of the line with slope of $-\frac{1}{2}$ that contains $(3, -7)$.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-7) = -\frac{1}{2}(x - 3)$ $m = -\frac{1}{2}, (x_1, y_1) = (3, -7)$

$y + 7 = -\frac{1}{2}(x - 3)$ Simplify.

The point-slope form of the equation of the line is $y + 7 = -\frac{1}{2}(x - 3)$.

CHECK Your Progress

2. Write an equation in point-slope form of the line with slope of 4 that contains $(-3, -6)$.

Both the slope-intercept form and the point-slope form require the slope of a line in order to write an equation. There are occasions when the slope of a line is not given. In cases such as these, use two points on the line to calculate the slope. Then use either the slope-intercept form or the point-slope form to write an equation.
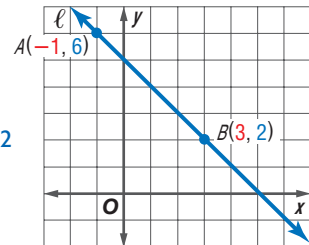
EXAMPLE   Two Points

3 Write an equation in slope-intercept form for line ℓ.

Find the slope of ℓ by using $A(-1, 6)$ and $B(3, 2)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \dfrac{2 - 6}{3 - (-1)}$ $x_1 = -1, x_2 = 3, y_1 = 6, y_2 = 2$

$= -\dfrac{4}{4}$ or -1 Simplify.

Now use the point-slope form and either point to write an equation.

Method 1 Use Point A.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 6 = -1[x - (-1)]$ $m = -1, (x_1, y_1) = (-1, 6)$

$y - 6 = -1(x + 1)$ Simplify.

$y - 6 = -x - 1$ Distributive Property

$y = -x + 5$ Add 6 to each side.

Method 2 Use Point B.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 2 = -1(x - 3)$	$m = -1, (x_1, y_1) = (3, 2)$
$y - 2 = -x + 3$	Distributive Property
$y = -x + 5$	Add 2 to each side.

The result is the same using either point.

CHECK Your Progress

3. Write an equation in slope-intercept form for the line that contains $(-2, 4)$ and $(8, 10)$.

EXAMPLE One Point and an Equation

4 Write an equation in slope-intercept form for a line containing (2, 0) that is perpendicular to the line with equation $y = -x + 5$.

Since the slope of the line $y = -x + 5$ is -1, the slope of a line perpendicular to it is 1.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 0 = 1(x - 2)$	$m = 1, (x_1, y_1) = (2, 0)$
$y = x - 2$	Distributive Property

CHECK Your Progress

4. Write an equation in slope-intercept form for a line containing $(-3, 6)$ that is parallel to the graph of $y = -\frac{3}{4}x + 3$.

Write Equations to Solve Problems Many real-world situations can be modeled using linear equations. In many business applications, the slope represents a *rate*.

Real-World EXAMPLE Write Linear Equations

5 **TEXT MESSAGING** Gracia's current wireless phone plan charges $39.95 per month for unlimited calls and $0.05 per text message.

a. Write an equation to represent the total monthly cost C for t text messages.

For each text message, the cost increases $0.05. So, the rate of change, or slope, is 0.05. The y-intercept is located where 0 messages are used, or $39.95.

$C = mt + b$	Slope-intercept form
$= 0.05t + 39.95$	$m = 0.05, b = 39.95$

The total monthly cost can be represented by the equation $C = 0.05t + 39.95$.

b. Compare her current plan to the plan presented at the beginning of the lesson. If she reads or sends an average of 150 text messages each month, which plan offers the better rate?

Evaluate each equation for $t = 150$.

Current plan: $C = 0.05t + 39.95$

$\qquad = 0.05(150) + 39.95 \qquad t = 150$

$\qquad = 47.45 \qquad$ Simplify.

Alternate plan: $C = 0.1t + 35$

$\qquad = 0.1(150) + 35 \qquad t = 150$

$\qquad = 50 \qquad$ Simplify.

Given her average usage, Gracia's current plan offers the better rate.

CHECK Your Progress

5. Suppose Gracia only sends or receives 50 text messages per month. Compare each plan. Which offers a better rate? Explain.

Online Personal Tutor at geometryonline.com

CHECK Your Understanding

Example 1
(p. 165)

Write an equation in slope-intercept form of the line having the given slope and y-intercept.

1. $m = \dfrac{1}{2}$

y-intercept: 4

2. $m = 3$

y-intercept: -4

3. $m = -\dfrac{3}{5}$

y-intercept at $(0, -2)$

Example 2
(p. 166)

Write an equation in point-slope form of the line having the given slope that contains the given point.

4. $m = \dfrac{3}{2}, (4, -1)$

5. $m = 3, (7, 5)$

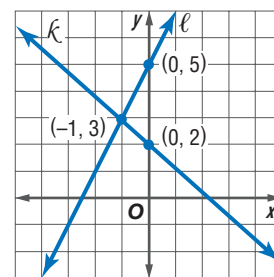**6.** $m = 1.25, (20, 137.5)$

Example 3
(pp. 166–167)

Write an equation in slope-intercept form for each line in the graph.

7. ℓ

8. k

Example 4
(p. 167)

9. the line parallel to ℓ that contains $(4, 4)$

10. the line perpendicular to ℓ that contains $(2, -1)$

Example 5
(pp. 167–168)

MUSIC For Exercises 11 and 12, use the following information.
Justin pays $5 per month for a subscription to an online music service. He pays $0.79 per song that he downloads. Another online music store offers 40 downloads per month for a monthly fee of $10.

11. Write an equation to represent the total monthly cost for each plan.

12. If Justin downloads 15 songs per month, should he keep his current plan, or change to the other plan? Explain.

Write an equation in slope-intercept form of the line having the given slope and y-intercept.

13. m: 3, y-intercept: -4 **14.** m: 2, $(0, 8)$ **15.** m: $\frac{5}{8}$, $(0, -6)$

16. m: $\frac{2}{9}$, y-intercept: $\frac{1}{3}$ **17.** m: -1, b: -3 **18.** m: $-\frac{1}{12}$, b: 1

Write an equation in point-slope form of the line with the given slope that contains the given point.

19. $m = 2$, $(3, 1)$ **20.** $m = -5$, $(4, 7)$ **21.** $m = -\frac{4}{5}$, $(-12, -5)$

22. $m = \frac{1}{16}$, $(3, 11)$ **23.** $m = 0.48$, $(5, 17.12)$ **24.** $m = -1.3$, $(10, 87.5)$

Write an equation in slope-intercept form for each line in the graph.

25. k

26. ℓ

27. m

28. n

29. perpendicular to line ℓ, contains $(-1, 6)$

30. parallel to line k, contains $(7, 0)$

31. parallel to line n, contains $(0, 0)$

32. perpendicular to line m, contains $(-3, -3)$
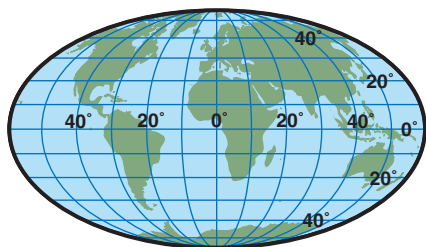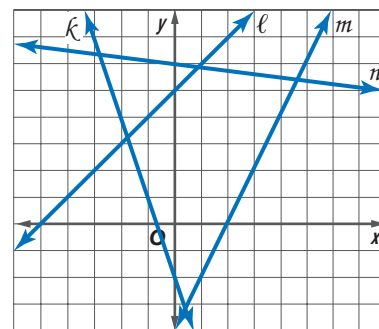
**BUSINESS** **For Exercises 33 and 34, use the following information.**
The Rainbow Paint Company sells an average of 750 gallons of paint each day.

33. The store has 10,800 gallons of paint in stock. Write an equation in slope-intercept form that describes how many gallons of paint will be on hand after x days if no new stock is added.

34. Draw a graph that represents the number of gallons of paint on hand at any given time.

MAPS **For Exercises 35 and 36, use the following information.**
Suppose a map of Pennsylvania is placed on a coordinate plane with the western corner of Lehigh County at the origin. Berks, Montgomery, and Lehigh Counties meet at $(80, -70)$, and Montgomery, Lehigh, and Bucks Counties meet at $(90, -80)$.

35. Write an equation in slope-intercept form that models the county line between Lehigh and Montgomery Counties.

36. The line separating Lehigh and Bucks Counties runs perpendicular to the Lehigh/Montgomery County line. Write an equation in slope-intercept form of the line that contains the Lehigh/Bucks County line.
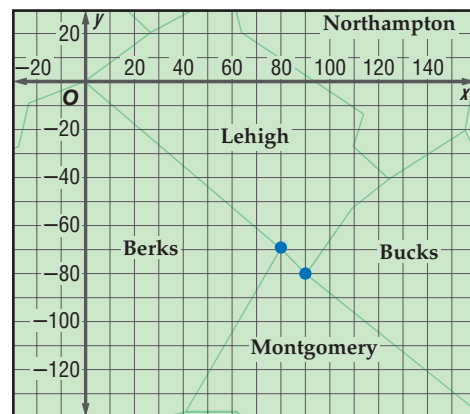
Write an equation in slope-intercept form for the line that satisfies the given conditions.

37. x-intercept $= 5$, y-intercept $= 3$

38. contains $(4, -1)$ and $(-2, -1)$

39. contains $(-5, -3)$ and $(10, -6)$

40. x-intercept $= 5$, y-intercept $= -1$

41. contains $(-6, 8)$ and $(-6, -4)$

42. contains $(-4, -1)$ and $(-8, -5)$

H.O.T. Problems

43. OPEN ENDED Write equations in slope-intercept form for two lines that contain $(-1, -5)$.

44. CHALLENGE The point-slope form of an equation of a line can be rewritten as $y = m(x - x_1) + y_1$. Describe how the graph of $y = m(x - x_1) + y_1$ is related to the graph of $y = mx$.

45. *Writing in Math* Use the information about wireless phone and text messages rates on page 165 to explain how the equation of a line can describe wireless telephone service. Include a description of how you can use equations to compare various plans.

STANDARDIZED TEST PRACTICE

46. REVIEW Jamie is collecting money to buy an $81 gift for her teacher. She has already contributed $24. She will collect $3 from each contributing student. If the equation below shows this relationship, from how many students must Jamie collect?

$$3s + 24 = 81$$

A 3 students   **C** 12 students

B 9 students   **D** 19 students

47. The graph of which equation passes through $(-3, -2)$ and is perpendicular to the graph of $y = \frac{3}{4}x + 8$?

F $y = -\frac{4}{3}x - 6$

G $y = -\frac{4}{3}x + 5$

H $y = \frac{3}{4}x + \frac{1}{4}$

J $y = -\frac{3}{4}x - 5$

Spiral Review

48. SOFTWARE In 2000, $498 million was spent on educational software. In 2004, the sales had dropped to $152 million. What is the rate of change between 2000 and 2004? (Lesson 3-3)

In the figure, $m\angle 1 = 58$, $m\angle 2 = 47$, and $m\angle 3 = 26$. Find the measure of each angle. (Lesson 3-2)
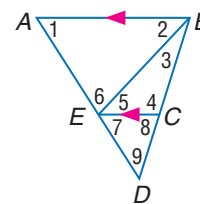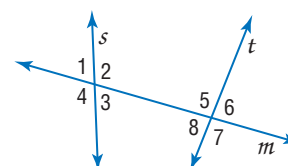
49. $\angle 7$ **50.** $\angle 5$ **51.** $\angle 6$

52. $\angle 4$ **53.** $\angle 8$ **54.** $\angle 9$

GET READY for the Next Lesson

PREREQUISITE SKILL Name the pairs of angles in the figure that meet each description. (Lesson 3-1)

55. consecutive interior angles

56. corresponding angles

57. alternate exterior angles

Geometry Lab
Equations of Perpendicular Bisectors

You can apply what you have learned about slope and equations of lines to geometric figures on a plane.

ACTIVITY

Find the equation of a line that is a perpendicular bisector of segment AB with endpoints $A(-3, 3)$ and $B(4, 0)$.

Step 1 A segment bisector contains the midpoint of the segment. Use the Midpoint Formula to find the midpoint M of $\overline{AB}$.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{-3 + 4}{2}, \frac{3 + 0}{2}\right)$$
$$= M\left(\frac{1}{2}, \frac{3}{2}\right)$$

Step 2 A perpendicular bisector is perpendicular to the segment through the midpoint. To find the slope of the bisector, first find the slope of $\overline{AB}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$
$$= \frac{0 - 3}{4 - (-3)} \qquad x_1 = -3, x_2 = 4, y_1 = 3, y_2 = 0$$
$$= -\frac{3}{7} \qquad \text{Simplify.}$$

Step 3 Now use the point-slope form to write the equation of the line. The slope of the bisector is $\frac{7}{3}$ since $-\frac{3}{7}\left(\frac{7}{3}\right) = -1$.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$
$$y - \frac{3}{2} = \frac{7}{3}\left(x - \frac{1}{2}\right) \qquad m = \frac{7}{3}, (x_1, y_1) = \left(\frac{1}{2}, \frac{3}{2}\right)$$
$$y - \frac{3}{2} = \frac{7}{3}x - \frac{7}{6} \qquad \text{Distributive Property}$$
$$y = \frac{7}{3}x + \frac{1}{3} \qquad \text{Add } \frac{3}{2} \text{ to each side.}$$
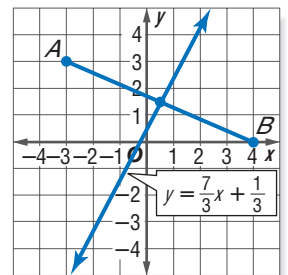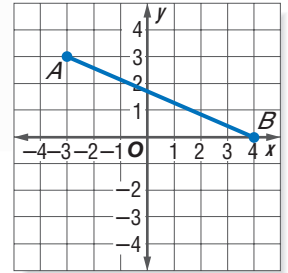
Exercises

Find the equation of the perpendicular bisector of $\overline{PQ}$ for the given endpoints.

1. $P(5, 2), Q(7, 4)$ **2.** $P(-3, 9), Q(-1, 5)$ **3.** $P(-6, -1), Q(8, 7)$

4. $P(-2, 1), Q(0, -3)$ **5.** $P(0, 1.6), Q(0.5, 2.1)$ **6.** $P(-7, 3), Q(5, 3)$

7. Extend what you have learned to find the equations of the lines that contain the sides of $\triangle XYZ$ with vertices $X(-2, 0)$, $Y(1, 3)$, and $Z(3, -1)$.

Proving Lines Parallel

GET READY for the Lesson

Have you ever been in a tall building and looked down at a parking lot? The parking lot is full of line segments that appear to be parallel. The workers who paint these lines must be certain that they are parallel.

Identify Parallel Lines When each stripe of a parking space intersects the center line, the angles formed are corresponding angles. If the lines are parallel, we know that the corresponding angles are congruent. Conversely, if the corresponding angles are congruent, then the lines must be parallel.

POSTULATE 3.4

If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

Abbreviation: If corr. ⊿ are ≅, then lines are ∥.

Examples: If ∠1 ≅ ∠5, ∠2 ≅ ∠6, ∠3 ≅ ∠7, or ∠4 ≅ ∠8, then $m \parallel n$.
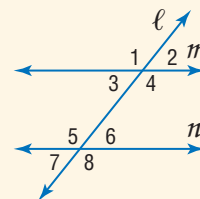
Postulate 3.4 justifies the construction of parallel lines.

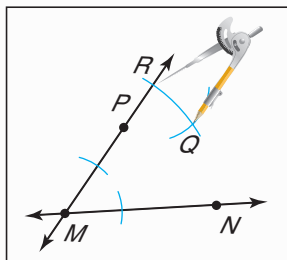**CONSTRUCTION**

Parallel Line Through a Point Not on Line

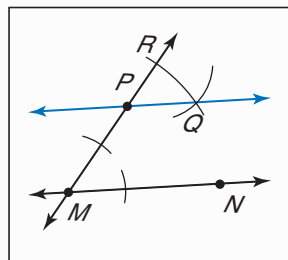**Step 1** Use a straightedge to draw $\overleftrightarrow{MN}$. Draw a point P that is not on $\overleftrightarrow{MN}$. Draw $\overleftrightarrow{PM}$.
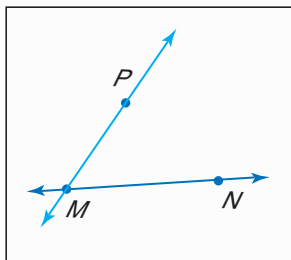
Step 2 Copy ∠PMN so that P is the vertex of the new angle. Label the intersection points Q and R.

Step 3 Draw $\overleftrightarrow{PQ}$. Because ∠RPQ ≅ ∠PMN by construction and they are corresponding angles, $\overrightarrow{PQ} \parallel \overleftrightarrow{MN}$.

The construction establishes that there is *at least* one line through P that is parallel to $\overleftrightarrow{MN}$. In 1795, Scottish physicist and mathematician John Playfair provided the modern version of Euclid's Parallel Postulate, which states there is *exactly* one line parallel to a line through a given point not on the line.

POSTULATE 3.5   *Parallel Postulate*

If given a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line.

Parallel lines with a transversal create many pairs of congruent angles. Conversely, those pairs of congruent angles can determine whether a pair of lines is parallel.

THEOREM   *Proving Lines Parallel*

	Theorems	Examples	Models
3.5	If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel. **Abbreviation:** *If alt. ext. ⊿ are ≅, then lines are ∥.*	If $\angle 1 \cong \angle 8$ or if $\angle 2 \cong \angle 7$, then $m \parallel n.$	
3.6	If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel. **Abbreviation:** *If cons. int. ⊿ are suppl., then lines are ∥.*	If $m\angle 3 + m\angle 5$ are supplementary or if $m\angle 4$ and $m\angle 6$ are supplementary, then $m \parallel n.$	
3.7	If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel. **Abbreviation:** *If alt. int. ⊿ are ≅, then lines are ∥.*	If $\angle 3 \cong \angle 6$ or if $\angle 4 \cong \angle 5$, then $m \parallel n.$	
3.8	In a plane, if two lines are perpendicular to the same line, then they are parallel. **Abbreviation:** *If 2 lines are ⊥ to the same line, then lines are ∥.*	If $\ell \perp m$ and $\ell \perp n$, then $m \parallel n.$	

You will prove Theorems 3.5, 3.6, 3.7, and 3.8 in Check Your Progress 3 and Exercises 21, 22, and 20, respectively.

EXAMPLE   **Identify Parallel Lines**

**1** In the figure, $\overline{BG}$ bisects $\angle ABH$. Determine which lines, if any, are parallel.

• The sum of the angle measures in a triangle must be 180, so $m\angle BDF = 180 - (45 + 65)$ or 70.

• Since $\angle BDF$ and $\angle BGH$ have the same measure, they are congruent.

• Congruent corresponding angles indicate parallel lines. So, $\overleftrightarrow{DF} \parallel \overleftrightarrow{GH}.$

- $\angle ABD \cong \angle DBF$, because $\overrightarrow{BG}$ bisects $\angle ABH$. So, $m\angle ABD = 45$.

- $\angle ABD$ and $\angle BDF$ are alternate interior angles, but they have different measures so they are not congruent.

- Thus, $\overleftrightarrow{AB}$ is not parallel to $\overleftrightarrow{DF}$ or $\overleftrightarrow{GH}$.

CHECK Your Progress

**1.** Given $\angle 2 \cong \angle 8$, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.
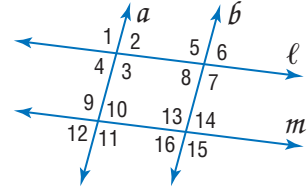
Angle relationships can be used to solve problems involving unknown values.

EXAMPLE Solve Problems with Parallel Lines

2 **ALGEBRA** Find x and $m\angle RSU$ so that $m \parallel n$.

Explore From the figure, you know that $m\angle RSU = 8x + 4$ and $m\angle STV = 9x - 11$. You also know that $\angle RSU$ and $\angle STV$ are corresponding angles.
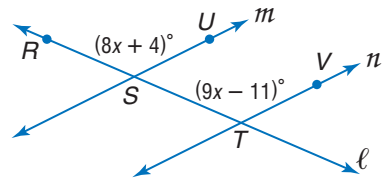
Plan For line m to be parallel to line n, the corresponding angles must be congruent. So, $m\angle RSU = m\angle STV$. Substitute the given angle measures into this equation and solve for x. Once you know the value of x, use substitution to find $m\angle RSU$.

Solve
$$m\angle RSU = m\angle STV \quad \text{Corresponding angles}$$
$$8x + 4 = 9x - 11 \quad \text{Substitution}$$
$$4 = x - 11 \quad \text{Subtract } 8x \text{ from each side.}$$
$$15 = x \quad \text{Add 11 to each side.}$$

Now use the value of x to find $m\angle RSU$.
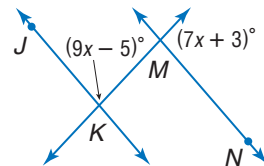
$$m\angle RSU = 8x + 4 \quad \text{Original equation}$$
$$= 8(15) + 4 \quad x = 15$$
$$= 124 \quad \text{Simplify.}$$

Check Verify the angle measure by using the value of x to find $m\angle STV$. That is, $9x - 11 = 9(15) - 11$ or 124. Since $m\angle RSU = m\angle STV$, $\angle RSU \cong \angle STV$ and $m \parallel n$.

Study Tip

Proving Lines Parallel

When proving lines parallel, be sure to check for congruent corresponding angles, alternate interior angles, alternate exterior angles, or supplementary consecutive interior angles.

CHECK Your Progress

2A. Find x so that $\overline{JK} \parallel \overline{MN}$.
2B. Find $m\angle JKM$.

Online Personal Tutor at geometryonline.com

Prove Lines Parallel The angle pair relationships formed by a transversal can be used to prove that two lines are parallel.
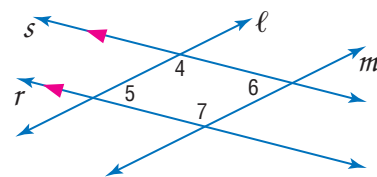
EXAMPLE Prove Lines Parallel

③ **Given:** $r \parallel s; \angle 5 \cong \angle 6$

Prove: $\ell \parallel m$

Proof:

Statements	Reasons
1. $r \parallel s; \angle 5 \cong \angle 6$	1. Given
2. $\angle 4$ and $\angle 5$ are supplementary.	2. Consecutive Interior Angle Theorem
3. $m\angle 4 + m\angle 5 = 180$	3. Definition of supplementary angles
4. $m\angle 5 = m\angle 6$	4. Definition of congruent angles
5. $m\angle 4 + m\angle 6 = 180$	5. Substitution
6. $\angle 4$ and $\angle 6$ are supplementary.	6. Definition of supplementary angles
7. $\ell \parallel m$	7. If cons. int. $\angle$ are suppl., then lines are $\parallel$.

CHECK Your Progress

3. PROOF Write a two-column proof of Theorem 3.5.

In Lesson 3-3, you learned that parallel lines have the same slope. You can use the slopes of lines to prove that lines are parallel.
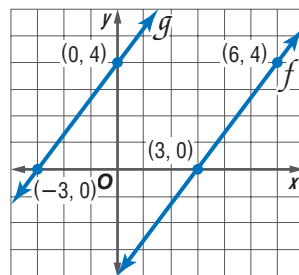
EXAMPLE Slope and Parallel Lines

④ **Determine whether** $g \parallel f$.

slope of f: $m = \dfrac{4 - 0}{6 - 3}$ or $\dfrac{4}{3}$

slope of g: $m = \dfrac{4 - 0}{0 - (-3)}$ or $\dfrac{4}{3}$

Since the slopes are the same, $g \parallel f$.
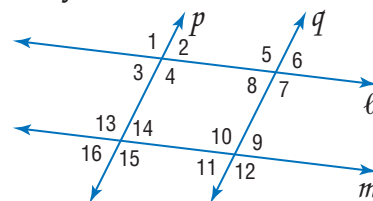
CHECK Your Progress

4. Line ℓ contains points at $(-5, 3)$ and $(0, 4)$. Line m contains points at $\left(2, -\dfrac{2}{3}\right)$ and $(12, 1)$. Determine whether $\ell \parallel m$.
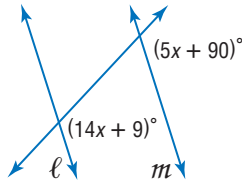
CHECK Your Understanding

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

1. $\angle 16 \cong \angle 3$

2. $\angle 4 \cong \angle 13$

3. $m\angle 14 + m\angle 10 = 180$
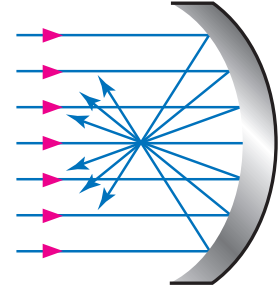
4. $\angle 1 \cong \angle 7$

Example 2
(p. 174)

5. Find x so that $\ell \parallel m$.
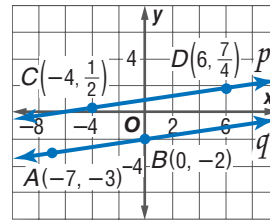
$(5x + 90)°$

$(14x + 9)°$

ℓ m

Example 3
(p. 175)

6. PHYSICS The Hubble Telescope gathers parallel light rays and directs them to a central focal point. Use a protractor to measure several of the angles shown in the diagram. Are the lines parallel? Explain how you know.

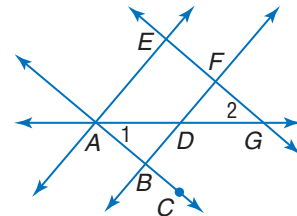Example 4
(p. 175)

7. Determine whether $p \parallel q$.

Exercises

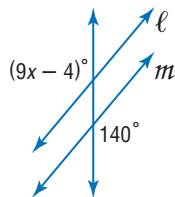| For Exercises	See Examples
8–11	1
12–17	2
18, 19	3
20-25	4

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

8. $\angle AEF \cong \angle BFG$

9. $\angle EAB \cong \angle DBC$

10. $\angle EFB \cong \angle CBF$
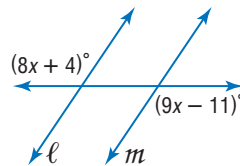
11. $m\angle GFD + m\angle CBD = 180$
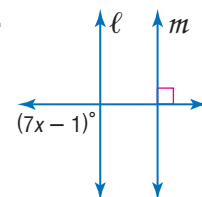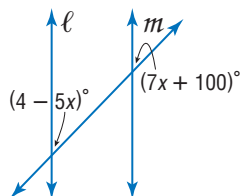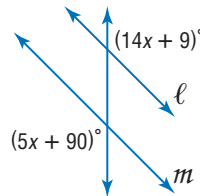

Find x so that $\ell \parallel m$.

12.

$(9x - 4)°$ ℓ
          m

$140°$

13.

$(8x + 4)°$

$(9x - 11)°$

ℓ m

14.

ℓ m

$(7x - 1)°$

15.

ℓ m

$(7x + 100)°$

$(4 - 5x)°$

16.

$(14x + 9)°$

ℓ

$(5x + 90)°$

m

17.

$(178 - 3x)°$

ℓ

$(7x - 38)°$

m

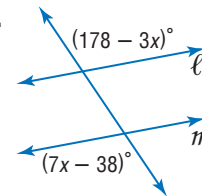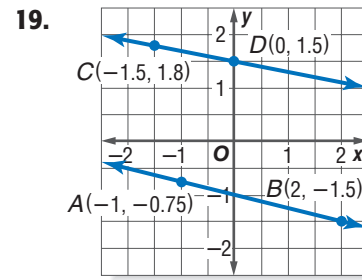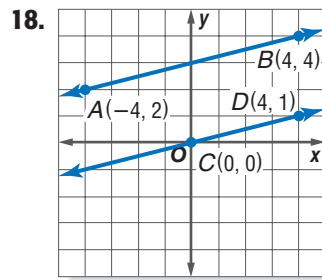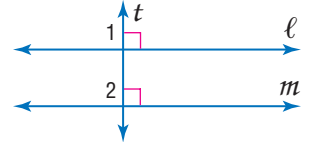**Determine whether each pair of lines is parallel. Explain why or why not.**

18.

19.

20. **PROOF** Copy and complete the proof of Theorem 3.8.

Given: $\ell \perp t$
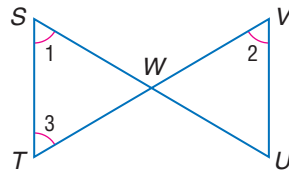$m \perp t$

Prove: $\ell \parallel m$

Proof:

Statements	Reasons
1. $\ell \perp t$, $m \perp t$	1. ___?___
2. $\angle 1$ and $\angle 2$ are right angles.	2. ___?___
3. $\angle 1 \cong \angle 2$	3. ___?___
4. $\ell \parallel m$	4. ___?___

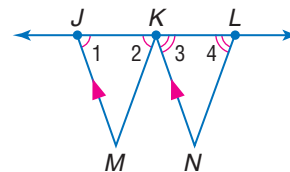21. **PROOF** Write a two-column proof of Theorem 3.6.

22. **PROOF** Write a paragraph proof of Theorem 3.7.

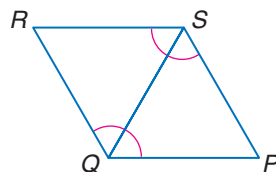**PROOF** Write a two-column proof for each of the following.

23. **Given:** $\angle 2 \cong \angle 1$
$\angle 1 \cong \angle 3$
Prove: $\overline{ST} \parallel \overline{UV}$

24. **Given:** $\overline{JM} \parallel \overline{KN}$
$\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
Prove: $\overline{KM} \parallel \overline{LN}$

25. **Given:** $\angle RSP \cong \angle PQR$
$\angle QRS$ and $\angle PQR$
are supplementary.
Prove: $\overline{PS} \parallel \overline{QR}$

26. **Given:** $\overline{AD} \perp \overline{CD}$
$\angle 1 \cong \angle 2$
Prove: $\overline{BC} \perp \overline{CD}$
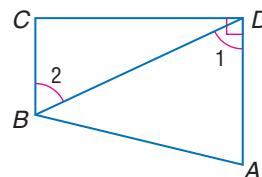
27. **RESEARCH** Use the Internet or other resource to find mathematicians like John Playfair who discovered new concepts and proved new theorems related to parallel lines. Briefly describe their discoveries. Include any factors that prompted their research, such as a real-world need or research in a different field.

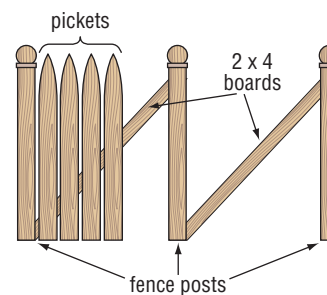**28. HOME IMPROVEMENT** To build a fence, Jim positioned the fence posts and then placed a 2 × 4 board at an angle between the fence posts. As he placed each picket, he made sure the angle that the picket made with the 2 × 4 was the same as the angle for the rest of the pickets. Why does this ensure that the pickets will be parallel?

pickets

2 x 4 boards

fence posts

29. FOOTBALL When striping the practice football field, Mr. Hawkinson first painted the sidelines. Next he marked off 10-yard increments on one sideline. He then constructed lines perpendicular to the sidelines at each 10-yard mark. Why does this guarantee that the 10-yard lines will be parallel?

30. CRAFTS Juan is making a stained glass piece. He cut the top and bottom pieces at a 30° angle. If the corners are right angles, explain how Juan knows that each pair of opposite sides is parallel.
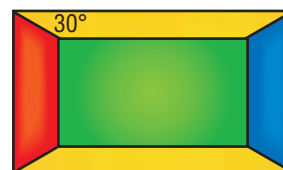
30°

31. FRAMING Wooden picture frames are often constructed using a miter box or miter saw. These tools allow you to cut at an angle of a given size. If each of the four pieces of framing material is cut at a 45° angle, will the sides of the frame be parallel? Explain your reasoning.

H.O.T. Problems

32. REASONING Summarize five different methods that can be used to prove that two lines are parallel.

33. REASONING Find a counterexample for the following statement.
If lines ℓ and m are cut by transversal t so that consecutive interior angles are congruent, then lines ℓ and m are parallel and t is perpendicular to both lines.

34. OPEN ENDED Describe two situations in your own life in which you encounter parallel lines. How could you verify that the lines are parallel?

35. CHALLENGE When Adeel was working on an art project, he drew a four-sided figure with two pairs of opposite parallel sides. He noticed some patterns relating to the angles in the figure. List as many patterns as you can about a 4-sided figure with two pairs of opposite parallel sides.

36. *Writing in Math* Use the information about parking lots on page 172 to explain how you know that the sides of a parking space are parallel. Include a comparison of the angles at which the lines forming the edges of a parking space strike the center line, and a description of the type of parking spaces that have sides that form congruent consecutive interior angles.

37. Which of the following facts would be sufficient to prove that line ℓ is parallel to $\overline{AC}$?
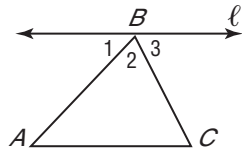
A $\angle 1 \cong \angle 3$

B $\angle 3 \cong \angle C$

C $\angle 1 \cong \angle C$

D $\angle 2 \cong \angle A$

38. REVIEW Kendra has at least one quarter, one dime, one nickel, and one penny. If she has three times as many pennies as nickels, the same number of nickels as dimes, and twice as many dimes as quarters, then what is the least amount of money she could have?

F $0.41

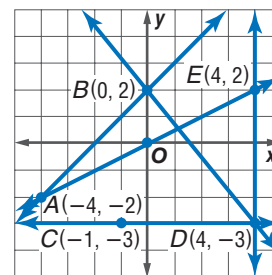**G** $0.48

H $0.58

J $0.61

Spiral Review

Write an equation in slope-intercept form for the line that satisfies the given conditions. (Lesson 3-4)

39. $m = 0.3$, y-intercept is -6

40. $m = \frac{1}{3}$, contains $(-3, -15)$

41. contains $(5, 7)$ and $(-3, 11)$

42. perpendicular to $y = \frac{1}{2}x - 4$, contains $(4, 1)$

Find the slope of each line. (Lesson 3-3)

43. $\overleftrightarrow{BD}$

44. $\overleftrightarrow{CD}$

45. $\overleftrightarrow{AB}$

46. $\overleftrightarrow{AE}$

47. any line parallel to $\overleftrightarrow{DE}$

48. any line perpendicular to $\overleftrightarrow{BD}$

49. CARPENTRY A carpenter must cut two pieces of wood at angles so that they fit together to form the corner of a picture frame. What type of angles must he use to make sure that a 90° corner results? (Lesson 1-5)

GET READY for the Next Lesson

PREREQUISITE SKILL Use the Distance Formula to find the distance between each pair of points. (Lesson 1-4)

50. $(2, 7), (7, 19)$

51. $(8, 0), (-1, 2)$

52. $(-6, -4), (-8, -2)$

Points of Intersection

You can use a TI-83/84 Plus graphing calculator to determine the points of intersection of a transversal and two parallel lines.

EXAMPLE

Parallel lines ℓ and m are cut by a transversal t. The equations of ℓ, m, and t are $y = \frac{1}{2}x - 4$, $y = \frac{1}{2}x + 6$, and $y = -2x + 1$, respectively. Use a graphing calculator to determine the points of intersection of t with ℓ and m.
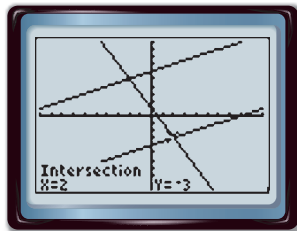
Step 1 Enter the equations in the Y= list and graph in the standard viewing window.

KEYSTROKES: [Y=] .5 [X,T,θ,n] [−] 4 [ENTER] .5 [X,T,θ,n] [+]

6 [ENTER] −2 [X,T,θ,n] [+] 1 [Zoom] 6

Step 2 Use the **CALC** menu to find the points of intersection.

- Find the intersection of ℓ and t.

  KEYSTROKES: [2nd] [CALC] 5 [ENTER] [▼]
  [ENTER] [ENTER]
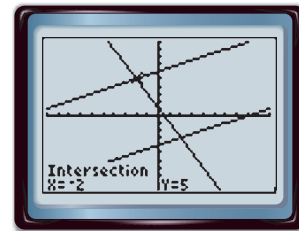
  [−10, 10] scl: 1 by [−10, 10] scl: 1

  Lines ℓ and t intersect at $(2, -3)$.

- Find the intersection of m and t.

  KEYSTROKES: [2nd] [CALC] 5 [▼] [ENTER]
  [ENTER] [ENTER]

  [−10, 10] scl: 1 by [−10, 10] scl: 1

  Lines m and t intersect at $(-2, 5)$.

EXERCISES

Parallel lines a and b are cut by a transversal t. Use a graphing calculator to determine the points of intersection of t with a and b. Round to the nearest tenth.

1. $a: y = 2x - 10$

$b: y = 2x - 2$

$t: y = -\frac{1}{2}x + 4$

2. $a: y = -x - 3$

$b: y = -x + 5$

$t: y = x - 6$

3. $a: y = 6$

$b: y = 0$

$t: x = -2$

4. $a: y = -3x + 1$

$b: y = -3x - 3$

$t: y = \frac{1}{3}x + 8$

5. $a: y = \frac{4}{5}x - 2$

$b: y = \frac{4}{5}x - 7$

$t: y = -\frac{5}{4}x$

6. $a: y = -\frac{1}{6}x + \frac{2}{3}$

$b: y = -\frac{1}{6}x + \frac{5}{12}$

$t: y = 6x + 2$

Other Calculator Keystrokes at geometryonline.com

Perpendiculars and Distance

Main Ideas

- Find the distance between a point and a line.
- Find the distance between parallel lines.

New Vocabulary

equidistant

GET READY for the Lesson

When installing shelves, it is important that the vertical brackets be parallel for the shelves to line up. One technique is to install the first bracket and then use a carpenter's square to measure and mark two or more points the same distance from the first bracket. You can then align the second bracket with those marks.
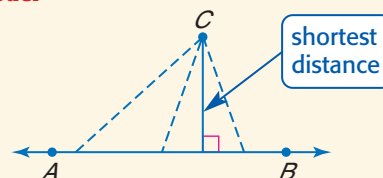
Distance from a Point to a Line In Lesson 3-5, you learned that if two lines are perpendicular to the same line, then they are parallel. The carpenter's square is used to construct a line perpendicular to each bracket. The space between each bracket is measured along the perpendicular segment. This is to ensure that the brackets are parallel. This is an example of using lines and perpendicular segments to determine distance. The shortest segment from a point to a line is the perpendicular segment from the point to the line.

KEY CONCEPT   *Distance Between a Point and a Line*

Words   The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point.

Model

EXAMPLE   **Distance from a Point to a Line**

1 Draw the segment that represents the distance from P to $\overleftrightarrow{AB}$.

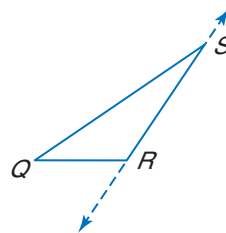Since the distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point, extend $\overline{AB}$ and draw $\overline{PQ}$ so that $\overline{PQ} \perp \overleftrightarrow{AB}$.
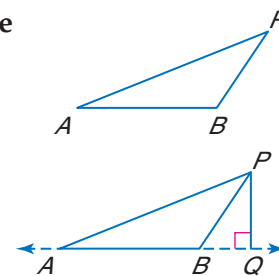
1. Draw the segment that represents the distance from Q to $\overleftrightarrow{RS}$.

When you draw a perpendicular segment from a point to a line, you can guarantee that it is perpendicular by using the construction of a line perpendicular to a line through a point not on that line.

EXAMPLE **Construct a Perpendicular Segment**

2 **COORDINATE GEOMETRY** Line ℓ contains points $(-6, -9)$ and $(0, -1)$. Construct a line perpendicular to line ℓ through $P(-7, -2)$ not on ℓ. Then find the distance from P to ℓ.

Step 1 Graph line ℓ and point P. Place the compass point at point P. Make the setting wide enough so that when an arc is drawn, it intersects ℓ in two places. Label these points of intersection A and B.
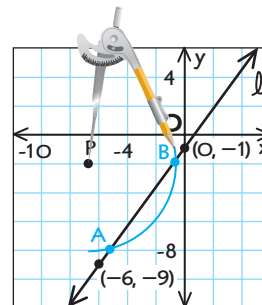
Step 2 Put the compass at point A and draw an arc below line ℓ. (*Hint:* Any compass setting greater than $\frac{1}{2}AB$ will work.)
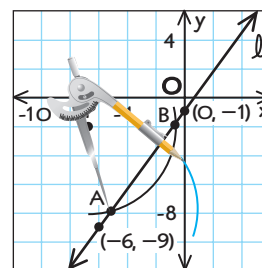
Step 3 Using the same compass setting, put the compass at point B and draw an arc to intersect the one drawn in step 2. Label the point of intersection Q.
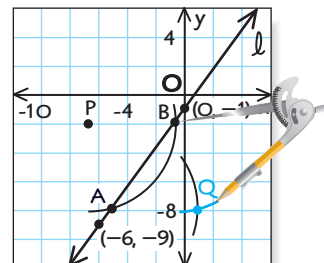
Study Tip

Distance
Note that the distance from a point to the x-axis can be determined by looking at the y-coordinate and the distance from a point to the y-axis can be determined by looking at the x-coordinate.

Step 4 Draw $\overleftrightarrow{PQ}$. $\overleftrightarrow{PQ} \perp \ell$. Label point R at the intersection of $\overleftrightarrow{PQ}$ and ℓ. *Use the slopes of $\overleftrightarrow{PQ}$ and ℓ to verify that the lines are perpendicular.*
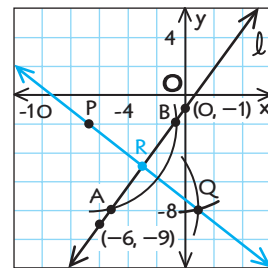
The segment constructed from point $P(-7, -2)$ perpendicular to the line ℓ, appears to intersect line ℓ at $R(-3, -5)$. Use the Distance Formula to find the distance between point P and line ℓ.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-7 - (-3))^2 + (-2 - (-5))^2}$$
$$= \sqrt{25} \text{ or } 5$$

The distance between P and ℓ is 5 units.

CHECK Your Progress

2. Line ℓ contains points $(1, 2)$ and $(5, 4)$. Construct a line perpendicular to ℓ through $P(1, 7)$. Then find the distance from P to ℓ.
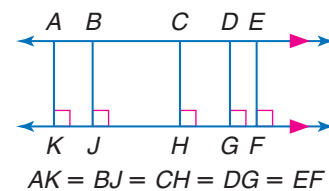
Distance Between Parallel Lines Two lines in a plane are parallel if they are everywhere **equidistant**. Equidistant means that the distance between two lines measured along a perpendicular line to the lines is always the same. The distance between parallel lines is the length of the perpendicular segment with endpoints that lie on each of the two lines.

$AK = BJ = CH = DG = EF$

KEY CONCEPT *Distance Between Parallel Lines*

The distance between two parallel lines is the distance between one of the lines and any point on the other line.

Recall from Lesson 1-1 that a *locus* is the set of all points that satisfy a given condition. Parallel lines can be described as the locus of points in a plane equidistant from a given line.
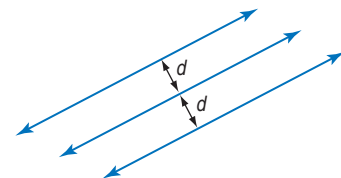
THEOREM 3.9

In a plane, if two lines are equidistant from a third line, then the two lines are parallel to each other.

You will prove Theorem 3.9 in Exercise 19.

3 **Find the distance between the parallel lines ℓ and n with equations $y = -\frac{1}{3}x - 3$ and $y = -\frac{1}{3}x + \frac{1}{3}$, respectively.**

You will need to solve a system of equations to find the endpoints of a segment that is perpendicular to both ℓ and n. The slope of lines ℓ and n is $-\frac{1}{3}$.

• First, write an equation of a line p perpendicular to ℓ and n. The slope of p is the opposite reciprocal of $-\frac{1}{3}$, or 3. Use the y-intercept of line ℓ, $(0, -3)$, as one of the endpoints of the perpendicular segment.
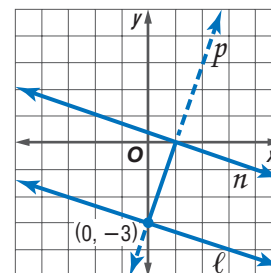

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - (-3) = 3(x - 0) \quad x_1 = 0, y_1 = -3, m = 3$$
$$y + 3 = 3x \quad \text{Simplify.}$$
$$y = 3x - 3 \quad \text{Subtract 3 from each side.}$$

• Next, use a system of equations to determine the point of intersection of lines n and p.

n: $y = -\frac{1}{3}x + \frac{1}{3}$ $-\frac{1}{3}x + \frac{1}{3} = 3x - 3$ Substitute $-\frac{1}{3}x + \frac{1}{3}$ for y in the second equation.

p: $y = 3x - 3$

$$-\frac{1}{3}x - 3x = -3 - \frac{1}{3} \quad \text{Group like terms on each side.}$$
$$-\frac{10}{3}x = -\frac{10}{3} \quad \text{Simplify on each side.}$$
$$x = 1 \quad \text{Divide each side by } -\frac{10}{3}.$$

Solve for y.   Substitute 1 for x in the equation for p.

$y = 3(1) - 3$ Simplify.

$= 0$

The point of intersection is $(1, 0)$.

• Then, use the Distance Formula to determine the distance between $(0, -3)$ and $(1, 0)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(0 - 1)^2 + (-3 - 0)^2} \quad x_2 = 0, x_1 = 1, y_2 = -3, y_1 = 0$$
$$= \sqrt{10} \quad \text{Simplify.}$$

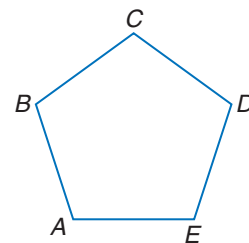The distance between the lines is $\sqrt{10}$ or about 3.16 units.

CHECK Your Progress

3. Find the distance between parallel lines a and b with equations $x + 3y = 6$ and $x + 3y = -14$, respectively.

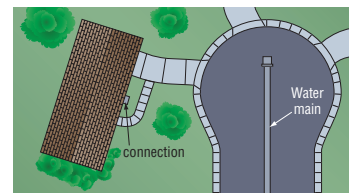**Online** Personal Tutor at geometryonline.com

Example 1
(p. 181)

1. Copy the figure. Draw the segment that represents the distance D to $\overleftrightarrow{AE}$.

2. UTILITIES Housing developers often locate the shortest distance from a house to the water main so that a minimum of pipe is required to connect the house to the water supply. Copy the diagram, and draw a possible location for the pipe.

Example 2
(pp. 182–183)

3. COORDINATE GEOMETRY Line ℓ contains points $(0, 0)$ and $(2, 4)$. Draw line ℓ. Construct a line perpendicular to ℓ through $A(2, -6)$. Then find the distance from A to ℓ.
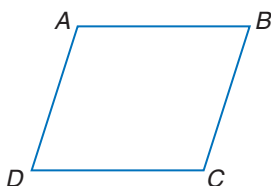
Example 3
(p. 184)

4. Find the distance between the pair of parallel lines with the given equations.
$$y = \frac{3}{4}x - 1$$
$$y = \frac{3}{4}x + \frac{1}{8}$$

Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
5–7	1
8, 9	2
10–18	3

Copy each figure. Draw the segment that represents the distance indicated.

5. C to $\overleftrightarrow{AD}$

6. K to $\overrightarrow{JL}$
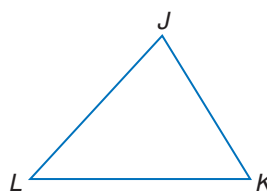
7. Q to $\overleftrightarrow{RS}$
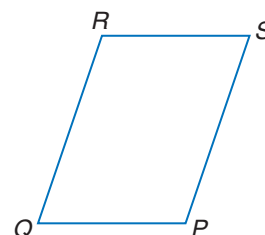
COORDINATE GEOMETRY Construct a line perpendicular to ℓ through P. Then find the distance from P to ℓ.

8. Line ℓ contains points $(-3, 0)$ and $(3, 0)$. Point P has coordinates $(4, 3)$.

9. Line ℓ contains points $(0, -2)$ and $(1, 3)$. Point P has coordinates $(-4, 4)$.

Find the distance between each pair of parallel lines with the given equations.

10. $y = -3$
  $y = 1$

11. $x = 4$
  $x = -2$

12. $y = 2x + 2$
  $y = 2x - 3$

13. $y = \frac{1}{3}x - 3$
  $y = \frac{1}{3}x + 2$

14. $x = 8.5$
  $x = -12.5$

15. $y = 15$
  $y = -4$

Find the distance between each pair of parallel lines with the given equations.

16. $y = 4x$
$y = 4x - 17$

17. $y = 2x - 3$
$2x - y = -4$

18. $y = -\frac{3}{4}x - 1$
$3x + 4y = 20$

19. PROOF Write a paragraph proof of Theorem 3.9.

Graph each line. Construct a perpendicular segment through the given point. Then find the distance from the point to the line.

20. $y = 5, (-2, 4)$

21. $y = 2x + 2, (-1, -5)$

22. $2x - 3y = -9, (2, 0)$

23. CONSTRUCTION When framing a wall during a construction project, carpenters often use a plumb line. A *plumb line* is a string with a weight called a *plumb bob* attached on one end. The plumb line is suspended from a point and then used to ensure that wall studs are vertical. How does the plumb line help to find the distance from a point to the floor?

CONSTRUCTIONS Line ℓ contains points $(-4, 3)$ and $(2, -3)$. Point P at $(-2, 1)$ is on line ℓ. Complete the following construction.

Step 1 Graph line ℓ and point P, and put the compass at point P. Using the same compass setting, draw arcs to the left and right of P. Label these points A and B.

Step 2 Open the compass to a setting greater than AP. Put the compass at point A and draw an arc above line ℓ.

Step 3 Using the same compass setting, put the compass at point B and draw an arc above line ℓ. Label the point of intersection Q. Then draw $\overleftrightarrow{PQ}$.
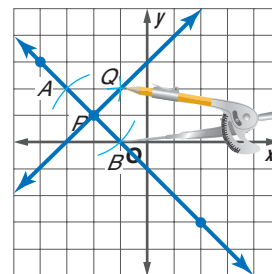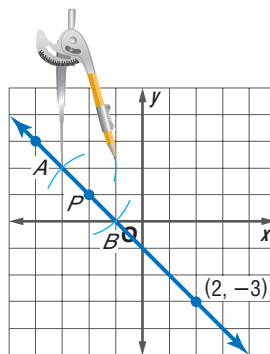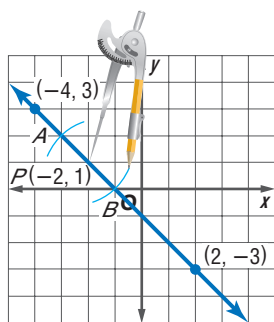
24. What is the relationship between line ℓ and $\overleftrightarrow{PQ}$? Verify your conjecture using the slopes of the two lines.

25. Repeat the construction above using a different line and point on that line.

26. REASONING Compare and contrast three different methods that you can use to show that two lines in a plane are parallel.

CHALLENGE For Exercises 27–32, draw a diagram that represents each description.

27. Point P is equidistant from two parallel lines.

28. Point P is equidistant from two intersecting lines.

29. Point P is equidistant from two parallel planes.

30. Point P is equidistant from two intersecting planes.

31. A line is equidistant from two parallel planes.

32. A plane is equidistant from two other planes that are parallel.

33. *Writing in Math* Refer to the information about shelving on page 181 to explain how the distance between parallel lines relates to hanging new shelves. Include an explanation of why marking several points equidistant from the first bracket will ensure that the brackets are parallel, and a description of other types of home improvement projects that require that two or more elements are parallel.

STANDARDIZED TEST PRACTICE

34. Segment AB is perpendicular to segment BD. Segment AB and segment CD bisect each other at point X. If $AB = 16$ and $CD = 20$, what is the measure of $\overline{BD}$?

A 6

B 8

C 10

D 18

35. REVIEW Pablo bought a sweater on sale for 25% off the original price and another 40% off the discounted price. If the sweater originally cost $48, what was the final sale price of the sweater?

F $14.40

G $21.60

H $31.20

J $36.00

Spiral Review

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer. (Lesson 3-5)

36. $\angle 5 \cong \angle 6$

37. $\angle 6 \cong \angle 2$

38. $\angle 1$ and $\angle 2$ are supplementary.
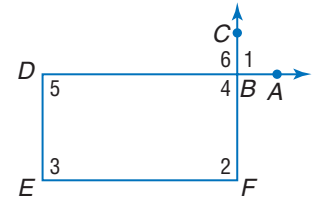
Write an equation in slope-intercept form for each line. (Lesson 3-4)

39. a **40.** b **41.** c

42. perpendicular to line a, contains $(-1, -4)$

43. parallel to line c, contains $(2, 5)$

44. COMPUTERS In 1999, 73% of American teenagers used the Internet. Five years later, this increased to 87%. If the rate of change is constant, estimate when 100% of American teenagers will use the Internet. (Lesson 3-3)
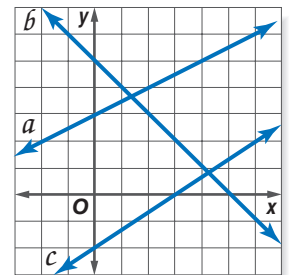
Cross-Curricular Project

Geometry and Earth Science

How's the Weather? It's time to complete your project. Use the information and data you have gathered about climate and locations on Earth to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.
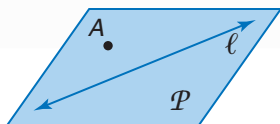
Cross-Curricular Project at geometryonline.com
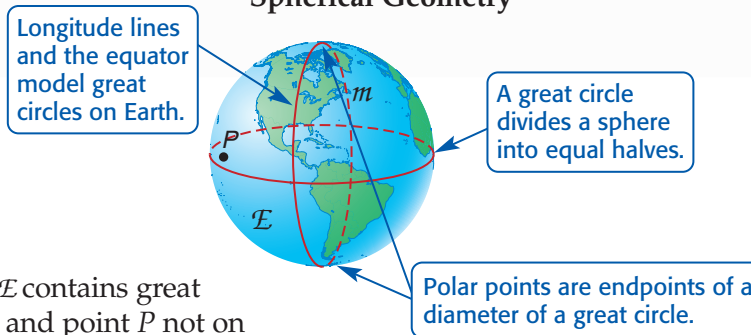
Geometry Lab
Non-Euclidean Geometry

So far in this text, we have studied **plane Euclidean geometry**, which is based on a system of points, lines, and planes. **Spherical geometry** is a system of points, great circles (lines), and spheres (planes). Spherical geometry is one type of **non-Euclidean geometry**. Much of spherical geometry was developed by early Babylonians, Arabs, and Greeks. Their study was based on the astronomy of Earth and their need to be able to measure time accurately.

Plane Euclidean Geometry

Plane P contains line ℓ and point A not on ℓ.

Spherical Geometry

Longitude lines and the equator model great circles on Earth.

A great circle divides a sphere into equal halves.

Polar points are endpoints of a diameter of a great circle.

Sphere $\mathcal{E}$ contains great circle m and point P not on m. m is a line on sphere $\mathcal{E}$.

The table below compares and contrasts *lines* in the system of plane Euclidean geometry and *lines* (great circles) in spherical geometry.

Plane Euclidean Geometry Lines on the Plane	Spherical Geometry Great Circles (Lines) on the Sphere
1. A line segment is the shortest path between two points. 2. There is a unique line passing through any two points. 3. A line goes on infinitely in two directions. 4. If three points are collinear, exactly one is between the other two. $A \qquad B \qquad C$ B is between A and C.	1. An arc of a great circle is the shortest path between two points. 2. There is a unique great circle passing through any pair of non-polar points. 3. A great circle is finite and returns to its original starting point. 4. If three points are collinear, any one of the three points is between the other two. A is between B and C. B is between A and C. C is between A and B.

In spherical geometry, Euclid's first four postulates and their related theorems hold true. However, theorems that depend on the parallel postulate (Postulate 5) may not be true.

In Euclidean geometry, parallel lines lie in the same plane and never intersect. In spherical geometry, the sphere is the plane, and a great circle represents a line. Every great circle containing A intersects ℓ. Thus, there exists no line through point A that is parallel to ℓ.
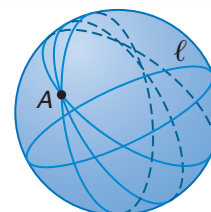
Every great circle of a sphere intersects all other great circles on that sphere in exactly two points. In the figure at the right, one possible line through point A intersects line ℓ at P and Q.
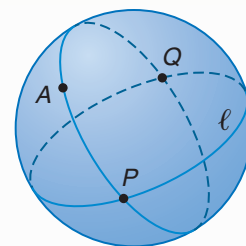
If two great circles divide a sphere into four congruent regions, the lines are perpendicular to each other at their intersection points. Each longitude circle on Earth intersects the equator at right angles.
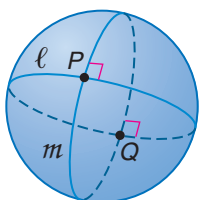
EXAMPLE   Compare Plane and Spherical Geometries

For each property listed from plane Euclidean geometry, write a corresponding statement for spherical geometry.
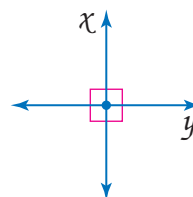
a. Perpendicular lines intersect at one point.
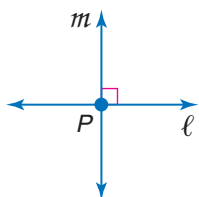
Perpendicular great circles intersect at two points.

b. Perpendicular lines form four right angles.

Perpendicular great circles form eight right angles.
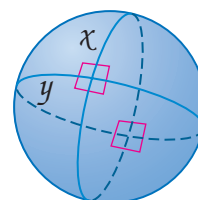
EXERCISES

For each property from plane Euclidean geometry, write a corresponding statement for spherical geometry.

1. A line goes on infinitely in two directions.

2. A line segment is the shortest path between two points.

3. Two distinct lines with no point of intersection are parallel.

4. Parallel lines have infinitely many common perpendicular lines.

If spherical points are restricted to be nonpolar points, determine if each statement from plane Euclidean geometry is also *true* in spherical geometry. If *false*, explain your reasoning.

5. Any two distinct points determine exactly one line.

6. If three points are collinear, exactly one point is between the other two.

7. Given line ℓ and point P not on ℓ, there exists exactly one line parallel to ℓ passing through P.

READING MATH

Necessary and Sufficient Conditions

We all know that water is a *necessary* condition for fish to survive. However, it is not a *sufficient* condition. For example, fish also need food to survive.

Necessary and sufficient conditions are important in mathematics. Consider the property of having four sides. While *having four sides* is a necessary condition for something being a square, that single condition is not, by itself, a sufficient condition to guarantee that it is a square. Trapezoids are four-sided shapes that are not squares.

Condition	Definition	Examples
necessary	A condition *A* is said to be *necessary* for a condition *B*, if and only if the falsity or nonexistence of *A* guarantees the falsity or nonexistence of *B*.	Having opposite sides parallel is a necessary condition for something being a square.
sufficient	A condition *A* is said to be *sufficient* for a condition *B*, if and only if the truth or existence of *A* guarantees the truth or existence of *B*.	Being a square is a sufficient condition for something being a rectangle.

Reading to Learn

**Determine whether each statement is *true* or *false*.
If false, give a counterexample.**

1. Being a square is a necessary condition for being a rectangle.

2. Being a rectangle is a necessary condition for being a square.

3. Being greater than 15 is a necessary condition for being less than 20.

4. Being less than 12 is a sufficient condition for being less than 20.

5. Walking on two legs is a sufficient condition for being a human being.

6. Breathing air is a necessary condition for being a human being.

7. Being an equilateral rectangle is both a necessary and sufficient condition for being a square.

Determine whether I is a *necessary* condition for II, a *sufficient* condition for II, or *both*. Explain.

8. I. Two points are given.
   II. An equation of a line can be written.

9. I. Two planes are parallel.
   II. Two planes do not intersect.

10. I. Two angles are congruent.
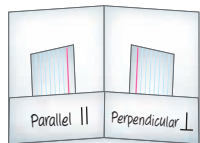   II. Two angles are alternate interior angles.

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.

Parallel ‖   Perpendicular ⊥

Key Concepts

Transversals (Lessons 3-1 and 3-2)

- If two parallel lines are cut by a transversal, then each of the following is true.
  - Each pair of alternate interior angles is congruent,
  - each pair of consecutive interior angles is supplementary, and
  - each pair of alternate exterior angles is congruent.

Slope (Lessons 3-3 and 3-4)

- The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

Proving Lines Parallel (Lesson 3-5)

- If two lines in a plane are cut by a transversal so that any of the following is true, then the two lines are parallel: a pair of alternate exterior angles is congruent, a pair of consecutive interior angles is supplementary, or a pair of alternate interior angles is congruent.

- In a plane, if two lines are perpendicular to the same line, then they are parallel.

Distance (Lesson 3-6)

- The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point.

- The distance between two parallel lines is the distance between one of the lines and any point on the other line.

Key Vocabulary

alternate exterior angles (p. 144)

alternate interior angles (p. 144)

consecutive interior angles (p. 144)

corresponding angles (p. 144)

equidistant (p. 183)

parallel lines (p. 142)

parallel planes (p. 142)

point-slope form (p. 166)

rate of change (p. 157)

slope (p. 156)

slope-intercept form (p. 165)

transversal (p. 143)

Vocabulary Check

**Refer to the figure and choose the term that best completes each sentence.**
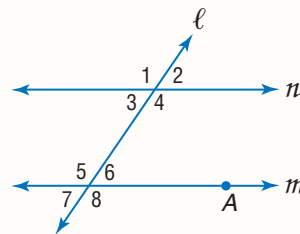
1. Angles 4 and 5 are (consecutive, alternate) interior angles.

2. The distance from point A to line n is the length of the segment (perpendicular, parallel) to line n through A.

3. If $\angle 4$ and $\angle 6$ are supplementary, lines m and n are said to be (parallel, intersecting) lines.

4. Line ℓ is a (slope-intercept, transversal) for lines n and m.

5. $\angle 1$ and $\angle 8$ are (alternate interior, alternate exterior) angles.

6. If $n \parallel m$, $\angle 6$ and $\angle 3$ are (supplementary, congruent).

7. Angles 5 and 3 are (consecutive, alternate) interior angles.

8. If $\angle 2 \cong \angle 7$, then lines n and m are (skew, parallel) lines.

Lesson-by-Lesson Review

**3-1** **Parallel Lines and Transversals** (pp. 142–147)

Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

9. $\angle 3$ and $\angle 6$

10. $\angle 5$ and $\angle 3$

11. $\angle 2$ and $\angle 7$

12. $\angle 4$ and $\angle 8$
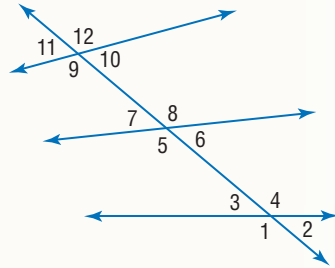
13. EAGLES The flight paths of two American bald eagles were tracked at an altitude of 8500 feet in a direction north to south and an altitude of 12,000 feet in a direction west to east, respectively. Describe the types of lines formed by the paths of these two eagles. Explain your reasoning.
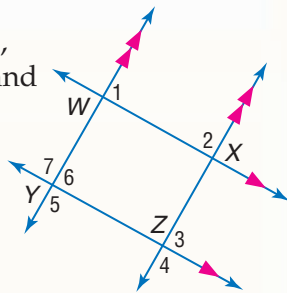
Example 1 Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.
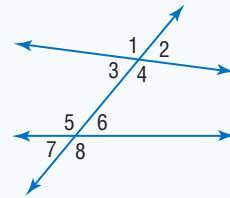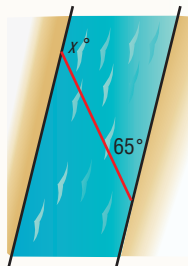
a. $\angle 7$ and $\angle 3$ corresponding

b. $\angle 4$ and $\angle 6$ consecutive interior

c. $\angle 7$ and $\angle 2$ alternate exterior

d. $\angle 3$ and $\angle 6$ alternate interior

3-2 **Angles and Parallel Lines** (pp. 149–154)

14. If $m\angle 1 = 3a + 40$, $m\angle 2 = 2a + 25$, and $m\angle 3 = 5b - 26$, find a and b.

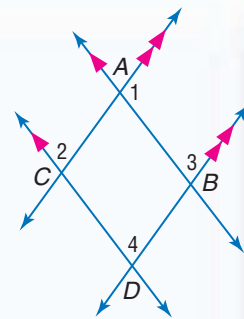**15. BOATING** To cross the river safely, Georgia angles her canoe 65° from the river's edge, as shown. At what angle x will she arrive on the other side of the river?

Example 2
If $m\angle 1 = 4p + 15$, and $m\angle 3 = 3p - 10$, find p.

Since $\overleftrightarrow{AC} \parallel \overrightarrow{BD}$, $\angle 1$ and $\angle 3$ are supplementary by the Consecutive Interior Angles Theorem.

$$m\angle 1 + m\angle 3 = 180 \quad \text{Def. of suppl. } \angle\text{s}$$
$$(4p + 15) + (3p - 10) = 180 \quad \text{Substitution}$$
$$7p + 5 = 180 \quad \text{Simplify.}$$
$$7p = 175 \quad \text{Subtract.}$$
$$p = 25 \quad \text{Divide.}$$

3-3 **Slopes of Lines** (pp. 156–163)

Graph the line that satisfies each condition.

16. contains (2, 3) and is parallel to $\overleftrightarrow{AB}$ with $A(-1, 2)$ and $B(1, 6)$

17. contains $(-2, -2)$ and is perpendicular to $\overleftrightarrow{PQ}$ with $P(5, 2)$ and $Q(3, -4)$

18. PAINTBALL During a game of paintball, Trevor and Carlos took different paths. If the field can be mapped on the coordinate plane, Trevor ran from $(-5, -3)$ to $(4, 3)$ and Carlos from $(2, -7)$ to $(-6, 5)$. Determine whether their paths are *parallel*, *perpendicular*, or *neither*.
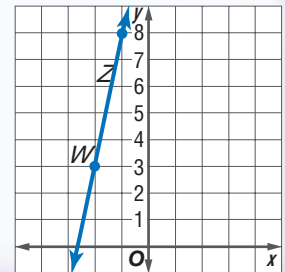
Example 3 Graph the line that contains $W(-2, 3)$ and is parallel to $\overleftrightarrow{XY}$ with $X(3, -4)$ and $Y(5, 6)$.

slope of $\overleftrightarrow{XY} = \dfrac{6 - (-4)}{5 - 3} = \dfrac{10}{2} = 5$

The slope of the line parallel to $\overleftrightarrow{XY}$ through $W(-2, 3)$ is also 5, since parallel lines have the same slope.

Graph the line.
Start at $(-2, 3)$.
Move up 5 units
and then move
right 1 unit. Label
the point Z.

Draw $\overleftrightarrow{WZ}$.

3-4 **Equations of Lines** (pp. 165–170)

Write an equation in the indicated form of the line that satisfies the given conditions.

19. $m = 2$, contains $(1, -5)$; point-slope

20. $m = -\dfrac{3}{2}$, contains $(2, -4)$; slope-intercept

21. contains $(-3, -7)$ and $(9, 1)$; point-slope

22. contains $(2, 5)$ and $(-2, -1)$; slope-intercept

23. DRIVING A car traveling at 30 meters per second begins to slow down or *decelerate* at a constant rate. After 2 seconds, its velocity is 16 meters per second. Write an equation that represents the car's velocity v after t seconds. Then use this equation to determine how long it will take the car to come to a complete stop.

Example 4 Write an equation in slope-intercept form of the line that passes through $(2, -4)$ and $(-3, 1)$.

Find the slope of the line.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$= \dfrac{1 - (-4)}{-3 - 2}$ $(x_1, y_1) = (2, -4)$, $(x_2, y_2) = (-3, 1)$

$= \dfrac{5}{-5}$ or -1 Simplify.

Now use the point-slope form and either point to write an equation.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-4) = -1(x - 2)$ $m = -1$, $(x_1, y_1) = (2, -4)$

$y + 4 = -x + 2$ Simplify.

$y = -x - 2$ Subtract 4 from each side.

3-5 **Proving Lines Parallel** (pp. 172–179)

Refer to the figure at the right. Determine which lines, if any, are parallel given the following information. State the postulate or theorem that justifies your answer.

24. $\angle GHL \cong \angle EJK$

25. $m\angle ADJ + m\angle DJE = 180$

26. OPTICAL ILLUSION Explain how you could use a protractor to prove that the lines in the optical illusion are parallel.
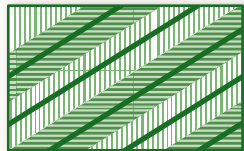
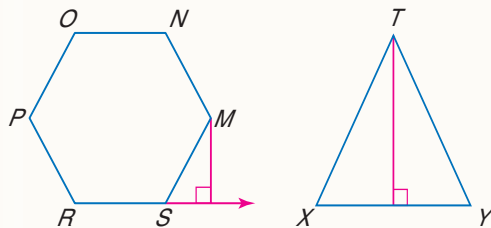**Example 5** Given that $\angle GHL \cong \angle ADE$, determine which lines, if any, are parallel.
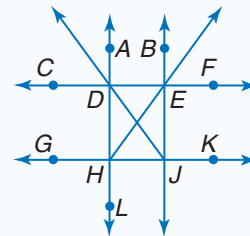
$\angle GHL$ and $\angle ADE$ are alternate exterior angles for $\overleftrightarrow{GK}$ and $\overleftrightarrow{CF}$. Since the angles are congruent, $\overleftrightarrow{GK}$ and $\overleftrightarrow{CF}$ are parallel by the Alternate Exterior Angles Theorem.

3-6 **Perpendiculars and Distance** (pp. 181–187)

Copy each figure. Draw the segment that represents the distance indicated.

27. M to $\overleftrightarrow{RS}$ **28.** T to $\overleftrightarrow{XY}$

29. NEBRASKA The northern and southern boundaries of the Nebraska Panhandle can be represented by lines with the equations $y = 90$ and $y = -48$. Find the approximate distance across the panhandle if the units on the map are measured in miles.
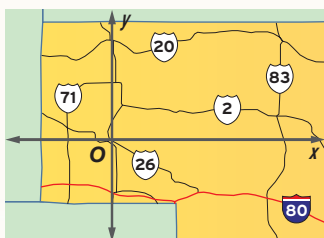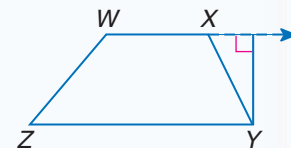
Example 6 Copy the figure. Draw the segment that represents the distance from Y to $\overleftrightarrow{WX}$.
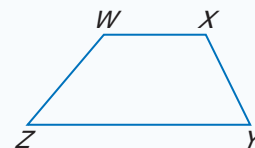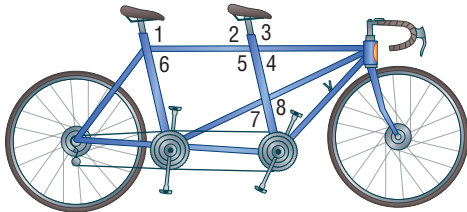
The distance from a line to a point not on the line is the length of the segment perpendicular to the line that passes through the point.

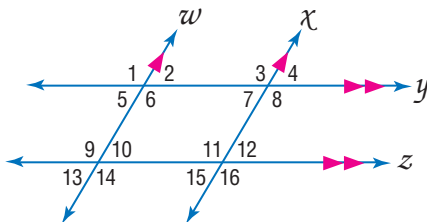Extend $\overline{WX}$ and draw the segment perpendicular to $\overline{WX}$ from Y.

1. MULTIPLE CHOICE The diagram shows the two posts on which seats are placed and several crossbars.

Which term *best* describes $\angle 6$ and $\angle 5$?

A alternate exterior angles

B alternate interior angles

C consecutive interior angles

D corresponding angles

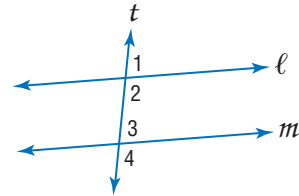**In the figure, $m\angle 12 = 64$. Find the measure of each angle.**

2. $\angle 8$ **3.** $\angle 13$

4. $\angle 7$ **5.** $\angle 11$

6. $\angle 3$ **7.** $\angle 4$

8. $\angle 9$ **9.** $\angle 5$

10. $\angle 16$ **11.** $\angle 14$

Graph the line that satisfies each condition.
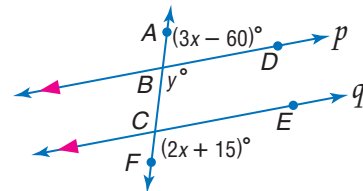
12. slope $= -1$, contains $P(-2, 1)$

13. contains $Q(-1, 3)$ and is perpendicular to $\overleftrightarrow{AB}$ with $A(-2, 0)$ and $B(4, 3)$

14. contains $M(1, -1)$ and is parallel to $\overleftrightarrow{FG}$ with $F(3, 5)$ and $G(-3, -1)$

15. slope $-\frac{4}{3}$, contains $K(3, -2)$

16. MULTIPLE CHOICE In the figure below, which can *not* be true if $m \parallel \ell$ and $m\angle 1 = 73$?

F $m\angle 4 > 73$

G $\angle 1 \cong \angle 4$

H $m\angle 2 + m\angle 3 = 180$

J $\angle 3 \cong \angle 1$

For Exercises 17–22, refer to the figure below. Find each value if $p \parallel q$.

17. x **18.** y

19. $m\angle FCE$ **20.** $m\angle ABD$

21. $m\angle BCE$ **22.** $m\angle CBD$

Find the distance between each pair of parallel lines.

23. $y = 2x - 1$, $y = 2x + 9$

24. $y = -x + 4$, $y = -x - 2$

25. COORDINATE GEOMETRY Detroit Road starts in the center of the city, and Lorain Road starts 4 miles west of the center of the city. Both roads run southeast. If these roads are put on a coordinate plane with the center of the city at $(0, 0)$, Lorain Road is represented by the equation $y = -x - 4$ and Detroit Road is represented by the equation $y = -x$. How far away is Lorain Road from Detroit Road?
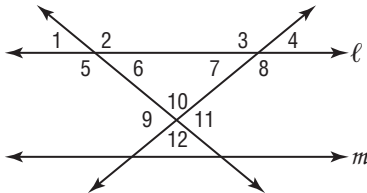
Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.
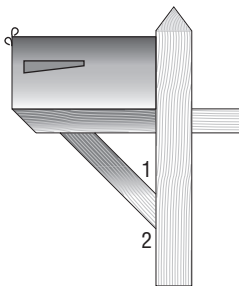
1. In the figure below, $\angle 3 \cong \angle 8$.

Which of the following conclusions does *not* have to be true?

A $\angle 4 \cong \angle 8$

B $\angle 4$ and $\angle 7$ are supplementary angles.

C Line ℓ is parallel to line m.

D $\angle 5$ and $\angle 6$ are supplementary angles.

2. In the diagram below of a mailbox post, which term describes $\angle 1$ and $\angle 2$?

F alternate exterior angles

G alternate interior angles

H consecutive interior angles

J corresponding angles

3. ALGEBRA Which problem situation can *not* be described by a linear function?

A The distance traveled at an average speed of 70 miles per hour for h hours.

B The area of an isosceles right triangle given the length of one leg.

C The amount of sales tax on a purchase if the rate is 6.5%.

D The gross weekly salary earned at an hourly rate of $5.85 for t hours.

4. In the accompanying diagram, parallel lines ℓ and m are cut by transversal t.
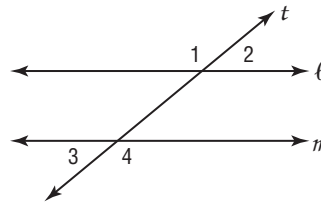
Which statement about angles 1 and 4 *must* be true?

F $\angle 1 \cong \angle 4$

G $\angle 1$ is the complement of $\angle 4$.

H $\angle 1$ is the supplement of $\angle 4$.

J $\angle 1$ and $\angle 4$ are acute angles.

5. What statement is needed in Step 2 to complete this proof?

Given: $\dfrac{4x-6}{3} = 10$

Prove: $x = 9$

Statements	Reasons
1. $\dfrac{4x-6}{3} = 10$	**1.** Given
2. ___?___	**2.** Multiplication Prop.
3. $4x - 6 = 30$	**3.** Simplify.
4. $4x = 36$	**4.** Addition Prop.
5. $x = 9$	**5.** Division Prop.

A $3\left(\dfrac{4x-6}{3}\right) = 10$
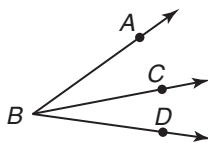
B $\dfrac{4x-6}{3} = 3(10)$

**C** $3\left(\dfrac{4x-6}{3}\right) = 3(10)$

D $4x - 6 = 30$

6. GRIDDABLE Point E is the midpoint of $\overline{DF}$. If $DE = 8x - 3$ and $EF = 3x + 7$, what is x?

Standardized Test Practice at **geometryonline.com**

7. If $\angle ABC \cong \angle CBD$, which statement *must* be
true?

F $\overline{BC}$ bisects $\angle ABD$.

G $\angle ABD$ is a right angle.

H $\angle ABC$ and $\angle CBD$ are supplementary.

J $\overline{AB}$ and $\overline{BD}$ are perpendicular.

8. ALGEBRA Which expression is equivalent to
$4y^3 8y^{-5}$?

A $32y^8$ **C** $32y^{-8}$

B $32y^{-2}$ **D** $32y^{-15}$

9. Based strictlly on this diagram, which is a
valid conclusion?
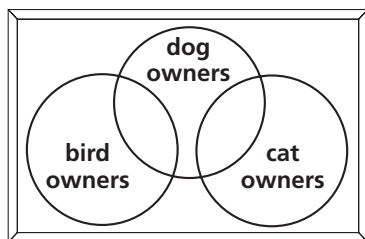
F No dog owners also own cats.

G No bird owners also own dogs.

H No cat owners also own birds.

**J** No pet owners own more than one pet.

TEST-TAKING TIP

Question 9 Remember that overlapping regions in a
Venn diagram represent common or shared elements
between sets.

10. Which of the following describes the line
containing the points $(2, 4)$ and $(0, -2)$?

A $y = \frac{1}{3}x - 4$ **C** $y = \frac{1}{3}x - 2$

B $y = -3x + 2$ **D** $y = 3x - 2$

11. Which property could justify the first step
in solving $3 \times \frac{14x + 6}{8} = 18$?

F Addition Property of Equality

G Division Property of Equality

H Substitution Property of Equality

J Transitive Property of Equality

12. If line ℓ is parallel to line m, which best
describes the construction below?
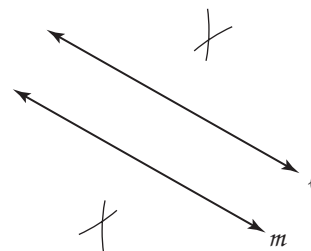
A a line perpendicular to lines ℓ and m

B a line parallel to lines ℓ and m

C a line intersecting line ℓ

D a line congruent to line m

Pre-AP

**Record your answer on a sheet of paper. Show
your work.**

13. To get a player out who was running from
third base to home, Kahlil threw the ball a
distance of 120 feet, from second base toward
home plate. Did the ball reach home plate?
If not, how far from the plate did it land?
Explain and show your calculations to justify
your answer.
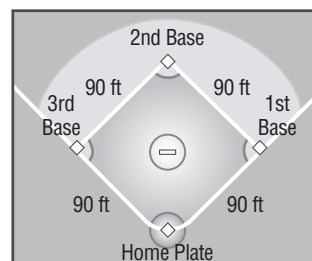
NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	3-2	3-1	786	3-2	2-4	1-5	3-5	794	2-2	3-4	1-3	2-1	1-3