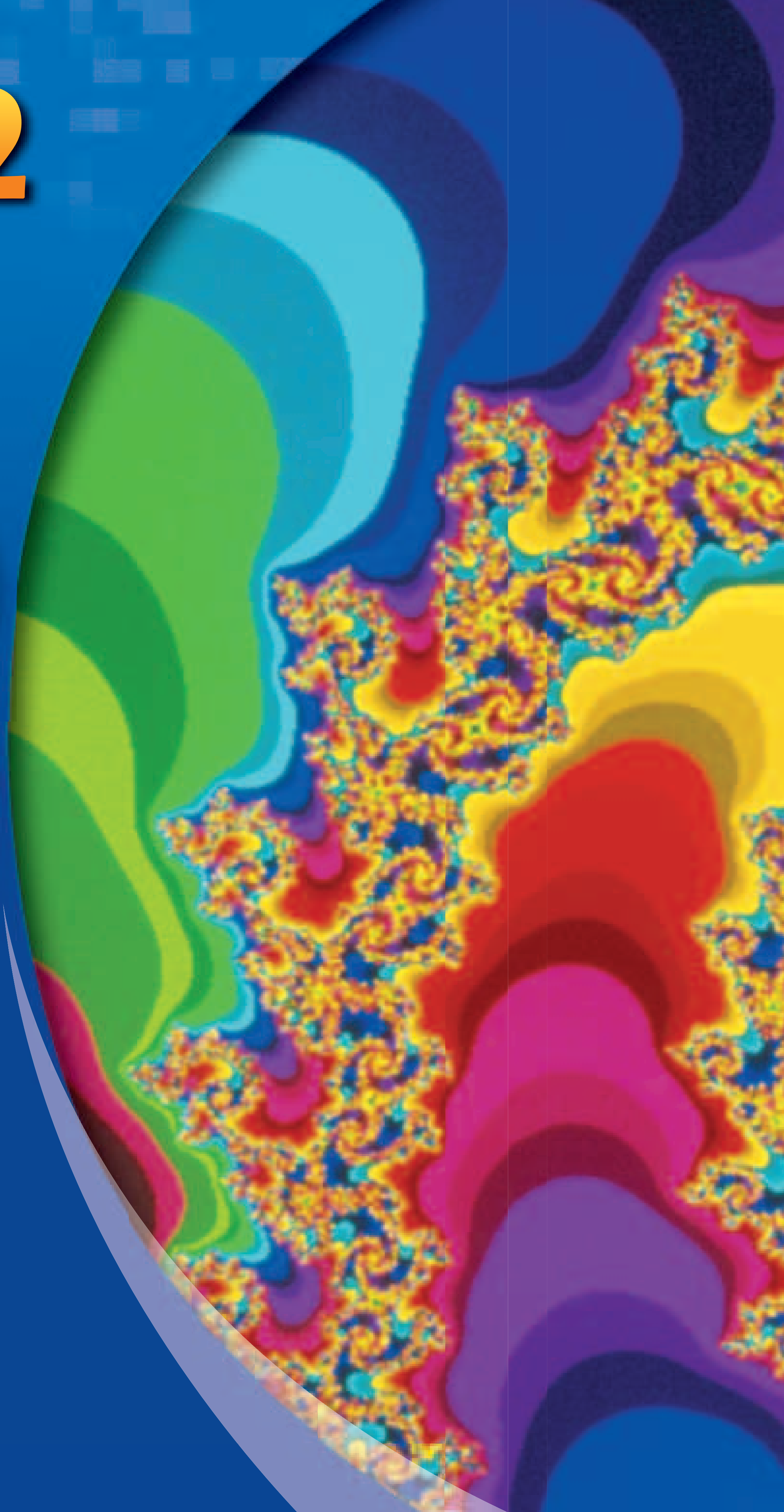
UNIT 2

Congruence



Focus

Use a variety of representations, tools, and technology to solve meaningful problems by representing and transforming figures and analyzing relationships.

CHAPTER 4
Congruent Triangles

BIG Idea Analyze geometric relationships in order to make and verify conjectures involving triangles.

BIG Idea Apply the concept of congruence to justify properties of figures and solve problems.

CHAPTER 5
Relationships in Triangles

BIG Idea Use a variety of representations to describe geometric relationships and solve problems involving triangles.

CHAPTER 6
Quadrilaterals

BIG Idea Analyze properties and describe relationships in quadrilaterals.

BIG Idea Apply logical reasoning to justify and prove mathematical statements involving quadrilaterals.

Gregory Sams/Photo Researchers

Cross-Curricular Project

Geometry and History

Who is behind this geometry idea anyway? Have you ever wondered who first developed some of the ideas you are learning in your geometry class? Many ideas we study were developed many years ago, but people today are also discovering new mathematics. Mathematicians continue to study fractals that were pioneered by Benoit Mandelbrot and Gaston Julia. In this project, you will be using the Internet to research a topic in geometry. You will then prepare a portfolio or poster to display your findings.

Math Online **Log on to geometryonline.com to begin.**

CHAPTER 4

Congruent Triangles

BIG Ideas

- Classify triangles.
- Apply the Angle Sum Theorem and the Exterior Angle Theorem.
- Identify corresponding parts of congruent triangles.
- Test for triangle congruence using SSS, SAS, ASA, and AAS.
- Use properties of isosceles and equilateral triangles.
- Write coordinate proofs.

Key Vocabulary

exterior angle (p. 211)

flow proof (p. 212)

corollary (p. 213)

congruent triangles (p. 217)

coordinate proof (p. 251)

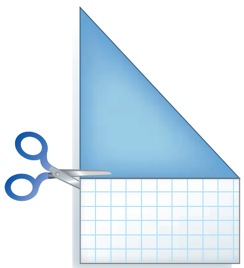
Real-World Link

Triangles Triangles with the same size and shape can be modeled by a pair of butterfly wings.

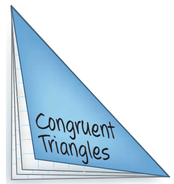
FOLDABLES Study Organizer

Congruent Triangles Make this Foldable to help you organize your notes. Begin with two sheets of grid paper and one sheet of construction paper.

1. **Stack** the grid paper on the construction paper. Fold diagonally to form a triangle and cut off the excess.



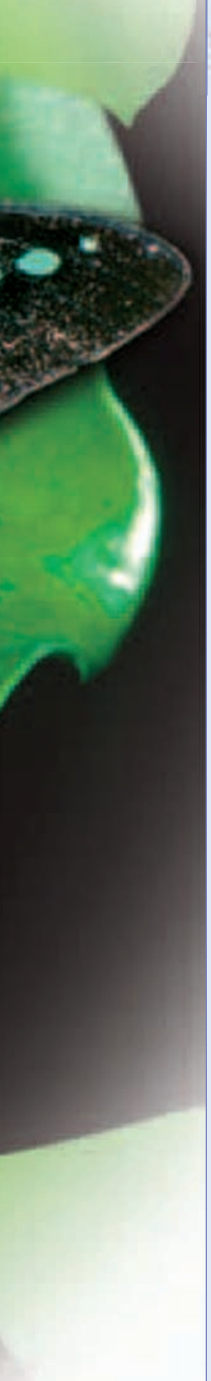
2. **Staple** the edge to form a booklet. Write the chapter title on the front and label each page with a lesson number and title.



Malcolm Thomas/www.ecopix.net

GET READY for Chapter 4

Diagnose Readiness You have two options for checking Prerequisite Skills.



Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

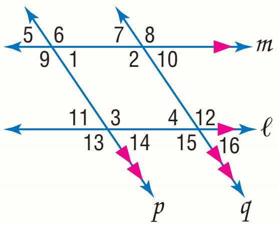
Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Solve each equation. (Prerequisite Skill)

1. $2x + 18 = 5$
2. $3m - 16 = 12$
3. $6 = 2a + \frac{1}{2}$
4. $\frac{2}{3}b + 9 = -15$
5. **FISH** Miranda bought 4 goldfish and \$5 worth of accessories. She spent a total of \$6 at the store. Write and solve an equation to find the amount for each goldfish. (Prerequisite Skill)

Name the indicated angles or pairs of angles if $p \parallel q$ and $m \parallel \ell$. (Lesson 3-1)



6. angles congruent to $\angle 8$
7. angles supplementary to $\angle 12$

Find the distance between each pair of points. Round to the nearest tenth. (Lesson 1-3)

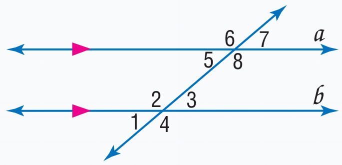
8. (6, 8), (−4, 3)
9. (11, −8), (−3, −4)
10. **MAPS** Jack laid a coordinate grid on a map where each block on the grid corresponds to a city block. If the coordinates of the football stadium are (15, −25) and the coordinates of Jack's house are (−8, 14), what is the distance between the stadium and Jack's house? Round to the nearest tenth. (Lesson 1-3)

QUICKReview

EXAMPLE 1 **Solve $\frac{7}{8}t + 4 = 18$.**

$\frac{7}{8}t + 4 = 18$	Write the equation.
$\frac{7}{8}t = 14$	Subtract.
$8\left(\frac{7}{8}t\right) = 14(8)$	Multiply.
$7t = 112$	Simplify.
$t = 16$	Divide each side by 7.

EXAMPLE 2 **Name the angles congruent to $\angle 6$ if $a \parallel b$.**



$\angle 8 \cong \angle 6$	Vertical Angle Theorem
$\angle 2 \cong \angle 6$	Corresponding Angles Postulate
$\angle 4 \cong \angle 6$	Alternate Exterior Angles Theorem

EXAMPLE 3 **Find the distance between (−1, 2) and (3, −4). Round to the nearest tenth.**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(3 - (-1))^2 + (-4 - 2)^2}$	$(x_1, y_1) = (-1, 2)$, $(x_2, y_2) = (3, -4)$
$= \sqrt{(4)^2 + (-6)^2}$	Subtract.
$= \sqrt{16 + 36}$	Simplify.
$= \sqrt{52}$	Add.
≈ 7.2	Use a calculator.

4-1 Classifying Triangles

Main Ideas

- Identify and classify triangles by angles.
- Identify and classify triangles by sides.

New Vocabulary

acute triangle
obtuse triangle
right triangle
equiangular triangle
scalene triangle
isosceles triangle
equilateral triangle

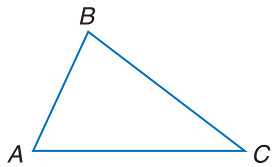
GET READY for the Lesson

Many structures use triangular shapes as braces for construction. The roof sections of houses are made of triangular trusses that support the roof and the house.

Classify Triangles by Angles

Triangle ABC, written $\triangle ABC$, has parts that are named using the letters A, B, and C.

- The sides of $\triangle ABC$ are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
- The vertices are A, B, and C.
- The angles are $\angle ABC$ or $\angle B$, $\angle BCA$ or $\angle C$, and $\angle BAC$ or $\angle A$.



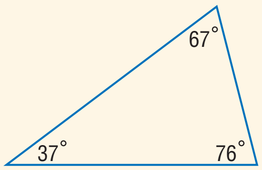
There are two ways to classify triangles. One way is by their angles. All triangles have at least two acute angles, but the third angle is used to classify the triangle.

Study Tip

Common Misconceptions
It is a common mistake to classify triangles by their angles in more than one way. These classifications are distinct groups. For example, a triangle cannot be right and acute.

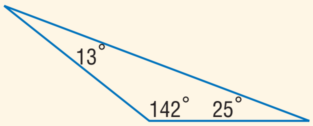
KEY CONCEPT *Classifying Triangles by Angle*

In an **acute triangle**, all of the angles are acute.



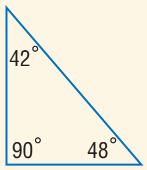
all angle measures < 90

In an **obtuse triangle**, one angle is obtuse.



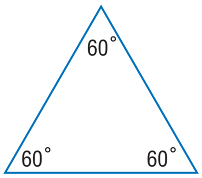
one angle measure > 90

In a **right triangle**, one angle is right.



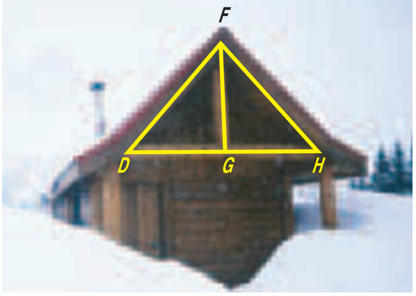
one angle measure = 90

An acute triangle with all angles congruent is an **equiangular triangle**.



Real-World EXAMPLE — Classify Triangles by Angles

1 **ARCHITECTURE** **The roof of this house is made up of three different triangles. Use a protractor to classify △*DFH*, △*DFG*, and △*HFG* as *acute*, *equiangular*, *obtuse*, or *right*.**

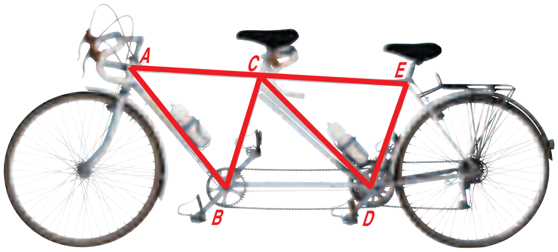


△*DFH* has all angles with measures less than 90, so it is an acute triangle. △*DFG* and △*HFG* both have one angle with measure equal to 90. Both of these are right triangles.

> **Study Tip**
>
> **Congruency**
> To indicate that sides of a triangle are congruent, an equal number of hash marks are drawn on the corresponding sides.

CHECK Your Progress

1. **BICYCLES** The frame of this tandem bicycle uses triangles. Use a protractor to classify △*ABC* and △*CDE*.

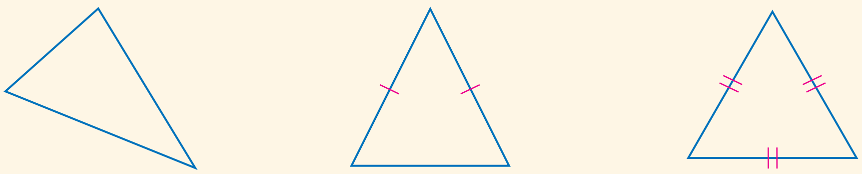


Classify Triangles by Sides Triangles can also be classified according to the number of congruent sides they have.

> **Study Tip**
>
> **Equilateral Triangles**
> An equilateral triangle is a special kind of isosceles triangle.

KEY CONCEPT — Classifying Triangles by Sides

No two sides of a **scalene triangle** are congruent.	At least two sides of an **isosceles triangle** are congruent.	All of the sides of an **equilateral triangle** are congruent.

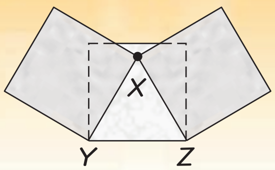


GEOMETRY LAB

Equilateral Triangles

MODEL

- Align three pieces of patty paper. Draw a dot at *X*.
- Fold the patty paper through *X* and *Y* and through *X* and *Z*.



ANALYZE

1. Is △*XYZ* equilateral? Explain.
2. Use three pieces of patty paper to make a triangle that is isosceles, but not equilateral.
3. Use three pieces of patty paper to make a scalene triangle.

EXAMPLE Classify Triangles by Sides

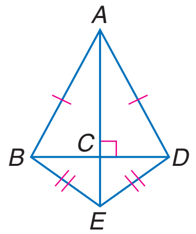
2 **Identify the indicated type of triangle in the figure.**

a. isosceles triangles

Isosceles triangles have at least two sides congruent. So, $\triangle ABD$ and $\triangle EBD$ are isosceles.

b. scalene triangles

Scalene triangles have no congruent sides. $\triangle AEB$, $\triangle AED$, $\triangle ACB$, $\triangle ACD$, $\triangle BCE$, and $\triangle DCE$ are scalene.

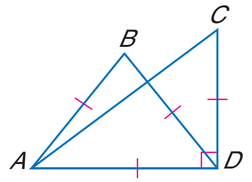


CHECK Your Progress

2 **Identify the indicated type of triangle in the figure.**

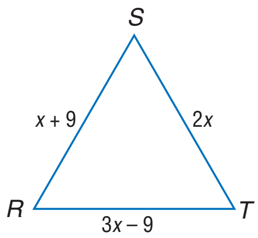
2A. equilateral

2B. isosceles



EXAMPLE Find Missing Values

3 **ALGEBRA Find x and the measure of each side of equilateral triangle RST.**



Since $\triangle RST$ is equilateral, $RS = ST$.

$x + 9 = 2x$ Substitution

$9 = x$ Subtract x from each side.

Next, substitute to find the length of each side.

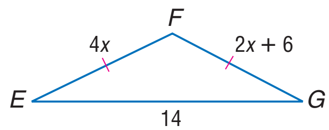
$RS = x + 9$ $\quad ST = 2x$ $\quad RT = 3x - 9$

$= 9 + 9$ or 18 $\quad = 2(9)$ or 18 $\quad = 3(9) - 9$ or 18

For $\triangle RST$, $x = 9$, and the measure of each side is 18.

CHECK Your Progress

3. Find x and the measure of the unknown sides of isosceles triangle EFG.



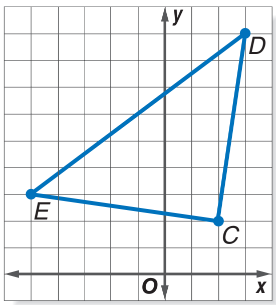
Study Tip

Look Back

To review the **Distance Formula**, see Lesson 1-3.

EXAMPLE Use the Distance Formula

4 **COORDINATE GEOMETRY Find the measures of the sides of $\triangle DEC$. Classify the triangle by sides.**



Use the Distance Formula to find the lengths of each side.

$$EC = \sqrt{(-5 - 2)^2 + (3 - 2)^2}$$
$$= \sqrt{49 + 1}$$
$$= \sqrt{50} \text{ or } 5\sqrt{2}$$

$$DC = \sqrt{(3-2)^2 + (9-2)^2}$$
$$= \sqrt{1 + 49}$$
$$= \sqrt{50} \text{ or } 5\sqrt{2}$$

$$ED = \sqrt{(-5-3)^2 + (3-9)^2}$$
$$= \sqrt{64 + 36}$$
$$= \sqrt{100} \text{ or } 10$$

Since $\overline{EC}$ and $\overline{DC}$ have the same length, $\triangle DEC$ is isosceles.

CHECK Your Progress

4. Find the measures of the sides of $\triangle HIJ$ with vertices $H(-3, 1)$, $I(0, 4)$, and $J(0, 1)$. Classify the triangle by sides.

Online Personal Tutor at geometryonline.com

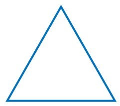
CHECK Your Understanding

Example 1 (p. 203)

Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1.

2.

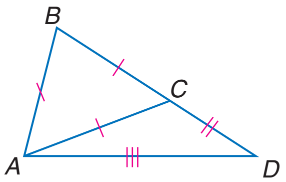


Example 2 (p. 204)

Identify the indicated type of triangle in the figure.

3. isosceles

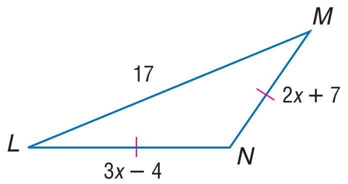
4. scalene



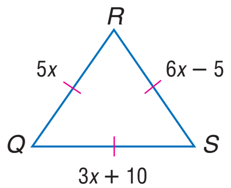
Example 3 (p. 204)

ALGEBRA Find x and the measures of the unknown sides of each triangle.

5.



6.



Example 4 (p. 204)

7. COORDINATE GEOMETRY Find the measures of the sides of $\triangle TWZ$ with vertices at $T(2, 6)$, $W(4, -5)$, and $Z(-3, 0)$. Classify the triangle by sides.

8. COORDINATE GEOMETRY Find the measures of the sides of $\triangle QRS$ with vertices at $Q(2, 1)$, $R(4, -3)$, and $S(-3, -2)$. Classify the triangle by sides.

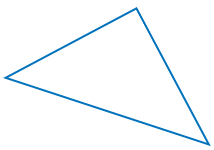
Exercises

HOMEWORK HELP

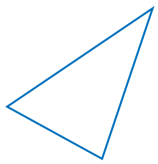
For Exercises	See Examples
9–12	1
13–14	2
15, 16	3
17–20	4

Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

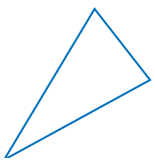
9.



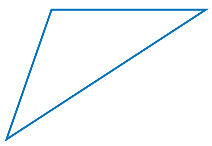
10.



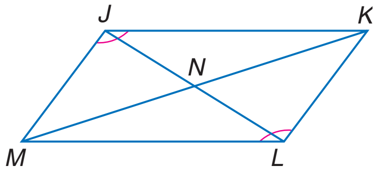
11.



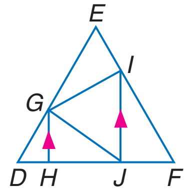
12.



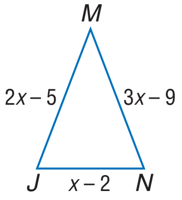
13. Identify the obtuse triangles if $\angle MJK \cong \angle KLM$, $m\angle MJK = 126$, and $m\angle JNM = 52$.



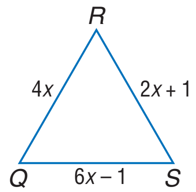
14. Identify the right triangles if $\overline{IJ} \parallel \overline{GH}$, $\overline{GH} \perp \overline{DF}$, and $\overline{GI} \perp \overline{EF}$.



15. **ALGEBRA** Find x, JM, MN, and JN if $\triangle JMN$ is an isosceles triangle with $\overline{JM} \cong \overline{MN}$.



16 **ALGEBRA** Find x, QR, RS, and QS if $\triangle QRS$ is an equilateral triangle.



COORDINATE GEOMETRY **Find the measures of the sides of $\triangle ABC$ and classify each triangle by its sides.**

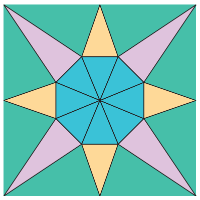
17. $A(5, 4)$, $B(3, -1)$, $C(7, -1)$

18. $A(-4, 1)$, $B(5, 6)$, $C(-3, -7)$

19. $A(-7, 9)$, $B(-7, -1)$, $C(4, -1)$

20. $A(-3, -1)$, $B(2, 1)$, $C(2, -3)$

21. **QUILTING** The star-shaped composite quilting square is made up of four different triangles. Use a ruler to classify the four triangles by sides.

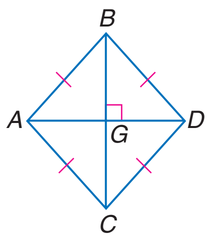


22. **ARCHITECTURE** The restored and decorated Victorian houses in San Francisco shown in the photograph are called the "Painted Ladies." Use a protractor to classify the triangles indicated in the photo by sides and angles.

Real-World Link
The Painted Ladies are located in Alamo Square. The area is one of 11 designated historic districts in San Francisco.

Source: www.sfvisitor.org

Identify the indicated triangles in the figure if $\overline{AB} \cong \overline{BD} \cong \overline{DC} \cong \overline{CA}$ and $\overline{BC} \perp \overline{AD}$.



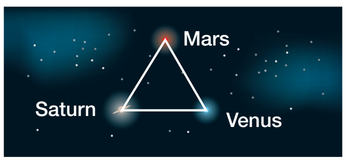
23. right

24. obtuse

25. scalene

26. isosceles

27. **ASTRONOMY** On May 5, 2002, Venus, Saturn, and Mars were aligned in a triangular formation. Use a protractor or ruler to classify the triangle formed by sides and angles.



28. **RESEARCH** Use the Internet or other resource to find out how astronomers can predict planetary alignment.

ALGEBRA **Find x and the measure of each side of the triangle.**

29. $\triangle GHJ$ is isosceles, with $\overline{HG} \cong \overline{JG}$, $GH = x + 7$, $GJ = 3x - 5$, and $HJ = x - 1$.

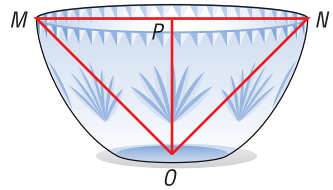
30. $\triangle MPN$ is equilateral with $MN = 3x - 6$, $MP = x + 4$, and $NP = 2x - 1$.

31. $\triangle QRS$ is equilateral. QR is two less than two times a number, RS is six more than the number, and QS is ten less than three times the number.

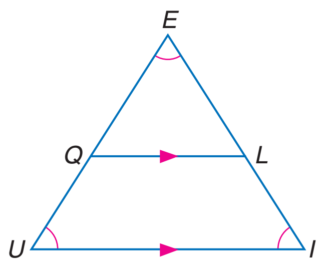
32. $\triangle JKL$ is isosceles with $\overline{KJ} \cong \overline{LJ}$. JL is five less than two times a number. JK is three more than the number. KL is one less than the number. Find the measure of each side.

33. ROAD TRIP The total distance from Charlotte to Raleigh to Winston-Salem and back to Charlotte is about 292 miles. The distance from Charlotte to Winston-Salem is 22 miles less than the distance from Raleigh to Winston-Salem. The distance from Charlotte to Raleigh is 60 miles greater than the distance from Winston-Salem to Charlotte. Classify the triangle that connects Charlotte, Raleigh, and Winston-Salem.

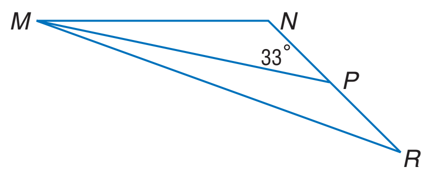
34. CRYSTAL The top of the crystal bowl pictured at the right is circular. The diameter at the top of the bowl is $\overline{MN}$. P is the midpoint of $\overline{MN}$, and $\overline{OP} \perp \overline{MN}$. If $MN = 24$ and $OP = 12$, determine whether $\triangle MPO$ and $\triangle NPO$ are equilateral.



35. PROOF Write a two-column proof to prove that $\triangle EQL$ is equiangular.

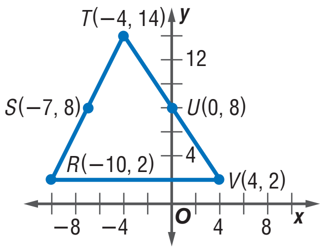


36. PROOF Write a paragraph proof to prove that $\triangle RPM$ is an obtuse triangle if $m\angle NPM = 33$.

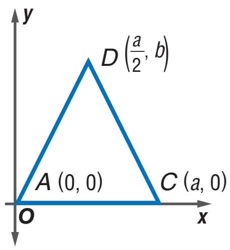


EXTRA PRACTICE
See pages 807, 831.
Math Online
Self-Check Quiz at geometryonline.com

37. COORDINATE GEOMETRY Show that S is the midpoint of $\overline{RT}$ and U is the midpoint of $\overline{TV}$.



38. COORDINATE GEOMETRY Show that $\triangle ADC$ is isosceles.



H.O.T. Problems

39. OPEN ENDED Draw an isosceles right triangle.

REASONING **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain.**

40. Equiangular triangles are also acute.

41. Right triangles are acute.

42. **CHALLENGE** $\overline{KL}$ is a segment representing one side of isosceles right triangle KLM with $K(2, 6)$, and $L(4, 2)$. $\angle KLM$ is a right angle, and $\overline{KL} \cong \overline{LM}$. Describe how to find the coordinates of M and name these coordinates.

43. ***Writing in Math*** Use the information on page 202 to explain why triangles are important in construction. Include a description of how to classify triangles and a justification of why you think one type of triangle might be used more often in architecture than other types.

STANDARDIZED TEST PRACTICE

44. Which type of triangle can serve as a counterexample to the conjecture below?

> If two angles of a triangle are acute, then the measure of the third angle must be greater than or equal to 90.

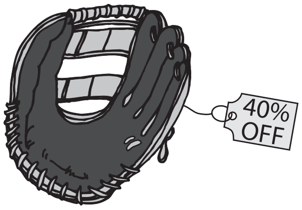
A equilateral

B obtuse

C right

D scalene

45. A baseball glove originally cost \$84.50. Jamal bought it at 40% off.



How much was deducted from the original price?

F \$50.70 **H** \$33.80

G \$44.50 **J** \$32.62

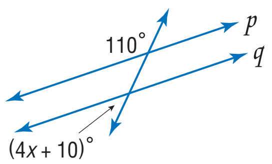
Spiral Review

Graph each line. Construct a perpendicular segment through the given point. Then find the distance from the point to the line. (Lesson 3-6)

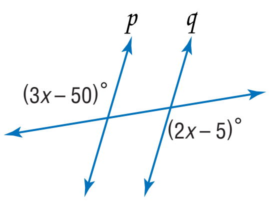
46. $y = x + 2$, $(2, -2)$ **47.** $x + y = 2$, $(3, 3)$ **48.** $y = 7$, $(6, -2)$

Find x so that $p \parallel q$. (Lesson 3-5)

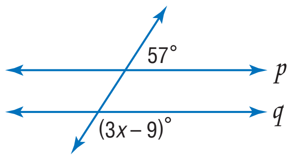
49.



50.



51.



GET READY for the Next Lesson

PREREQUISITE SKILL **In the figure, $\overline{AB} \parallel \overline{RQ}$, $\overline{BC} \parallel \overline{PR}$, and $\overline{AC} \parallel \overline{PQ}$. Name the indicated angles or pairs of angles.** (Lessons 3-1 and 3-2)

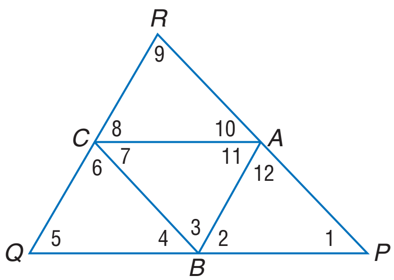
52. three pairs of alternate interior angles

53. six pairs of corresponding angles

54. all angles congruent to $\angle 3$

55. all angles congruent to $\angle 7$

56. all angles congruent to $\angle 11$

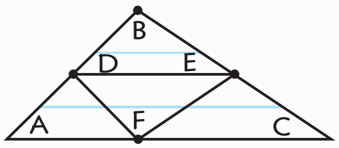


EXPLORE 4-2

Geometry Lab
Angles of Triangles

ACTIVITY 1

Find the relationship among the measures of the interior angles of a triangle.



Step 1 Draw an obtuse triangle and cut it out. Label the vertices A, B, and C.

Step 2 Find the midpoint of $\overline{AB}$ by matching A to B. Label this point D.

Step 3 Find the midpoint of $\overline{BC}$ by matching B to C. Label this point E.

Step 4 Draw $\overline{DE}$.

Step 5 Fold $\triangle ABC$ along $\overline{DE}$. Label the point where B touches $\overline{AC}$ as F.

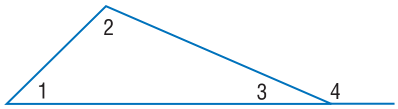
Step 6 Draw $\overline{DF}$ and $\overline{FE}$. Measure each angle.

ANALYZE THE MODEL

Describe the relationship between each pair.

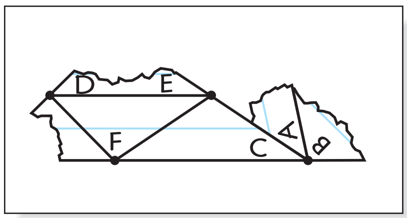
1. $\angle A$ and $\angle DFA$ 2. $\angle B$ and $\angle DFE$ 3. $\angle C$ and $\angle EFC$
4. What is the sum of the measures of $\angle DFA$, $\angle DFE$, and $\angle EFC$?
5. What is the sum of the measures of $\angle A$, $\angle B$, and $\angle C$?
6. **Make a conjecture** about the sum of the measures of the angles of any triangle.

In the figure at the right, $\angle 4$ is called an *exterior angle* of the triangle. $\angle 1$ and $\angle 2$ are the *remote interior angles* of $\angle 4$.



ACTIVITY 2

Find the relationship among the interior and exterior angles of a triangle.



Step 1 Trace $\triangle ABC$ from Activity 1 onto a piece of paper. Label the vertices.

Step 2 Extend $\overline{AC}$ to draw an exterior angle at C.

Step 3 Tear $\angle A$ and $\angle B$ off the triangle from Activity 1.

Step 4 Place $\angle A$ and $\angle B$ over the exterior angle.

ANALYZE THE RESULTS

7. **Make a conjecture** about the relationship of $\angle A$, $\angle B$, and the exterior angle at C.
8. Repeat the steps for the exterior angles of $\angle A$ and $\angle B$.
9. Is your conjecture true for all exterior angles of a triangle?
10. Repeat Activity 2 with an acute triangle and with a right triangle.
11. **Make a conjecture** about the measure of an exterior angle and the sum of the measures of its remote interior angles.

4-2 Angles of Triangles

Main Ideas

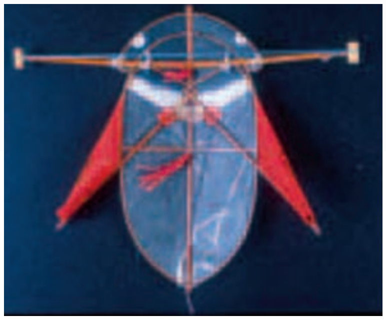
- Apply the Angle Sum Theorem.
- Apply the Exterior Angle Theorem.

New Vocabulary

exterior angle
remote interior angles
flow proof
corollary

GET READY for the Lesson

The Drachen Foundation coordinates the annual Miniature Kite Contest. In a recent year, the kite in the photograph won second place in the Most Beautiful Kite category. The overall dimensions are 10.5 centimeters by 9.5 centimeters. The wings of the beetle are triangular.

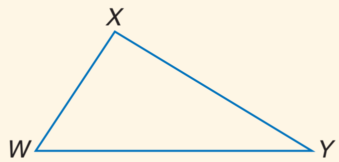


Angle Sum Theorem If the measures of two of the angles of a triangle are known, how can the measure of the third angle be determined? The Angle Sum Theorem explains that the sum of the measures of the angles of any triangle is always 180.

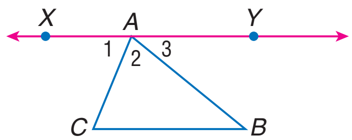
THEOREM 4.1 Angle Sum

The sum of the measures of the angles of a triangle is 180.

Example: $m\angle W + m\angle X + m\angle Y = 180$



PROOF Angle Sum Theorem



Given: $\triangle ABC$

Prove: $m\angle C + m\angle 2 + m\angle B = 180$

Proof:

Statements	Reasons
1. $\triangle ABC$	1. Given
2. Draw $\overleftrightarrow{XY}$ through A parallel to $\overline{CB}$.	2. Parallel Postulate
3. $\angle 1$ and $\angle CAY$ form a linear pair.	3. Def. of a linear pair
4. $\angle 1$ and $\angle CAY$ are supplementary.	4. If 2 ∠s form a linear pair, they are supplementary.
5. $m\angle 1 + m\angle CAY = 180$	5. Def. of suppl. ∠s
6. $m\angle CAY = m\angle 2 + m\angle 3$	6. Angle Addition Postulate
7. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	7. Substitution
8. $\angle 1 \cong \angle C$, $\angle 3 \cong \angle B$	8. Alt. Int. ∠s Theorem
9. $m\angle 1 = m\angle C$, $m\angle 3 = m\angle B$	9. Def. of ≅ ∠s
10. $m\angle C + m\angle 2 + m\angle B = 180$	10. Substitution

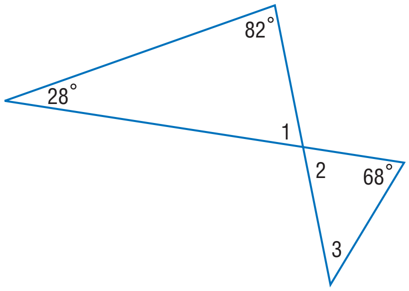
Study Tip

Auxiliary Lines
Recall that sometimes extra lines have to be drawn to complete a proof. These are called *auxiliary lines*.

If we know the measures of two angles of a triangle, we can find the measure of the third.

EXAMPLE Interior Angles

1 **Find the missing angle measures.**



Find $m\angle 1$ first because the measures of two angles of the triangle are known.

$m\angle 1 + 28 + 82 = 180$ Angle Sum Theorem

$m\angle 1 + 110 = 180$ Simplify.

$m\angle 1 = 70$ Subtract 110 from each side.

$\angle 1$ and $\angle 2$ are congruent vertical angles. So $m\angle 2 = 70$.

$m\angle 3 + 68 + 70 = 180$ Angle Sum Theorem

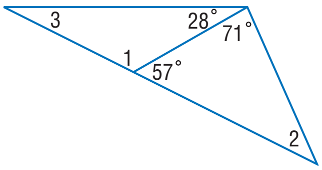
$m\angle 3 + 138 = 180$ Simplify.

$m\angle 3 = 42$ Subtract 138 from each side.

Therefore, $m\angle 1 = 70$, $m\angle 2 = 70$, and $m\angle 3 = 42$.

CHECK Your Progress

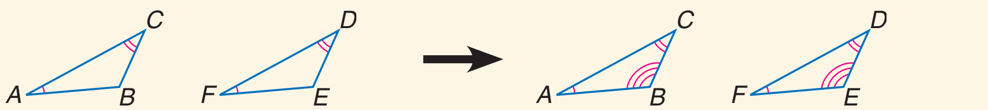
1.



The Angle Sum Theorem leads to a useful theorem about the angles in two triangles.

THEOREM 4.2 *Third Angle Theorem*

If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.



Example: If $\angle A \cong \angle F$ and $\angle C \cong \angle D$, then $\angle B \cong \angle E$.

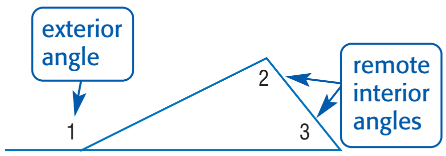
You will prove this theorem in Exercise 34.

Vocabulary Link

Remote
Everyday Use located far away; distant in space

Interior
Everyday Use the internal portion or area

Exterior Angle Theorem Each angle of a triangle has an exterior angle. An **exterior angle** is formed by one side of a triangle and the extension of another side. The interior angles of the triangle not adjacent to a given exterior angle are called **remote interior angles** of the exterior angle.

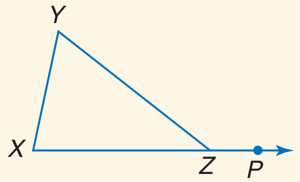


Extra Examples at geometryonline.com

THEOREM 4.3 — *Exterior Angle Theorem*

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Example: $m\angle X + m\angle Y = m\angle YZP$



Study Tip

Flow Proof

Write each statement and reason on an index card. Then organize the index cards in logical order.

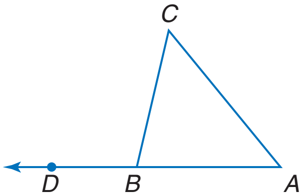
We will use a flow proof to prove this theorem. A **flow proof** organizes a series of statements in logical order, starting with the given statements. Each statement is written in a box with the reason verifying the statement written below the box. Arrows are used to indicate how the statements relate to each other.

PROOF — Exterior Angle Theorem

Write a flow proof of the Exterior Angle Theorem.

Given: $\triangle ABC$

Prove: $m\angle CBD = m\angle A + m\angle C$

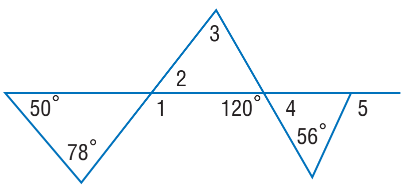


Flow Proof:

- $\triangle ABC$ — Given
- $\angle CBD$ and $\angle ABC$ form a linear pair. — Definition of linear pair
- $\angle CBD$ and $\angle ABC$ are supplementary. — If 2 ∠s form a linear pair, they are supplementary.
- $m\angle A + m\angle ABC + \angle C = 180$ — Angle Sum Theorem
- $m\angle CBD + m\angle ABC = 180$ — Definition of supplementary
- $m\angle A + m\angle ABC + m\angle C = m\angle CBD + m\angle ABC$ — Substitution Property
- $m\angle A + m\angle C = m\angle CBD$ — Subtraction Property

EXAMPLE 2 — Exterior Angles

Find the measure of each angle.



a. $m\angle 1$

$m\angle 1 = 50 + 78$ — Exterior Angle Theorem

$= 128$ — Simplify.

b. $m\angle 2$

$m\angle 1 + m\angle 2 = 180$ — If 2 ∠s form a linear pair, they are suppl.

$128 + m\angle 2 = 180$ — Substitution

$m\angle 2 = 52$ — Subtract 128 from each side.

c. $m\angle 3$

$m\angle 2 + m\angle 3 = 120$ Exterior Angle Theorem

$52 + m\angle 3 = 120$ Substitution

$m\angle 3 = 68$ Subtract 52 from each side.

Therefore, $m\angle 1 = 128$, $m\angle 2 = 52$, and $m\angle 3 = 68$.

CHECK Your Progress

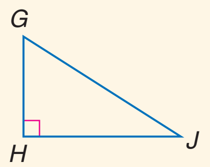
2A. $m\angle 4$ **2B.** $m\angle 5$

Online **Personal Tutor at** geometryonline.com

A statement that can be easily proved using a theorem is often called a **corollary** of that theorem. A corollary, just like a theorem, can be used as a reason in a proof.

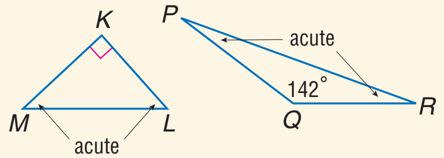
COROLLARIES

4.1 The acute angles of a right triangle are complementary.



Example: $m\angle G + m\angle J = 90$

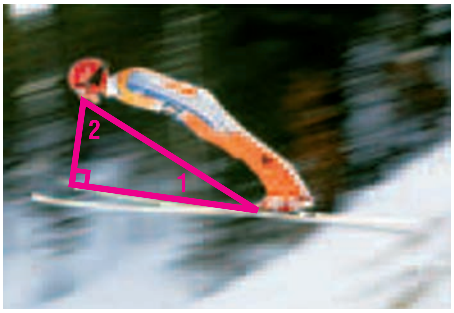
4.2 There can be at most one right or obtuse angle in a triangle.



You will prove Corollaries 4.1 and 4.2 in Exercises 32 and 33.

Real-World EXAMPLE Right Angles

3 **SKI JUMPING** **Ski jumper Simon Ammann of Switzerland forms a right triangle with his skis and his line of sight. Find $m\angle 2$ if $m\angle 1$ is 27.**



Use Corollary 4.1 to write an equation.

$m\angle 1 + m\angle 2 = 90$

$27 + m\angle 2 = 90$ Substitution

$m\angle 2 = 63$ Subtract 27 from each side.

CHECK Your Progress

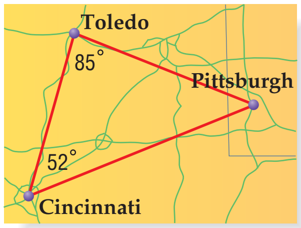
3. WIND SURFING A windsurfing sail is generally a right triangle. One of the angles that is not the right angle has a measure of 68°. What is the measure of the other nonright angle?

CHECK Your Understanding

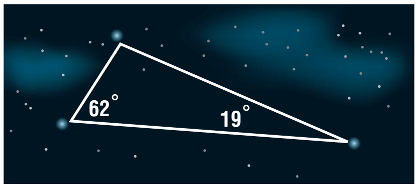
Example 1 (p. 211)

Find the missing angle measure.

1.



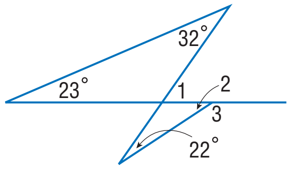
2.



Example 2 (p. 212)

Find each measure.

3. $m\angle 1$ **4.** $m\angle 2$ **5.** $m\angle 3$

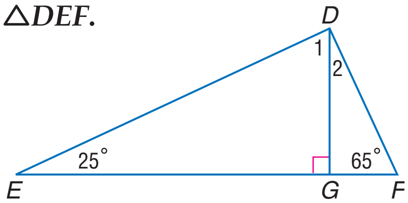


Example 3 (p. 213)

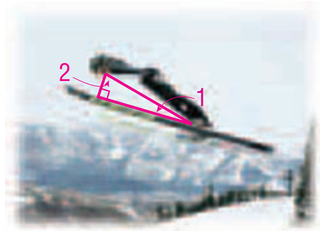
Find each measure in $\triangle DEF$.

6. $m\angle 1$

7. $m\angle 2$



8. SKI JUMPING American ski jumper Jessica Jerome forms a right angle with her skis. If $m\angle 2 = 70$, find $m\angle 1$.

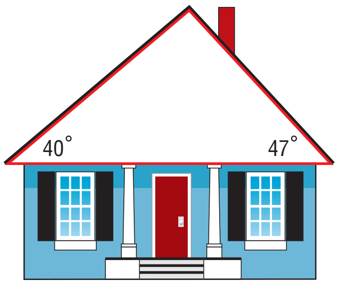


Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–12	1
13–18	2
19–22	3

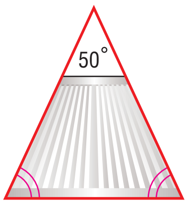
Find the missing angle measures.

9.

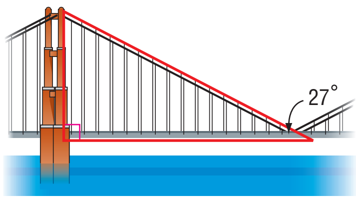


10.

11.



12.

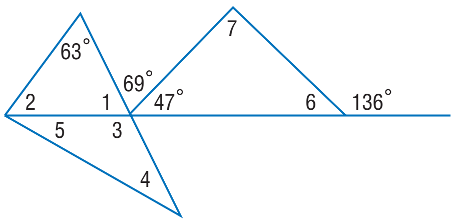


Find each measure if $m\angle 4 = m\angle 5$.

13. $m\angle 1$ **14.** $m\angle 2$

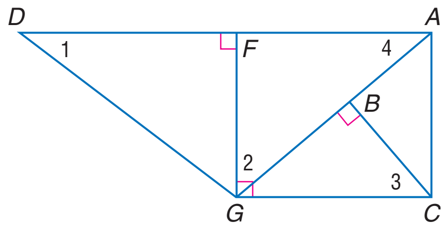
15. $m\angle 3$ **16.** $m\angle 4$

17. $m\angle 5$ **18.** $m\angle 6$



Doug Pensinger/Getty Images

Find each measure if $m\angle DGF = 53$ and $m\angle AGC = 40$.



19. $m\angle 1$

20. $m\angle 2$

21. $m\angle 3$

22. $m\angle 4$

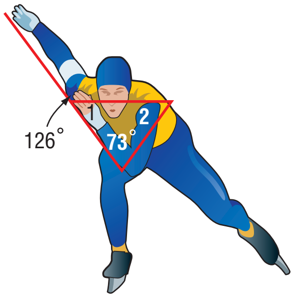
SPEED SKATING **For Exercises 23–26, use the following information.**
Speed skater Catriona Lemay Doan of Canada forms at least two sets of triangles and exterior angles as she skates. Use the measures of given angles to find each measure.

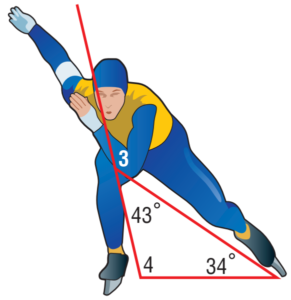
23. $m\angle 1$

24. $m\angle 2$

25. $m\angle 3$

26. $m\angle 4$





Real-World Link
Catriona Lemay Doan is the first Canadian to win a Gold medal in the same event in two consecutive Olympic games.

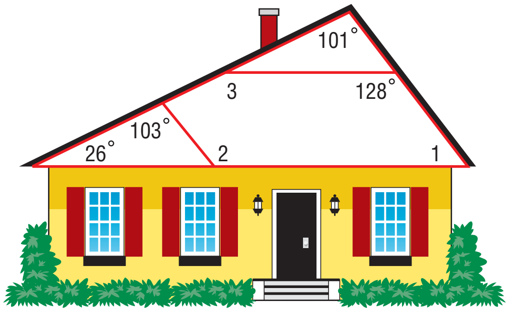
Source: catrionalemaydoan.com

HOUSING **For Exercises 27–29, use the following information.**
The two braces for the roof of a house form triangles. Find each measure.

27. $m\angle 1$

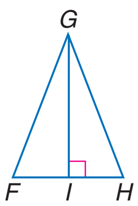
28. $m\angle 2$

29. $m\angle 3$

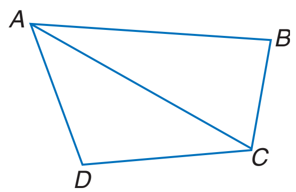


PROOF **For Exercises 30–34, write the specified type of proof.**

30. flow proof
Given: $\angle FGI \cong \angle IGH$
$\overline{GI} \perp \overline{FH}$
Prove: $\angle F \cong \angle H$



31. two-column proof
Given: $ABCD$ is a quadrilateral.
Prove: $m\angle DAB + m\angle B + m\angle BCD + m\angle D = 360$



32. flow proof of Corollary 4.1

33. paragraph proof of Corollary 4.2

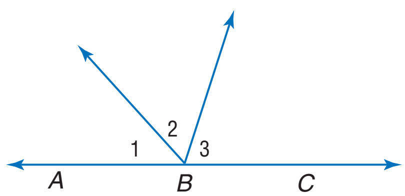
34. two-column proof of Theorem 4.2

EXTRA PRACTICE
See pages 807, 831.
Math Online
Self-Check Quiz at geometryonline.com

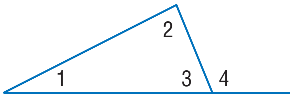
H.O.T. Problems

35. **OPEN ENDED** Draw a triangle. Label one exterior angle and its remote interior angles.

36. **CHALLENGE** $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. The measures of $\angle 1$, $\angle 2$, and $\angle 3$ are in a 4:5:6 ratio. Find the measure of each angle.



37. **FIND THE ERROR** Najee and Kara are discussing the Exterior Angle Theorem. Who is correct? Explain.



Najee	Kara
$m\angle 1 + m\angle 2 = m\angle 4$	$m\angle 1 + m\angle 2 + m\angle 4 = 180$

38. ***Writing in Math*** Use the information about kites provided on page 210 to explain how the angles of triangles are used to make kites. Include an explanation of how you can find the measure of a third angle if two angles of two triangles are congruent. Also include a description of the properties of two angles in a triangle if the measure of the third is 90°.

STANDARDIZED TEST PRACTICE

39. Two angles of a triangle have measures of 35° and 80°. Which of the following could *not* be a measure of an exterior angle of the triangle?

A 165°

B 145°

C 115°

D 100°

40. Which equation is equivalent to $7x - 3(2 - 5x) = 8x$?

F $2x - 6 = 8x$

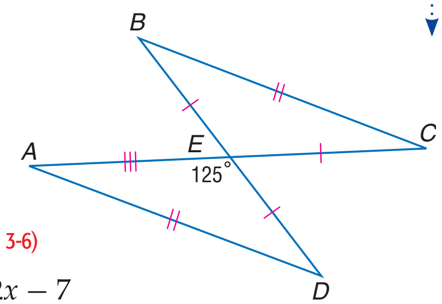
G $22x - 6 = 8x$

H $-8x - 6 = 8x$

J $22x + 6 = 8x$

Spiral Review

Identify the indicated triangles if $\overline{BC} \cong \overline{AD}$, $\overline{EB} \cong \overline{EC}$, $\overline{AC}$ bisects $\overline{BD}$, and $m\angle AED = 125$. (Lesson 4-1)



41. scalene

42. obtuse

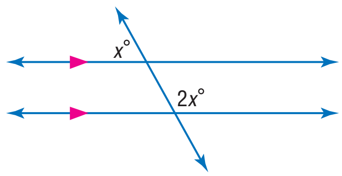
43. isosceles

Find the distance between each pair of parallel lines. (Lesson 3-6)

44. $y = x + 6$, $y = x - 10$

45. $y = -2x + 3$, $y = -2x - 7$

46. **MODEL TRAINS** Regan is going to set up two parallel train tracks with a third track running diagonally across the first two. To properly place a switch, she needs the angle between the diagonal and top of the second track to be twice as large as the angle between the diagonal and top of the first track. What is the value of x? (Lesson 3-2)



GET READY for the Next Lesson

PREREQUISITE SKILL List the property of congruence used for each statement. (Lessons 2-5 and 2-6)

47. $\angle 1 \cong \angle 1$ and $\overline{AB} \cong \overline{AB}$.

48. If $\overline{AB} \cong \overline{XY}$, then $\overline{XY} \cong \overline{AB}$.

49. If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

50. If $\angle 2 \cong \angle 3$ and $\angle 3 \cong \angle 4$, then $\angle 2 \cong \angle 4$.

4-3 Congruent Triangles

Main Ideas

- Name and label corresponding parts of congruent triangles.
- Identify congruence transformations.

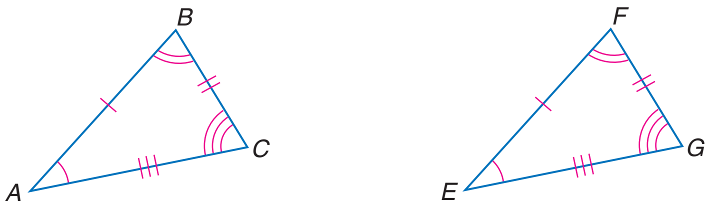
New Vocabulary

congruent triangles
congruence transformations

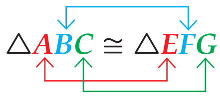
GET READY for the Lesson

The Kaibab suspension bridge near Bright Angel Campground, Arizona, carries the Kaibab Trail across the Colorado River. Steel beams, stained a special color to blend in with the natural scenery of the Grand Canyon, are arranged along the side of the bridge in a triangular web. Triangles spread weight and stress evenly throughout the bridge.

Corresponding Parts of Congruent Triangles Triangles that are the same size and shape are **congruent triangles**. Each triangle has three angles and three sides. If all six of the corresponding parts of two triangles are congruent, then the triangles are congruent.



If $\triangle ABC$ is congruent to $\triangle EFG$, the vertices of the two triangles correspond in the same order as the letters naming the triangles.



$$\triangle ABC \cong \triangle EFG$$

This correspondence of vertices can be used to name the corresponding congruent sides and angles of the two triangles.

$\angle A \cong \angle E$ $\quad \angle B \cong \angle F$ $\quad \angle C \cong \angle G$

$\overline{AB} \cong \overline{EF}$ $\quad \overline{BC} \cong \overline{FG}$ $\quad \overline{AC} \cong \overline{EG}$

The corresponding sides and angles can be determined from any congruence statement by following the order of the letters.

Study Tip

Congruent Parts

In congruent triangles, congruent sides are opposite congruent angles.

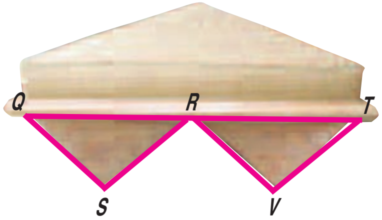
KEY CONCEPT — *Definition of Congruent Triangles (CPCTC)*

Two triangles are congruent if and only if their corresponding parts are congruent.

CPCTC stands for *corresponding parts of congruent triangles are congruent*. "If and only if" is used to show that both the conditional and its converse are true.

Real-World EXAMPLE **Corresponding Congruent Parts**

1 **FURNITURE DESIGN** **The legs of this stool form two triangles. Suppose the measures in inches are $QR = 12$, $RS = 23$, $QS = 24$, $RT = 12$, $TV = 24$, and $RV = 23$.**



a. Name the corresponding congruent angles and sides.

$\angle Q \cong \angle T$ $\quad$ $\angle QRS \cong \angle TRV$ $\quad$ $\angle S \cong \angle V$

$\overline{QR} \cong \overline{TR}$ $\quad$ $\overline{RS} \cong \overline{RV}$ $\quad$ $\overline{QS} \cong \overline{TV}$

b. Name the congruent triangles.

$\triangle QRS \cong \triangle TRV$

CHECK Your Progress

The measures of the sides of triangles *PDQ* and *OEC* are $PD = 5$, $DQ = 7$, $PQ = 11$; $EC = 7$, $OC = 5$, and $OE = 11$.

1A. Name the corresponding congruent angles and sides.

1B. Name the congruent triangles.

Like congruence of segments and angles, congruence of triangles is reflexive, symmetric, and transitive.

THEOREM 4.4 — Properties of Triangle Congruence

Congruence of triangles is reflexive, symmetric, and transitive.

Reflexive

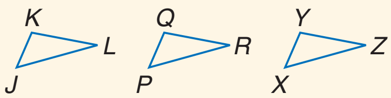
$\triangle JKL \cong \triangle JKL$

Symmetric

If $\triangle JKL \cong \triangle PQR$, then $\triangle PQR \cong \triangle JKL$.

Transitive

If $\triangle JKL \cong \triangle PQR$, and $\triangle PQR \cong \triangle XYZ$, then $\triangle JKL \cong \triangle XYZ$.



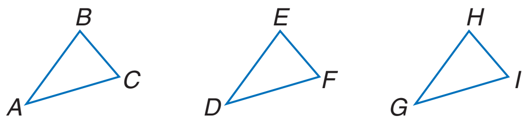
You will prove the symmetric and reflexive parts of Theorem 4.4 in Exercises 30 and 32, respectively.

Proof **Theorem 4.4 (Transitive)**

Given: $\triangle ABC \cong \triangle DEF$

$\triangle DEF \cong \triangle GHI$

Prove: $\triangle ABC \cong \triangle GHI$



Proof: You are given that $\triangle ABC \cong \triangle DEF$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. You are also given that $\triangle DEF \cong \triangle GHI$. So $\angle D \cong \angle G$, $\angle E \cong \angle H$, $\angle F \cong \angle I$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, and $\overline{DF} \cong \overline{GI}$, by CPCTC. Therefore, $\angle A \cong \angle G$, $\angle B \cong \angle H$, $\angle C \cong \angle I$, $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$, and $\overline{AC} \cong \overline{GI}$ because congruence of angles and segments is transitive. Thus, $\triangle ABC \cong \triangle GHI$ by the definition of congruent triangles.

Dwayne Resnick

Study Tip

Naming Congruent Triangles

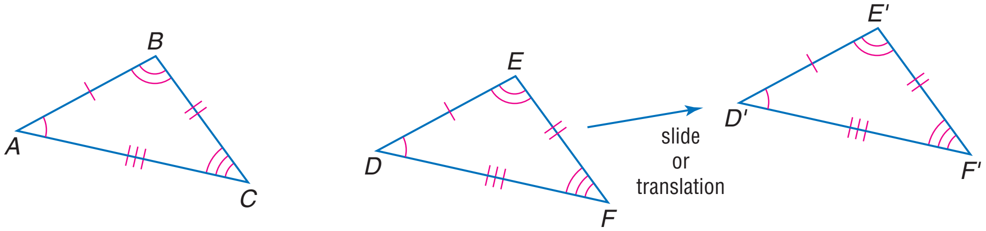
There are six ways to name each pair of congruent triangles.

Study Tip

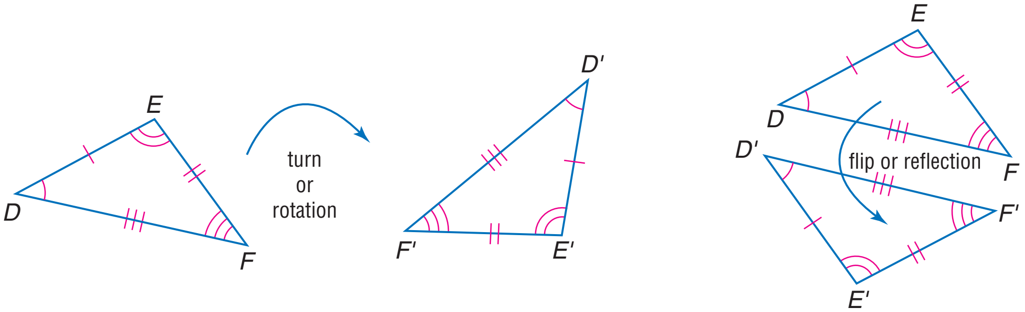
Transformations

Not all transformations preserve congruence. Only transformations that do not change the size or shape of the figure are congruence transformations. You will learn more about transformations in Chapter 9.

Identify Congruence Transformations In the figures below, $\triangle ABC$ is congruent to $\triangle DEF$. If you *slide*, or *translate*, $\triangle DEF$ up and to the right, $\triangle DEF$ is still congruent to $\triangle ABC$.



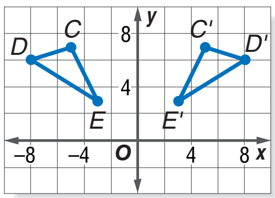
The congruency does not change whether you *turn*, or *rotate*, $\triangle DEF$ or *flip*, or *reflect*, $\triangle DEF$. $\triangle ABC$ is still congruent to $\triangle DEF$.



If you slide, flip, or turn a triangle, the size and shape do not change. These three transformations are called **congruence transformations**.

EXAMPLE 2 Transformations in the Coordinate Plane

COORDINATE GEOMETRY The vertices of $\triangle CDE$ are $C(-5, 7)$, $D(-8, 6)$, and $E(-3, 3)$. The vertices of $\triangle C'D'E'$ are $C'(5, 7)$, $D'(8, 6)$, and $E'(3, 3)$.



a. Verify that $\triangle CDE \cong \triangle C'D'E'$.

Use the Distance Formula to find the length of each side in the triangles.

$$DC = \sqrt{[-8-(-5)]^2 + (6-7)^2} = \sqrt{9+1} \text{ or } \sqrt{10}$$

$$D'C' = \sqrt{(8-5)^2 + (6-7)^2} = \sqrt{9+1} \text{ or } \sqrt{10}$$

$$DE = \sqrt{[-8-(-3)]^2 + (6-3)^2} = \sqrt{25+9} \text{ or } \sqrt{34}$$

$$D'E' = \sqrt{(8-3)^2 + (6-3)^2} = \sqrt{25+9} \text{ or } \sqrt{34}$$

$$CE = \sqrt{[-5-(-3)]^2 + (7-3)^2} = \sqrt{4+16} = \sqrt{20} \text{ or } 2\sqrt{5}$$

$$C'E' = \sqrt{(5-3)^2 + (7-3)^2} = \sqrt{4+16} = \sqrt{20} \text{ or } 2\sqrt{5}$$

By the definition of congruence, $\overline{DC} \cong \overline{D'C'}$, $\overline{DE} \cong \overline{D'E'}$, and $\overline{CE} \cong \overline{C'E'}$. Use a protractor to measure the angles of the triangles. You will find that the measures are the same.

In conclusion, because $\overline{DC} \cong \overline{D'C'}$, $\overline{DE} \cong \overline{D'E'}$, and $\overline{CE} \cong \overline{C'E'}$, $\angle D \cong \angle D'$, $\angle C \cong \angle C'$, and $\angle E \cong \angle E'$, $\triangle CDE \cong \triangle C'D'E'$.

(continued on the next page)

b. Name the congruence transformation for $\triangle CDE$ and $\triangle C'D'E'$.

$\triangle C'D'E'$ is a flip, or reflection, of $\triangle CDE$.

CHECK Your Progress

COORDINATE GEOMETRY **The vertices of $\triangle LMN$ are $L(1, 1)$, $M(3, 5)$, and $N(5, 1)$. The vertices of $\triangle L'M'N'$ are $L'(-1, -1)$, $M'(-3, -5)$, and $N'(-5, -1)$.**

2A. Verify that $\triangle LMN \cong L'M'N'$.

2B. Name the congruence transformation for $\triangle LMN$ and $\triangle L'M'N'$.

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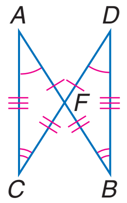
CHECK Your Understanding

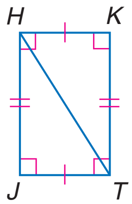
Example 1 (p. 218)

Identify the corresponding congruent angles and sides and the congruent triangles in each figure.

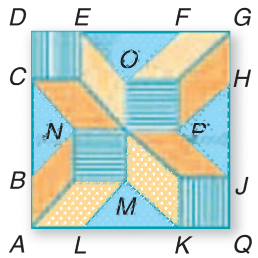
1.



2.



3. QUILTING In the quilt design, assume that angles and segments that appear to be congruent are congruent. Indicate which triangles are congruent.



Example 2 (p. 219)

4. The vertices of $\triangle SUV$ and $\triangle S'U'V'$ are $S(0, 4)$, $U(0, 0)$, $V(2, 2)$, $S'(0, -4)$, $U'(0, 0)$, and $V'(-2, -2)$. Verify that the triangles are congruent and then name the congruence transformation.

5. The vertices of $\triangle QRT$ and $\triangle Q'R'T'$ are $Q(-4, 3)$, $Q'(4, 3)$, $R(-4, -2)$, $R'(4, -2)$, $T(-1, -2)$, and $T'(1, -2)$.Verify that $\triangle QRT \cong \triangle Q'R'T'$. Then name the congruence transformation.

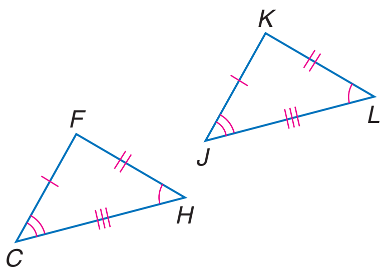
Exercises

HOMEWORK HELP

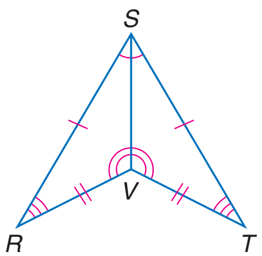
For Exercises	See Examples
6–9	1
10–13	2

Identify the congruent angles and sides and the congruent triangles in each figure.

6.

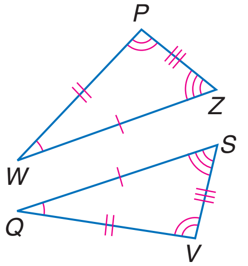


7.

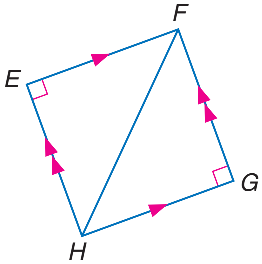


Identify the congruent angles and sides and the congruent triangles in each figure.

8.

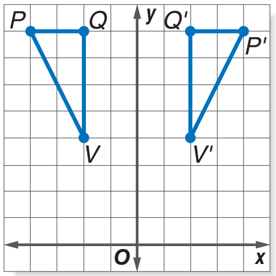


9.

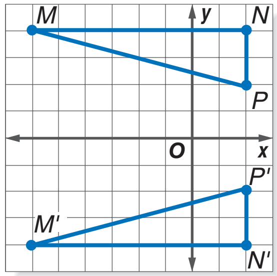


Verify each congruence and name the congruence transformation.

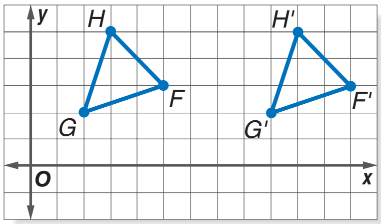
10. $\triangle PQV \cong \triangle P'Q'V'$



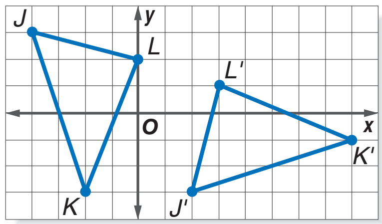
11. $\triangle MNP \cong \triangle M'N'P'$



12. $\triangle GHF \cong \triangle G'H'F'$



13. $\triangle JKL \cong \triangle J'K'L'$



Name the congruent angles and sides for each pair of congruent triangles.

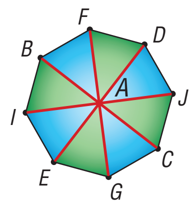
14. $\triangle TUV \cong \triangle XYZ$

15. $\triangle CDG \cong \triangle RSW$

16. $\triangle BCF \cong \triangle DGH$

17. $\triangle ADG \cong \triangle HKL$

18. UMBRELLAS Umbrellas usually have eight triangular sections with ribs of equal length. Are the statements $\triangle JAD \cong \triangle IAE$ and $\triangle JAD \cong \triangle EAI$ both correct? Explain.



Real-World Link

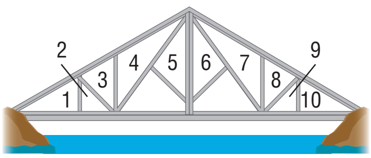
A mosaic is composed of glass, marble, or ceramic pieces often arranged in a pattern. The pieces, or *tesserae*, are set in cement. Mosaics are used to decorate walls, floors, and gardens.

Source: www.dimosaic.com

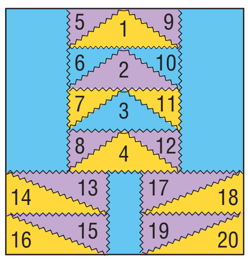
19. MOSAICS The figure at the left is the center of a Roman mosaic. If the bases of the triangles are each the same length, what else do you need to know to conclude that the four triangles surrounding the square are congruent?

Assume that segments and angles that appear to be congruent in each figure are congruent. Indicate which triangles are congruent.

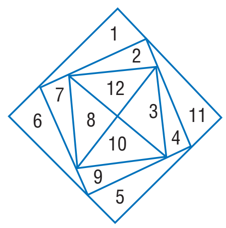
20.



21.



22.



North Carolina Museum of Art, Raleigh, Gift of Mr. & Mrs. Gordon Hanes

Determine whether each statement is *true* or *false*. Draw an example or counterexample for each.

23. Two triangles with corresponding congruent angles are congruent.

24. Two triangles with angles and sides congruent are congruent.

ALGEBRA For Exercises 25 and 26, use the following information.
$\triangle QRS \cong \triangle GHJ$, $RS = 12$, $QR = 10$, $QS = 6$, and $HJ = 2x - 4$.

25. Draw and label a figure to show the congruent triangles.

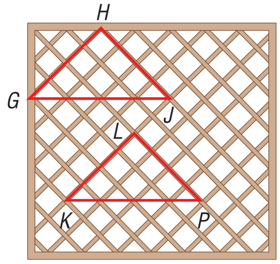
26. Find x.

ALGEBRA For Exercises 27 and 28, use the following information.
$\triangle JKL \cong \triangle DEF$, $m\angle J = 36$, $m\angle E = 64$, and $m\angle F = 3x + 52$.

27. Draw and label a figure to show the congruent triangles.

28. Find x.

29. GARDENING This garden lattice will be covered with morning glories in the summer. Malina wants to save two triangular areas for artwork. If $\triangle GHJ \cong \triangle KLP$, name the corresponding congruent angles and sides.

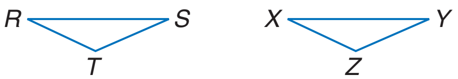


30. PROOF Put the statements used to prove the statement below in the correct order. Provide the reasons for each statement.

Congruence of triangles is symmetric.

Given: $\triangle RST \cong \triangle XYZ$

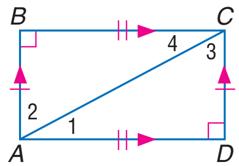
Prove: $\triangle XYZ \cong \triangle RST$



Proof:

- $\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$ — ___?___
- $\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$ — ___?___
- $\triangle RST \cong \triangle XYZ$ — ___?___
- $\triangle XYZ \cong \triangle RST$ — ___?___

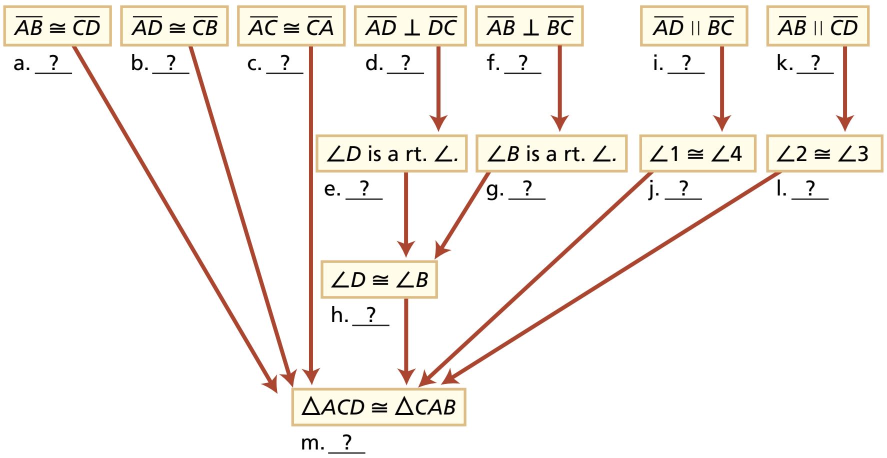
31. PROOF Copy the flow proof and provide the reasons for each statement.



Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$, $\overline{AD} \perp \overline{DC}$, $\overline{AB} \perp \overline{BC}$, $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ACD \cong \triangle CAB$

Proof:



EXTRA PRACTICE
See pages 807, 831.

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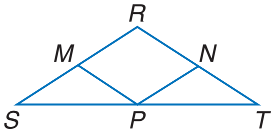
Self-Check Quiz at
geometryonline.com

32. **PROOF** Write a flow proof to prove that congruence of triangles is reflexive. (Theorem 4.4)

H.O.T. Problems

33. **OPEN ENDED** Find a real-world picture of congruent triangles and explain how you know that the triangles are congruent.

34. **CHALLENGE** $\triangle RST$ is isosceles with $RS = RT$, M, N, and P are midpoints of the respective sides, $\angle S \cong \angle MPS$, and $\overline{NP} \cong \overline{MP}$. What else do you need to know to prove that $\triangle SMP \cong \triangle TNP$?



35. ***Writing in Math*** Use the information on page 217 to explain why triangles are used in the design and construction of bridges.

STANDARDIZED TEST PRACTICE

36. Triangle ABC is congruent to $\triangle HIJ$. The vertices of $\triangle ABC$ are $A(-1, 2)$, $B(0, 3)$, and $C(2, -2)$. What is the measure of side $\overline{HJ}$?

A $\sqrt{2}$ **C** 5

B 3 **D** cannot be determined

37. **REVIEW** Which is a factor of $x^2 + 19x - 42$?

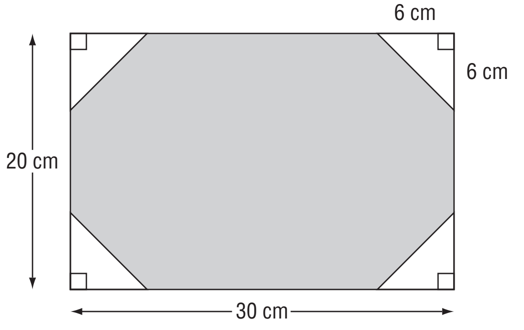
F $x + 14$

G $x + 2$

H $x - 14$

J $x - 2$

38. Bryssa cut four congruent triangles off the corners of a rectangle to make an octagon as shown below.



What is the area of the octagon?

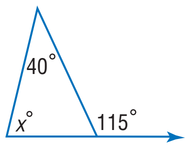
A 456 cm^2 **C** 552 cm^2

B 528 cm^2 **D** 564 cm^2

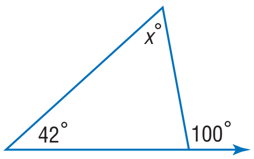
Spiral Review

Find x. (Lesson 4-2)

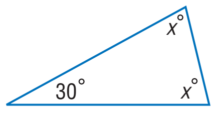
39.



40.



41.



Find x and the measure of each side of the triangle. (Lesson 4-1)

42. $\triangle BCD$ is isosceles with $\overline{BC} \cong \overline{CD}$, $BC = 2x + 4$, $BD = x + 2$ and $CD = 10$.

43. Triangle HKT is equilateral with $HK = x + 7$ and $HT = 4x - 8$.

GET READY for the Next Lesson

PREREQUISITE SKILL **Find the distance between each pair of points.** (Lesson 1-3)

44. $(-1, 7), (1, 6)$

45. $(8, 2), (4, -2)$

46. $(3, 5), (5, 2)$

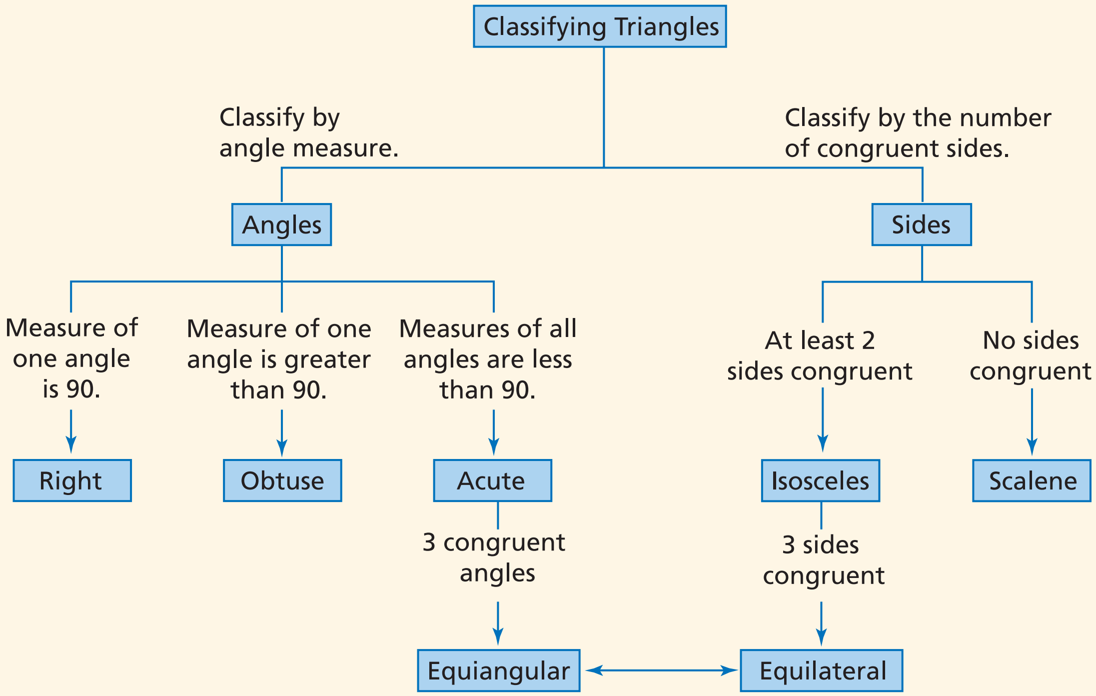
47. $(0, -6), (-3, -1)$

READING MATH

Making Concept Maps

When studying a chapter, it is wise to record the main topics and vocabulary you encounter. In this chapter, some of the new vocabulary words were *triangle*, *acute triangle*, *obtuse triangle*, *right triangle*, *equiangular triangle*, *scalene triangle*, *isosceles triangle*, and *equilateral triangle*. The triangles are all related by the size of the angles or the number of congruent sides.

A graphic organizer called a *concept map* is a convenient way to show these relationships. A concept map is shown below for the different types of triangles. The main ideas are in boxes. Any information that describes how to move from one box to the next is placed along the arrows.



Reading to Learn

1. Describe how to use the concept map to classify triangles by their side lengths.
2. In $\triangle ABC$, $m\angle A = 48$, $m\angle B = 41$, and $m\angle C = 91$. Use the concept map to classify $\triangle ABC$.
3. Identify the type of triangle that is linked to both classifications.

4-4 Proving Congruence—SSS, SAS

Main Ideas

- Use the SSS Postulate to test for triangle congruence.
- Use the SAS Postulate to test for triangle congruence.

New Vocabulary

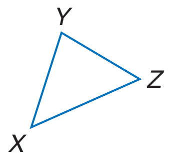
included angle

GET READY for the Lesson

Around 120 B.C., Greek developers and land owners used the properties of geometry to accurately and precisely divide plots of land. Since that time, surveying has been used in areas such as map making and engineering. To check a measurement, land surveyors mark out a right triangle and then mark a second triangle that is congruent to the first.

SSS Postulate Is it always necessary to show that all of the corresponding parts of two triangles are congruent to prove that the triangles are congruent? In this lesson, we will explore two other methods to prove that triangles are congruent.

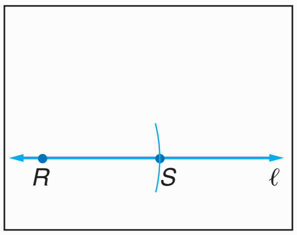
Use the following construction to construct a triangle with sides that are congruent to a given $\triangle XYZ$.



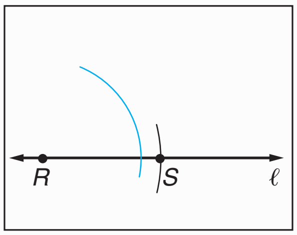
CONSTRUCTION

Congruent Triangles Using Sides

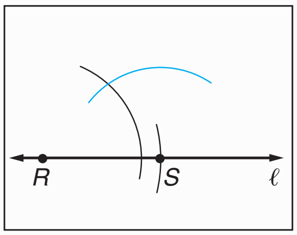
Step 1 Use a straightedge to draw any line ℓ, and select a point R. Use a compass to construct $\overline{RS}$ on ℓ, such that $\overline{RS} \cong \overline{XZ}$.



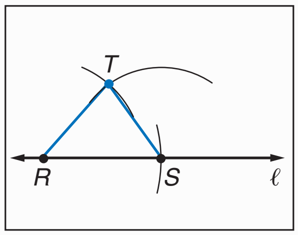
Step 2 Using R as the center, draw an arc with radius equal to XY.



Step 3 Using S as the center, draw an arc with radius equal to YZ.



Step 4 Let T be the point of intersection of the two arcs. Draw $\overline{RT}$ and $\overline{ST}$ to form $\triangle RST$.



Step 5 Cut out $\triangle RST$ and place it over $\triangle XYZ$. How does $\triangle RST$ compare to $\triangle XYZ$?

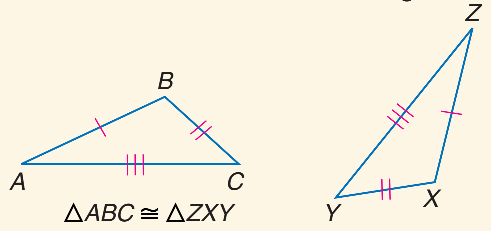
Getty Images

If the corresponding sides of two triangles are congruent, then the triangles are congruent. This is the Side-Side-Side Postulate and is written as SSS.

POSTULATE 4.1 — Side-Side-Side Congruence

If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.

Abbreviation: *SSS*



$\triangle ABC \cong \triangle ZXY$

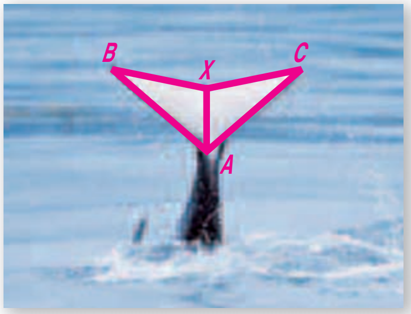
Real-World Link

Orca whales are commonly called "killer whales" because of their predatory nature. They are the largest members of the dolphin family. An average male is about 19–22 feet long and weighs between 8000 and 12,000 pounds.

Source: seaworld.org

Real-World EXAMPLE 1 — Use SSS in Proofs

MARINE BIOLOGY **The tail of an orca whale can be viewed as two triangles that share a common side. Write a two-column proof to prove that $\triangle BXA \cong \triangle CXA$ if $\overline{AB} \cong \overline{AC}$ and $\overline{BX} \cong \overline{CX}$.**



Given: $\overline{AB} \cong \overline{AC}$; $\overline{BX} \cong \overline{CX}$

Prove: $\triangle BXA \cong \triangle CXA$

Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$; $\overline{BX} \cong \overline{CX}$	**1.** Given
2. $\overline{AX} \cong \overline{AX}$	**2.** Reflexive Property
3. $\triangle BXA \cong \triangle CXA$	**3.** SSS

CHECK Your Progress

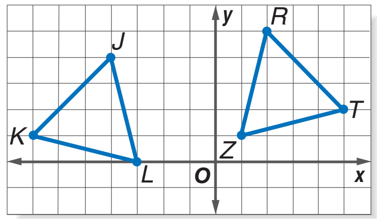
1A. A "Caution, Floor Slippery When Wet" sign is composed of three triangles. If $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{DC}$, prove that $\triangle ACB \cong \triangle ACD$.

1B. Triangle QRS is an isosceles triangle with $\overline{QR} \cong \overline{RS}$. If there exists a line $\overline{RT}$ that bisects $\angle QRS$ and $\overline{QS}$, show that $\triangle QRT \cong \triangle SRT$.

You can use the Distance Formula and postulates about triangle congruence to relate figures on the coordinate plane.

EXAMPLE SSS on the Coordinate Plane

2 **COORDINATE GEOMETRY** **Determine whether $\triangle RTZ \cong \triangle JKL$ for $R(2, 5)$, $Z(1, 1)$, $T(5, 2)$, $L(-3, 0)$, $K(-7, 1)$, and $J(-4, 4)$. Explain.**



Use the Distance Formula to show that the corresponding sides are congruent.

$$RT = \sqrt{(2-5)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18} \text{ or } 3\sqrt{2}$$

$$JK = \sqrt{[-4-(-7)]^2 + (4-1)^2} = \sqrt{9+9} = \sqrt{18} \text{ or } 3\sqrt{2}$$

$$TZ = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{16+1} \text{ or } \sqrt{17}$$

$$KL = \sqrt{[-7-(-3)]^2 + (1-0)^2} = \sqrt{16+1} \text{ or } \sqrt{17}$$

$$RZ = \sqrt{(2-1)^2 + (5-1)^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$$JL = \sqrt{[-4-(-3)]^2 + (4-0)^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$RT = JK$, $TZ = KL$, and $RZ = JL$. By definition of congruent segments, all corresponding segments are congruent. Therefore, $\triangle RTZ \cong \triangle JKL$ by SSS.

CHECK Your Progress

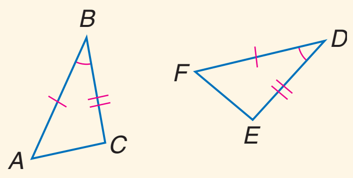
2. Determine whether triangles ABC and TDS with vertices $A(1, 1)$, $B(3, 2)$, $C(2, 5)$, $T(1, -1)$, $D(3, -3)$, and $S(2, -5)$ are congruent. Justify your reasoning.

SAS Postulate Suppose you are given the measures of two sides and the angle they form, called the **included angle**. These conditions describe a unique triangle. Two triangles in which corresponding sides and the included pairs of angles are congruent provide another way to show that triangles are congruent.

POSTULATE 4.2 *Side-Angle-Side Congruence*

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Abbreviation: *SAS*



$\triangle ABC \cong \triangle FDE$

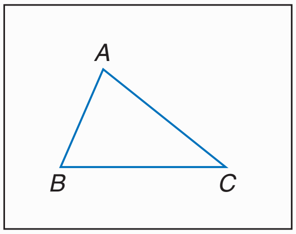
Concepts in Motion
Animation
geometryonline.com

You can also construct congruent triangles given two sides and the included angle.

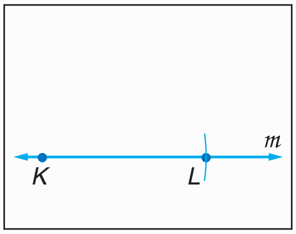
CONSTRUCTION

Congruent Triangles Using Two Sides and the Included Angle

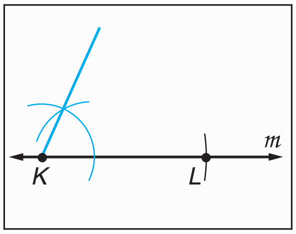
Step 1 Draw a triangle and label its vertices *A*, *B*, and *C*.



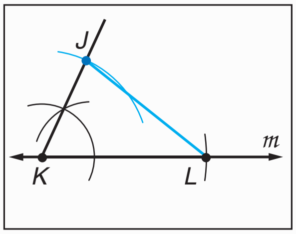
Step 2 Select a point *K* on line *m*. Use a compass to construct $\overline{KL}$ on *m* such that $\overline{KL} \cong \overline{BC}$.



Step 3 Construct an angle congruent to $\angle B$ using $\overrightarrow{KL}$ as a side of the angle and point *K* as the vertex.



Step 4 Construct $\overline{JK}$ such that $\overline{JK} \cong \overline{AB}$. Draw $\overline{JL}$ to complete $\triangle JKL$.



Step 5 Cut out $\triangle JKL$ and place it over $\triangle ABC$. How does $\triangle JKL$ compare to $\triangle ABC$?

Study Tip

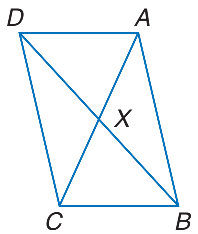
Flow Proofs
Flow proofs can be written vertically or horizontally.

EXAMPLE Use SAS in Proofs

3 **Write a flow proof.**

Given: X is the midpoint of $\overline{BD}$.
X is the midpoint of $\overline{AC}$.

Prove: $\triangle DXC \cong \triangle BXA$

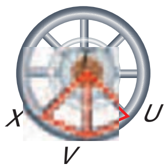


Flow Proof:

- X is the midpoint of $\overline{DB}$. (Given) → $\overline{DX} \cong \overline{BX}$ (Midpoint Theorem) → $\triangle DXC \cong \triangle BXA$
- X is the midpoint of $\overline{AC}$. (Given) → $\overline{CX} \cong \overline{AX}$ (Midpoint Theorem) → $\triangle DXC \cong \triangle BXA$
- $\angle DXC \cong \angle BXA$ (Vertical ∠s are ≅.) → $\triangle DXC \cong \triangle BXA$ (SAS)

CHECK Your Progress

3. The spokes used in a captain's wheel divide the wheel into eight parts. If $\overline{TU} \cong \overline{TX}$ and $\angle XTV \cong \angle UTV$, show that $\triangle XTV \cong \triangle UTV$.



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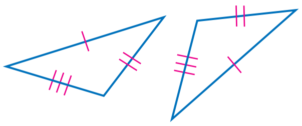
EXAMPLE 4 Identify Congruent Triangles

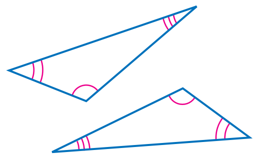
Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

a.



Each pair of corresponding sides are congruent. The triangles are congruent by the SSS Postulate.

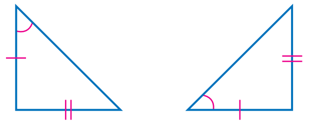
b.



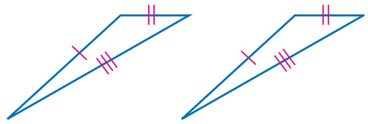
The triangles have three pairs of corresponding angles congruent. This does not match the SSS or the SAS Postulate. It is *not possible* to prove them congruent.

CHECK Your Progress

4A.



4B.



CHECK Your Understanding

Example 1 (p. 226)

1. JETS The United States Navy Flight Demonstration Squadron, the Blue Angels, fly in a formation that can be viewed as two triangles with a common side. Write a two-column proof to prove that $\triangle SRT \cong \triangle QRT$ if T is the midpoint of $\overline{SQ}$ and $\overline{SR} \cong \overline{QR}$.

Example 2 (p. 227)

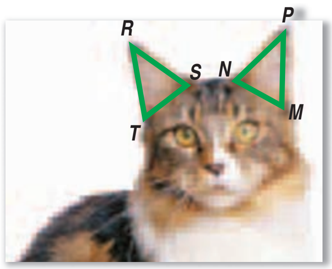
Determine whether $\triangle EFG \cong \triangle MNP$ given the coordinates of the vertices. Explain.

2. $E(-4, -3)$, $F(-2, 1)$, $G(-2, -3)$, $M(4, -3)$, $N(2, 1)$, $P(2, -3)$

3. $E(-2, -2)$, $F(-4, 6)$, $G(-3, 1)$, $M(2, 2)$, $N(4, 6)$, $P(3, 1)$

Example 3 (p. 228)

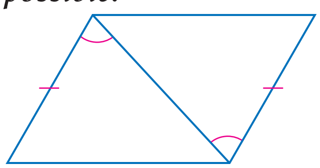
4. CATS A cat's ear is triangular in shape. Write a proof to prove $\triangle RST \cong \triangle PNM$ if $\overline{RS} \cong \overline{PN}$, $\overline{RT} \cong \overline{PM}$, $\angle S \cong \angle N$, and $\angle T \cong \angle M$.



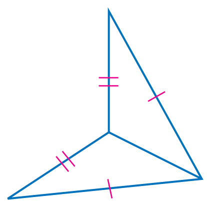
Example 4 (p. 229)

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

5.



6.

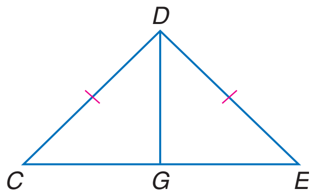


Exercises

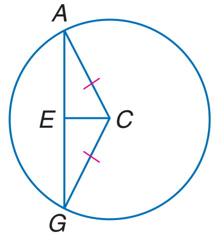
HOMEWORK HELP	
For Exercises	See Examples
7, 8	1
9–12	2
13, 14	3
15–18	4

PROOF **For Exercises 7 and 8, write a two-column proof.**

7. **Given:** $\triangle CDE$ is an isosceles triangle. G is the midpoint of $\overline{CE}$.
Prove: $\triangle CDG \cong \triangle EDG$



8. **Given:** $\overline{AC} \cong \overline{GC}$
$\overline{EC}$ bisects $\overline{AG}$.
Prove: $\triangle GEC \cong \triangle AEC$

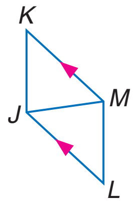


Determine whether $\triangle JKL \cong \triangle FGH$ given the coordinates of the vertices. Explain.

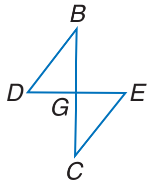
9. $J(2, 5), K(5, 2), L(1, 1), F(-4, 4), G(-7, 1), H(-3, 0)$
10. $J(-1, 1), K(-2, -2), L(-5, -1), F(2, -1), G(3, -2), H(2, 5)$
11. $J(-1, -1), K(0, 6), L(2, 3), F(3, 1), G(5, 3), H(8, 1)$
12. $J(3, 9), K(4, 6), L(1, 5), F(1, 7), G(2, 4), H(-1, 3)$

PROOF **For Exercises 13 and 14, write the specified type of proof.**

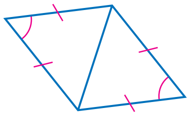
13. flow proof
Given: $\overline{KM} \parallel \overline{LJ}$, $\overline{KM} \cong \overline{LJ}$
Prove: $\triangle JKM \cong \triangle MLJ$

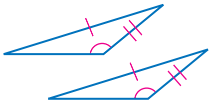


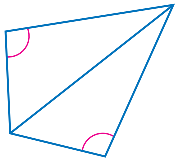
14. two-column proof
Given: $\overline{DE}$ and $\overline{BC}$ bisect each other.
Prove: $\triangle DGB \cong \triangle EGC$

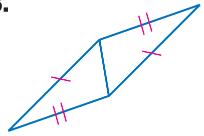


Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

15. 

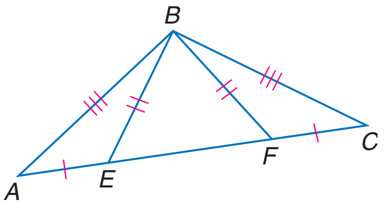
16. 

17. 

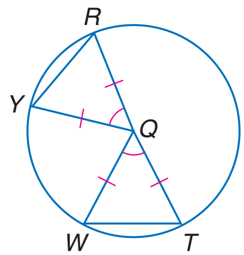
18. 

PROOF **For Exercises 19 and 20, write a flow proof.**

19. **Given:** $\overline{AE} \cong \overline{CF}$, $\overline{AB} \cong \overline{CB}$, $\overline{BE} \cong \overline{BF}$
Prove: $\triangle AFB \cong \triangle CEB$



20. **Given:** $\overline{RQ} \cong \overline{TQ} \cong \overline{YQ} \cong \overline{WQ}$
$\angle RQY \cong \angle WQT$
Prove: $\triangle QWT \cong \triangle QYR$

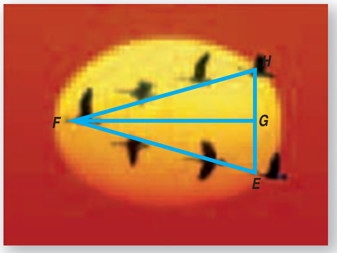


Real-World Link

The infield is a square 90 feet on each side.

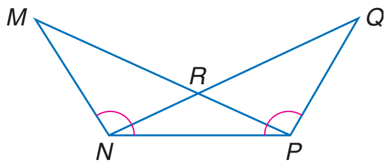
Source: mlb.com

21. GEESE A flock of geese flies in formation. Write a proof to prove that $\triangle EFG \cong \triangle HFG$ if $\overline{EF} \cong \overline{HF}$ and G is the midpoint of $\overline{EH}$.

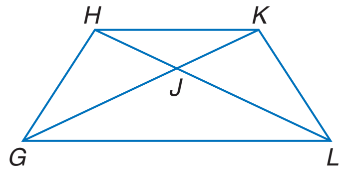


PROOF **For Exercises 22 and 23, write a two-column proof.**

22. Given: $\triangle MRN \cong \triangle QRP$
$\angle MNP \cong \angle QPN$
Prove: $\triangle MNP \cong \triangle QPN$



23. Given: $\triangle GHJ \cong \triangle LKJ$
Prove: $\triangle GHL \cong \triangle LKG$



BASEBALL **For Exercises 24 and 25, use the following information.**
A baseball diamond is a square with four right angles and all sides congruent.

24. Write a two-column proof to prove that the distance from first base to third base is the same as the distance from home plate to second base.

25. Write a two-column proof to prove that the angle formed between second base, home plate, and third base is the same as the angle formed between second base, home plate, and first base.

EXTRA PRACTICE
See pages 818, 831.

Math Online
Self-Check Quiz at geometryonline.com

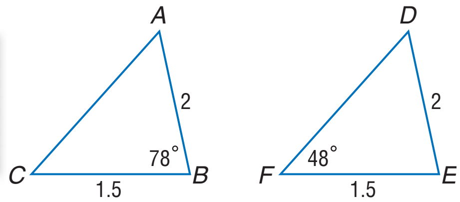
H.O.T. Problems

26. REASONING Explain how the SSS postulate can be used to prove that two triangles are congruent.

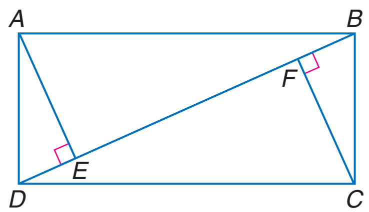
27. OPEN ENDED Find two triangles in a newspaper or magazine and show that they are congruent.

28. FIND THE ERROR Carmelita and Jonathan are trying to determine whether $\triangle ABC$ is congruent to $\triangle DEF$. Who is correct and why?

Carmelita	Jonathan
$\triangle ABC \cong \triangle DEF$ by SAS	Congruence cannot be determined.



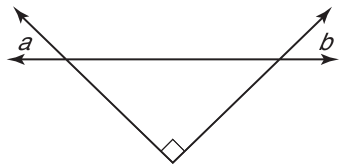
29. CHALLENGE Devise a plan and write a two-column proof for the following.
Given: $\overline{DE} \cong \overline{FB}$, $\overline{AE} \cong \overline{FC}$,
$\overline{AE} \perp \overline{DB}$, $\overline{CF} \perp \overline{DB}$
Prove: $\triangle ABD \cong \triangle CDB$



30. *Writing in Math* Describe two different methods that could be used to prove that two triangles are congruent.

STANDARDIZED TEST PRACTICE

31. Which of the following statements about the figure is true?



A $a + b < 90$ **C** $a + b = 90$

B $a + b > 90$ **D** $a + b = 45$

32. REVIEW The Murphy family just drove 300 miles to visit their grandparents. Mr. Murphy drove 70 mph for 65% of the trip and 35 mph or less for 20% of the trip that was left. Assuming that Mr. Murphy never went over 70 mph, how many miles did he travel at a speed between 35 and 70 mph?

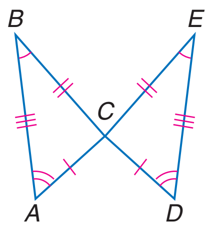
F 195 **H** 21

G 84 **J** 18

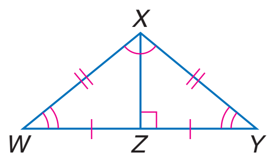
Spiral Review

Identify the congruent triangles in each figure. (Lesson 4-3)

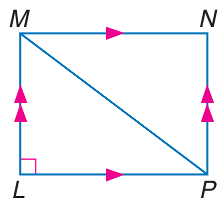
33.



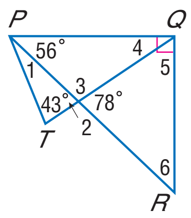
34.



35.



Find each measure if $\overline{PQ} \perp \overline{QR}$. (Lesson 4-2)



36. $m\angle 2$ **37.** $m\angle 3$

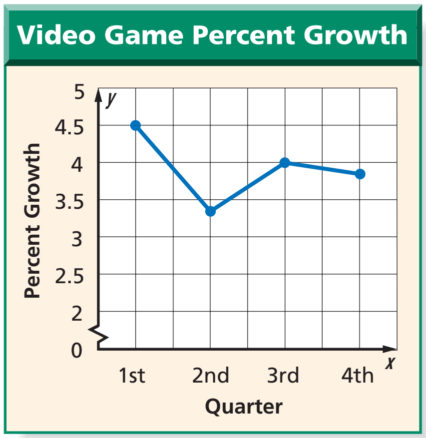
38. $m\angle 5$ **39.** $m\angle 4$

40. $m\angle 1$ **41.** $m\angle 6$

ANALYZE GRAPHS **For Exercises 42 and 43, use the graph of sales of a certain video game system in a recent year.** (Lesson 3-3)

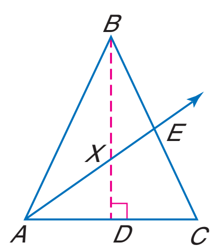
42. Find the rate of change from first quarter to the second quarter.

43. Which had the greater rate of change: first quarter to second quarter, or third to fourth?



GET READY for the Next Lesson

PREREQUISITE SKILL **$\overline{BD}$ and $\overline{AE}$ are angle bisectors and segment bisectors. Name the indicated segments and angles.** (Lessons 1-5 and 1-6)



44. segment congruent to $\overline{EC}$

45. angle congruent to $\angle ABD$

46. angle congruent to $\angle BDC$

47. segment congruent to $\overline{AD}$

48. angle congruent to $\angle BAE$

49. angle congruent to $\angle BXA$

4 Mid-Chapter Quiz

Lessons 4-1 through 4-4

1. **MULTIPLE CHOICE** Classify $\triangle ABC$ with vertices $A(-1, 1)$, $B(1, 3)$, and $C(3, -1)$. (Lesson 4-1)

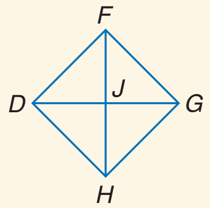
A scalene acute

B equilateral

C isosceles acute

D isosceles right

2. Identify the isosceles triangles in the figure, if $\overline{FH}$ and $\overline{DG}$ are congruent perpendicular bisectors. (Lesson 4-1)



$\triangle ABC$ is equilateral with $AB = 2x$, $BC = 4x - 7$, and $AC = x + 3.5$. (Lesson 4-1)

3. Find x.

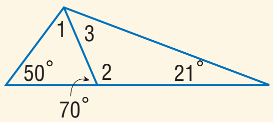
4. Find the measure of each side.

Find the measure of each angle listed below. (Lesson 4-2)

5. $m\angle 1$

6. $m\angle 2$

7. $m\angle 3$

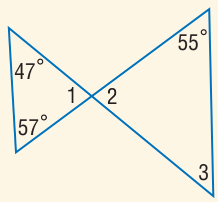


Find each measure. (Lesson 4-2)

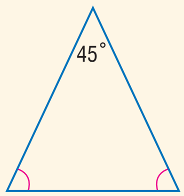
8. $m\angle 1$

9. $m\angle 2$

10. $m\angle 3$



11. Find the missing angle measures. (Lesson 4-2)



12. If $\triangle MNP \cong \triangle JKL$, name the corresponding congruent angles and sides. (Lesson 4-3)

13. **MULTIPLE CHOICE** **Given:** $\triangle ABC \cong \triangle XYZ$. Which of the following *must* be true? (Lesson 4-3)

F $\angle A \cong \angle Y$

G $\overline{AC} \cong \overline{XZ}$

H $\overline{AB} \cong \overline{YZ}$

J $\angle Z \cong \angle B$

COORDINATE GEOMETRY **The vertices of $\triangle JKL$ are $J(7, 7)$, $K(3, 7)$, $L(7, 1)$. The vertices of $\triangle J'K'L'$ are $J'(7, -7)$, $K'(3, -7)$, $L'(7, -1)$.** (Lesson 4-3)

14. Verify that $\triangle JKL \cong \triangle J'K'L'$.

15. Name the congruence transformation for $\triangle JKL$ and $\triangle J'K'L'$.

16. Determine whether $\triangle JML \cong \triangle BDG$ given that $J(-4, 5)$, $M(-2, 6)$, $L(-1, 1)$, $B(-3, -4)$, $D(-4, -2)$, and $G(1, -1)$. (Lesson 4-4)

Determine whether $\triangle XYZ \cong \triangle TUV$ given the coordinates of the vertices. Explain. (Lesson 4-4)

17. $X(0, 0)$, $Y(3, 3)$, $Z(0, 3)$, $T(-6, -6)$, $U(-3, -3)$, $V(-3, -6)$

18. $X(7, 0)$, $Y(5, 4)$, $Z(1, 1)$, $T(-5, -4)$, $U(-3, 4)$, $V(1,1)$

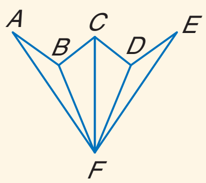
19. $X(9, 6)$, $Y(3, 7)$, $Z(9, -6)$, $T(-10, 7)$, $U(-4, 7)$, $V(-10, -7)$

Write a two-column proof. (Lesson 4-4)

20. **Given:** $\triangle ABF \cong \triangle EDF$

$\overline{CF}$ is angle bisector of $\angle DFB$.

Prove: $\triangle BCF \cong \triangle DCF$.



4-5 Proving Congruence—ASA, AAS

Main Ideas

- Use the ASA Postulate to test for triangle congruence.
- Use the AAS Theorem to test for triangle congruence.

New Vocabulary

included side

GET READY for the Lesson

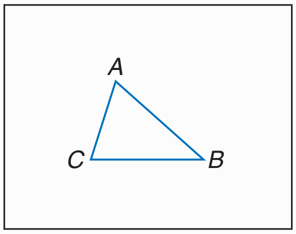
The Bank of China Tower in Hong Kong has triangular trusses for structural support. These trusses form congruent triangles. In this lesson, we will explore two additional methods of proving triangles congruent.

ASA Postulate Suppose you were given the measures of two angles of a triangle and the side between them, the **included side**. Do these measures form a unique triangle?

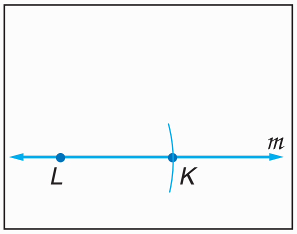
CONSTRUCTION

Congruent Triangles Using Two Angles and Included Side

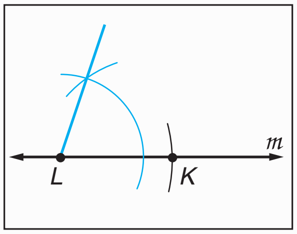
Step 1 Draw a triangle and label its vertices *A*, *B*, and *C*.



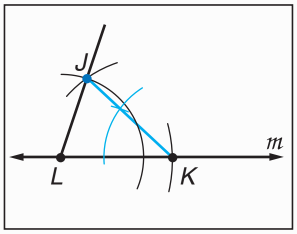
Step 2 Draw any line *m* and select a point *L*. Construct $\overline{LK}$ such that $\overline{LK} \cong \overline{CB}$.



Step 3 Construct an angle congruent to $\angle C$ at *L* using $\overrightarrow{LK}$ as a side of the angle.



Step 4 Construct an angle congruent to $\angle B$ at *K* using $\overrightarrow{LK}$ as a side of the angle. Label the point where the new sides of the angles meet *J*.



Step 5 Cut out $\triangle JKL$ and place it over $\triangle ABC$. How does $\triangle JKL$ compare to $\triangle ABC$?

This construction leads to the Angle-Side-Angle Postulate, written as ASA.

Sylvain Grandadam/Photo Researchers

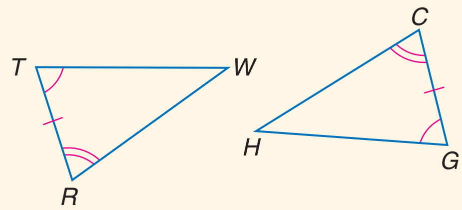
Reading Math

Included Side The *included side* refers to the side that each of the angles share.

POSTULATE 4.3 Angle-Side-Angle Congruence

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

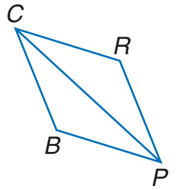
Abbreviation: *ASA*



$\triangle RTW \cong \triangle CGH$

EXAMPLE 1 Use ASA in Proofs

Write a paragraph proof.

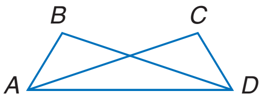


Given: $\overline{CP}$ bisects $\angle BCR$ and $\angle BPR$.

Prove: $\triangle BCP \cong \triangle RCP$

Proof: Since $\overline{CP}$ bisects $\angle BCR$ and $\angle BPR$, $\angle BCP \cong \angle RCP$ and $\angle BPC \cong \angle RPC$. $\overline{CP} \cong \overline{CP}$ by the Reflexive Property. By ASA, $\triangle BCP \cong \triangle RCP$.

CHECK Your Progress



1. **Given:** $\angle CAD \cong \angle BDA$ and $\angle CDA \cong \angle BAD$
   Prove: $\triangle ABD \cong \triangle DCA$

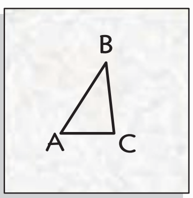
AAS Theorem Suppose you are given the measures of two angles and a nonincluded side. Is this information sufficient to prove two triangles congruent?

GEOMETRY LAB

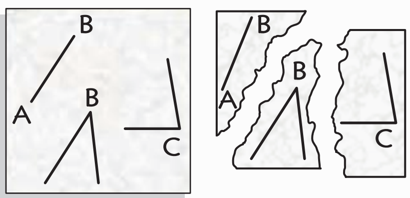
Angle-Angle-Side Congruence

MODEL

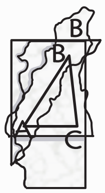
Step 1 Draw a triangle on a piece of patty paper. Label the vertices *A*, *B*, and *C*.



Step 2 Copy $\overline{AB}$, $\angle B$, and $\angle C$ on another piece of patty paper and cut them out.



Step 3 Assemble them to form a triangle in which the side is not the included side of the angles.



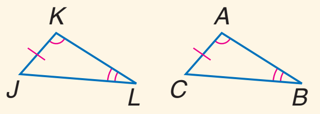
ANALYZE

1. Place the original $\triangle ABC$ over the assembled figure. How do the two triangles compare?
2. **Make a conjecture** about two triangles with two angles and the nonincluded side of one triangle congruent to two angles and the nonincluded side of the other triangle.

This lab leads to the Angle-Angle-Side Theorem, written as AAS.

THEOREM 4.5 **Angle-Angle-Side Congruence**

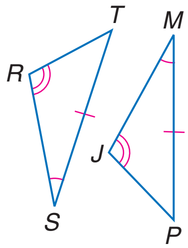
If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.



Abbreviation: *AAS*

Example: $\triangle JKL \cong \triangle CAB$

PROOF Theorem 4.5



Given: $\angle M \cong \angle S$, $\angle J \cong \angle R$, $\overline{MP} \cong \overline{ST}$

Prove: $\triangle JMP \cong \triangle RST$

Proof:

Statements	Reasons
1. $\angle M \cong \angle S$, $\angle J \cong \angle R$, $\overline{MP} \cong \overline{ST}$	**1.** Given
2. $\angle P \cong \angle T$	**2.** Third Angle Theorem
3. $\triangle JMP \cong \triangle RST$	**3.** ASA

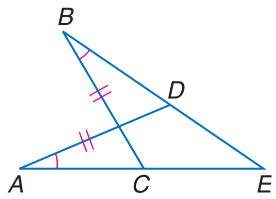
EXAMPLE Use AAS in Proofs

Study Tip

Overlapping Triangles

When triangles overlap, it is a good idea to draw each triangle separately and label the congruent parts.

2 Write a flow proof.



Given: $\angle EAD \cong \angle EBC$

$\overline{AD} \cong \overline{BC}$

Prove: $\overline{AE} \cong \overline{BE}$

Flow Proof:

$\angle EAD \cong \angle EBC$ — Given

$\overline{AD} \cong \overline{BC}$ — Given

$\angle E \cong \angle E$ — Reflexive Property

→ $\triangle ADE \cong \triangle BCE$ — AAS

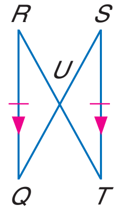
→ $\overline{AE} \cong \overline{BE}$ — CPCTC

CHECK Your Progress

2. Write a flow proof.

Given: $\overline{RQ} \cong \overline{ST}$ and $\overline{RQ} \parallel \overline{ST}$

Prove: $\triangle RUQ \cong \triangle TUS$



You have learned several methods for proving triangle congruence. The Concept Summary lists ways to help you determine which method to use.

CONCEPT SUMMARY

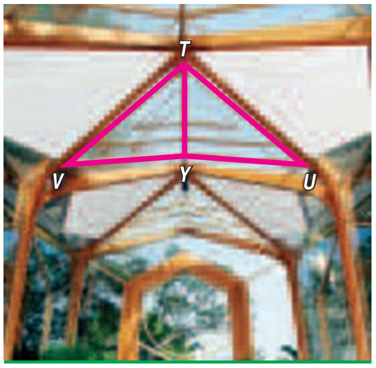
Method	Use when. . .
Definition of Congruent Triangles	All corresponding parts of one triangle are congruent to the corresponding parts of the other triangle.
SSS	The three sides of one triangle are congruent to the three sides of the other triangle.
SAS	Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
ASA	Two angles and the included side of one triangle are congruent to two angles and the included side of the other triangle.
AAS	Two angles and a nonincluded side of one triangle are congruent to two angles and side of the other triangle.

Real-World Career
Architect
About 28% of architects are self-employed. Architects design a variety of buildings including offices, retail spaces, and schools.

Math Online For more information, go to geometryonline.com.

Real-World EXAMPLE Determine if Triangles Are Congruent

3 ARCHITECTURE **This glass chapel was designed by Frank Lloyd Wright's son, Lloyd Wright. Suppose the redwood supports, $\overline{TU}$ and $\overline{TV}$, measure 3 feet, $TY = 1.6$ feet, and $m\angle U$ and $m\angle V$ are 31. Determine whether $\triangle TYU \cong \triangle TYV$. Justify your answer.**

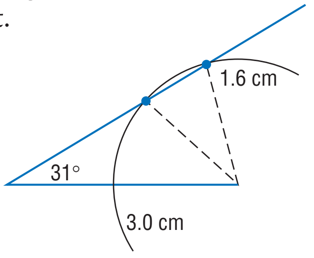


Explore We are given three measurements of each triangle. We need to determine whether the two triangles are congruent.

Plan Since $m\angle U = m\angle V$, $\angle U \cong \angle V$. Likewise, $TU = TV$ so $\overline{TU} \cong \overline{TV}$, and $TY = TY$ so $\overline{TY} \cong \overline{TY}$. Check each possibility using the five methods you know.

Solve We are given information about side-side-angle (SSA). This is not a method to prove two triangles congruent.

Check Use a compass, protractor, and ruler to draw a triangle with the given measurements. For space purposes, use centimeters instead of feet.



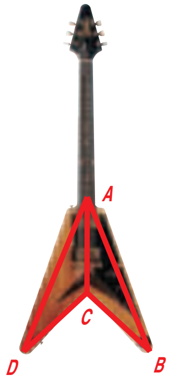
- Draw a segment 3.0 centimeters long.
- At one end, draw an angle of 31°. Extend the line longer than 3.0 centimeters.
- At the other end, draw an arc with a radius of 1.6 centimeters such that it intersects the line.

Notice that there are two possible segments that could determine the triangle. Since the given measurements do not lead to a unique triangle, we cannot show that the triangles are congruent.

(continued on the next page)

CHECK Your Progress

3. A flying V guitar is made up of two triangles. If $AB = 27$ inches, $AD = 27$ inches, $DC = 7$ inches, and $CB = 7$ inches, determine whether $\triangle ADC \cong \triangle ABC$. Explain.



Concepts in Motion
Interactive Lab
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Online Personal Tutor at geometryonline.com

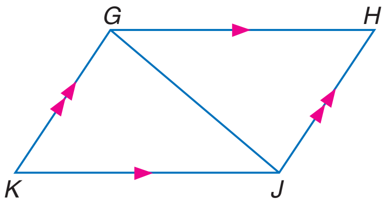
CHECK Your Understanding

Example 1 (p. 235)

PROOF For Exercises 1–4, write the specified type of proof.

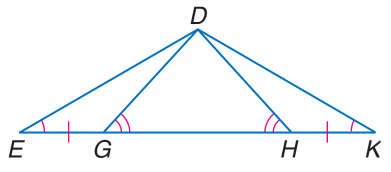
1. flow proof

Given: $\overline{GH} \parallel \overline{KJ}$, $\overline{GK} \parallel \overline{HJ}$
Prove: $\triangle GJK \cong \triangle JGH$



2. paragraph proof

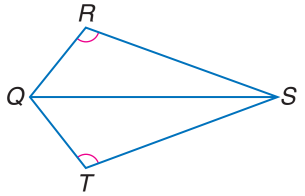
Given: $\angle E \cong \angle K$, $\angle DGH \cong \angle DHG$
$\overline{EG} \cong \overline{KH}$
Prove: $\triangle EGD \cong \triangle KHD$



Example 2 (p. 236)

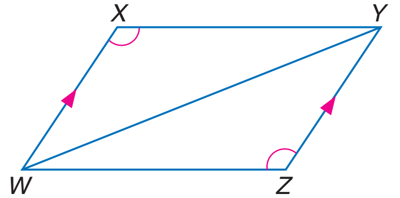
3. paragraph proof

Given: $\overline{QS}$ bisects $\angle RST$; $\angle R \cong \angle T$
Prove: $\triangle QRS \cong \triangle QTS$



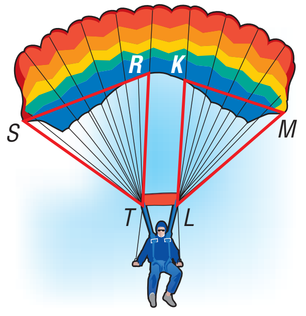
4. flow proof

Given: $\overline{XW} \parallel \overline{YZ}$, $\angle X \cong \angle Z$
Prove: $\triangle WXY \cong \triangle YZW$



Example 3 (p. 237)

5. **PARACHUTES** Suppose $\overline{ST}$ and $\overline{ML}$ each measure seven feet, $\overline{SR}$ and $\overline{MK}$ each measure 5.5 feet, and $m\angle T = m\angle L = 49$. Determine whether $\triangle SRT \cong \triangle MKL$. Justify your answer.



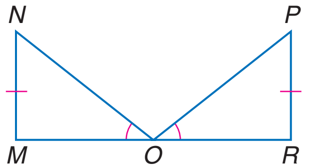
Exercises

HOMEWORK HELP	
For Exercises	See Examples
6, 7	1
8, 9	2
10, 11	3

Write a paragraph proof.

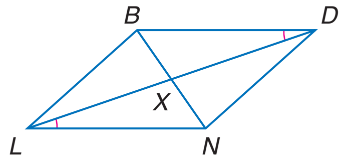
6. Given: $\angle NOM \cong \angle POR$, $\overline{NM} \perp \overline{MR}$, $\overline{PR} \perp \overline{MR}$, $\overline{NM} \cong \overline{PR}$

Prove: $\overline{MO} \cong \overline{OR}$



7. Given: $\overline{DL}$ bisects $\overline{BN}$. $\angle XLN \cong \angle XDB$

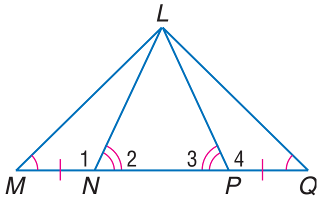
Prove: $\overline{LN} \cong \overline{DB}$



Write a flow proof.

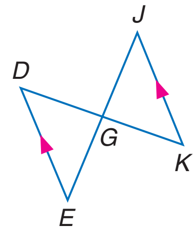
8. Given: $\overline{MN} \cong \overline{PQ}$, $\angle M \cong \angle Q$, $\angle 2 \cong \angle 3$

Prove: $\triangle MLP \cong \triangle QLN$



9. Given: $\overline{DE} \parallel \overline{JK}$, $\overline{DK}$ bisects $\overline{JE}$.

Prove: $\triangle EGD \cong \triangle JGK$



GARDENING **For Exercises 10 and 11, use the following information.**
Beth is planning a garden. She wants the triangular sections $\triangle CFD$ and $\triangle HFG$ to be congruent. F is the midpoint of $\overline{DG}$, and $DG = 16$ feet.

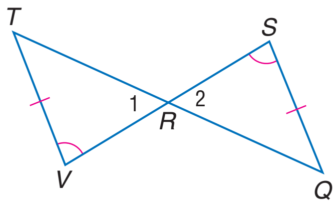
10. Suppose $\overline{CD}$ and $\overline{GH}$ each measure 4 feet and the measure of $\angle CFD$ is 29. Determine whether $\triangle CFD \cong \triangle HFG$. Justify your answer.

11. Suppose F is the midpoint of $\overline{CH}$, and $\overline{CH} \cong \overline{DG}$. Determine whether $\triangle CFD \cong \triangle HFG$. Justify your answer.

Write a flow proof.

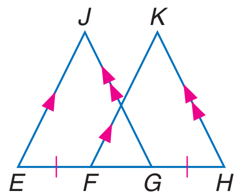
12. Given: $\angle V \cong \angle S$, $\overline{TV} \cong \overline{QS}$

Prove: $\overline{VR} \cong \overline{SR}$



13. Given: $\overline{EJ} \parallel \overline{FK}$, $\overline{JG} \parallel \overline{KH}$, $\overline{EF} \cong \overline{GH}$

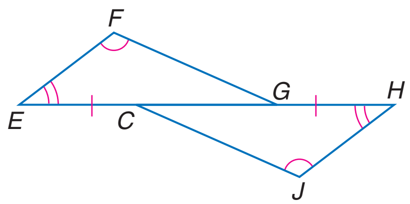
Prove: $\triangle EJG \cong \triangle FKH$



Write a paragraph proof.

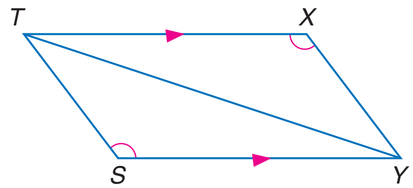
14. Given: $\angle F \cong \angle J$, $\angle E \cong \angle H$, $\overline{EC} \cong \overline{GH}$

Prove: $\overline{EF} \cong \overline{HJ}$



15. Given: $\overline{TX} \parallel \overline{SY}$, $\angle TXY \cong \angle TSY$

Prove: $\triangle TSY \cong \triangle YXT$



Real-World Link

The largest kite ever flown was 210 feet long and 72 feet wide.

Source: *Guinness Book of World Records*

EXTRA PRACTICE

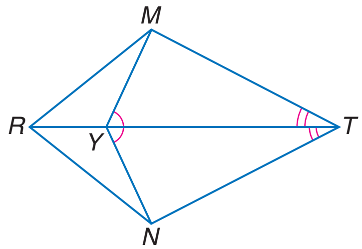
See pages 808, 831.

Math Online

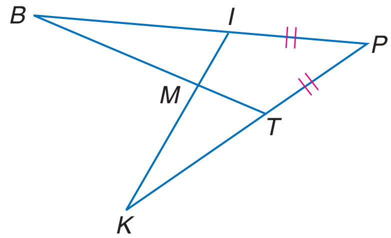
Self-Check Quiz at geometryonline.com

PROOF Write a two-column proof.

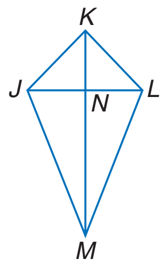
16. Given: $\angle MYT \cong \angle NYT$, $\angle MTY \cong \angle NTY$
Prove: $\triangle RYM \cong \triangle RYN$



17. Given: $\triangle BMI \cong \triangle KMT$, $\overline{IP} \cong \overline{PT}$
Prove: $\triangle IPK \cong \triangle TPB$



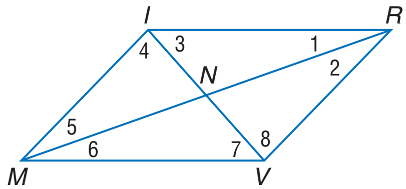
KITES **For Exercises 18 and 19, use the following information.**
Austin is making a kite. Suppose JL is two feet, JM is 2.7 feet, and the measure of $\angle NJM$ is 68.



18. If N is the midpoint of $\overline{JL}$ and $\overline{KM} \perp \overline{JL}$, determine whether $\triangle JKN \cong \triangle LKN$. Justify your answer.

19. If $\overline{JM} \cong \overline{LM}$ and $\angle NJM \cong \angle NLM$, determine whether $\triangle JNM \cong \triangle LNM$. Justify your answer.

Complete each congruence statement and the postulate or theorem that applies.



20. If $\overline{IM} \cong \overline{RV}$ and $\angle 2 \cong \angle 5$, then $\triangle INM \cong \triangle$ __?__ by __?__.

21. If $\overline{IR} \parallel \overline{MV}$ and $\overline{IR} \cong \overline{MV}$, then $\triangle IRN \cong \triangle$ __?__ by __?__.

H.O.T. Problems

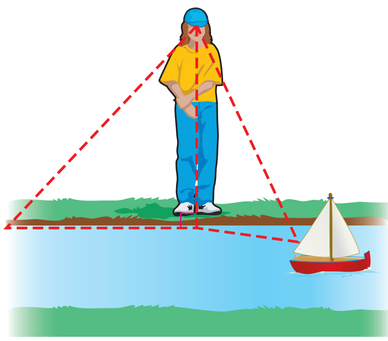
22. **Which One Doesn't Belong?** Identify the term that does not belong with the others. Explain your reasoning.

ASA	SSS	SSA	AAS

23. **REASONING** Find a counterexample to show why AAA (Angle-Angle-Angle) cannot be used to prove congruence in triangles.

24. **OPEN ENDED** Draw and label two triangles that could be proved congruent by SAS.

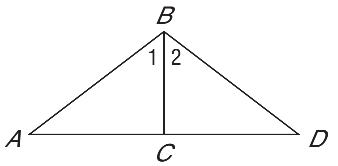
25. **CHALLENGE** Neva wants to estimate the distance between herself and a toy boat. She adjusts the visor of her cap so that it is in line with her line of sight to the toy boat. She keeps her neck stiff and turns her body to establish a line of sight to a point on the ground. Then she paces out the distance to the new point. Is the distance from the toy boat the same as the distance she just paced out? Explain your reasoning.



26. ***Writing in Math*** Use the information about construction on page 234 to explain how congruent triangles are used in construction. Include why it is important to use congruent triangles for support.

STANDARDIZED TEST PRACTICE

27. Given: $\overline{BC}$ is perpendicular to $\overline{AD}$; $\angle 1 \cong \angle 2$.



Which theorem or postulate could be used to prove $\triangle ABC \cong \triangle DBC$?

A AAS
B ASA
C SAS
D SSS

28. REVIEW Which expression can be used to find the values of $s(n)$ in the table?

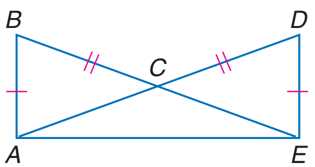
n	−8	−4	−1	0	1
$s(n)$	1.00	2.00	2.75	3.00	3.25

F $-2n + 3$
G $-n + 7$
H $\frac{1}{4}n + 3$
J $\frac{1}{2}n + 5$

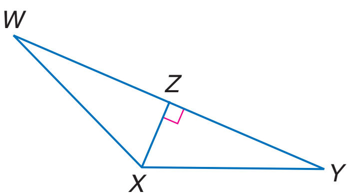
Spiral Review

Write a flow proof. (Lesson 4-4)

29. Given: $\overline{BA} \cong \overline{DE}$, $\overline{DA} \cong \overline{BE}$
Prove: $\triangle BEA \cong \triangle DAE$

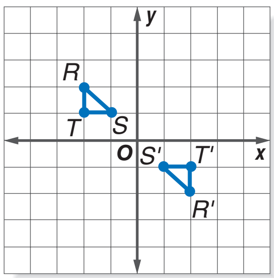


30. Given: $\overline{XZ} \perp \overline{WY}$, $\overline{XZ}$ bisects $\overline{WY}$.
Prove: $\triangle WZX \cong \triangle YZX$

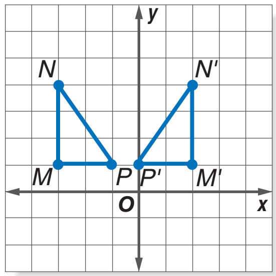


Verify congruence and name the congruence transformation. (Lesson 4-3)

31. $\triangle RTS \cong \triangle R'T'S'$



32. $\triangle MNP \cong \triangle M'N'P'$



Write each statement in if-then form. (Lesson 2-3)

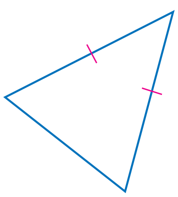
33. Happy people rarely correct their faults.

34. A champion is afraid of losing.

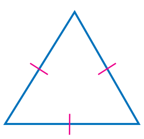
GET READY for the Next Lesson

PREREQUISITE SKILL **Classify each triangle according to its sides.** (Lesson 4-1)

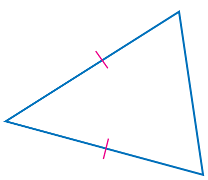
35.



36.



37.



EXTEND 4-5

Geometry Lab

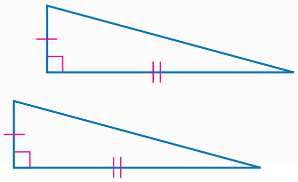
Congruence in Right Triangles

In Lessons 4-4 and 4-5, you learned theorems and postulates to prove triangles congruent. Do these theorems and postulates apply to right triangles?

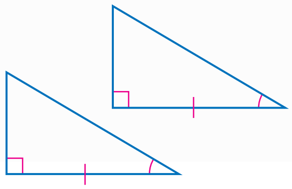
ACTIVITY 1 Triangle Congruence

Study each pair of right triangles.

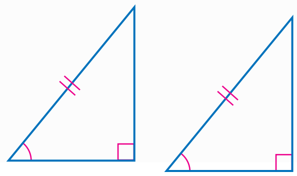
a.



b.



c.



ANALYZE THE RESULTS

1. Is each pair of triangles congruent? If so, which congruence theorem or postulate applies?

2. Rewrite the congruence rules from Exercise 1 using *leg*, (L), or *hypotenuse*, (H), to replace *side*. Omit the A for any right angle since we know that all right triangles contain a right angle and all right angles are congruent.

3. **MAKE A CONJECTURE** If you know that the corresponding legs of two right triangles are congruent, what other information do you need to declare the triangles congruent? Explain.

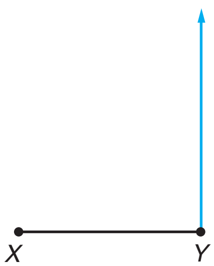
In Lesson 4-5, you learned that SSA is not a valid test for determining triangle congruence. Can SSA be used to prove right triangles congruent?

ACTIVITY 2 SSA and Right Triangles

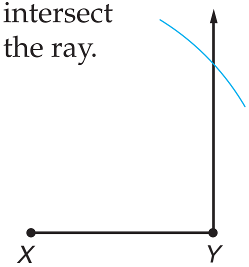
How many right triangles exist that have a hypotenuse of 10 centimeters and a leg of 7 centimeters?

Step 1
Draw $\overline{XY}$ so that $XY = 7$ centimeters.

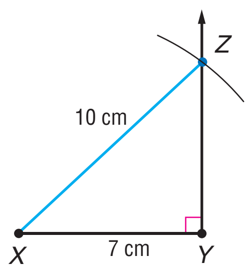
Step 2
Use a protractor to draw a ray from Y that is perpendicular to $\overline{XY}$.



Step 3
Open your compass to a width of 10 centimeters. Place the point at X and draw a long arc to intersect the ray.



Step 4
Label the intersection Z and draw $\overline{XZ}$ to complete $\triangle XYZ$.

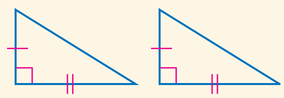
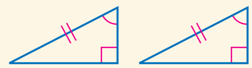
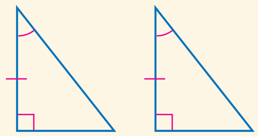
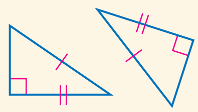


ANALYZE THE RESULTS

4. Does the model yield a unique triangle?
5. Can you use the lengths of the hypotenuse and a leg to show right triangles are congruent?
6. **Make a conjecture** about the case of SSA that exists for right triangles.

The two activities provide evidence for four ways to prove right triangles congruent.

KEY CONCEPT — *Right Triangle Congruence*

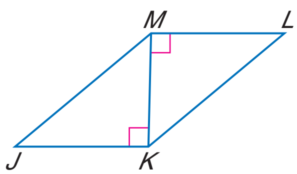
Theorems	Abbreviation	Example
4.6 Leg-Leg Congruence If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.	**LL**	
4.7 Hypotenuse-Angle Congruence If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent.	**HA**	
4.8 Leg-Angle Congruence If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.	**LA**	
Postulate		
4.4 Hypotenuse-Leg Congruence If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.	**HL**	

EXERCISES

PROOF Write a paragraph proof of each theorem.

7. Theorem 4.6
8. Theorem 4.7
9. Theorem 4.8 (*Hint*: There are two possible cases.)

Use the figure to write a two-column proof.



10. **Given:** $\overline{ML} \perp \overline{MK}$, $\overline{JK} \perp \overline{KM}$
$\angle J \cong \angle L$
Prove: $\overline{JM} \cong \overline{KL}$

11. **Given:** $\overline{JK} \perp \overline{KM}$, $\overline{JM} \cong \overline{KL}$
$\overline{ML} \parallel \overline{JK}$
Prove: $\overline{ML} \cong \overline{JK}$

4-6 Isosceles Triangles

Main Ideas

- Use properties of isosceles triangles.
- Use properties of equilateral triangles.

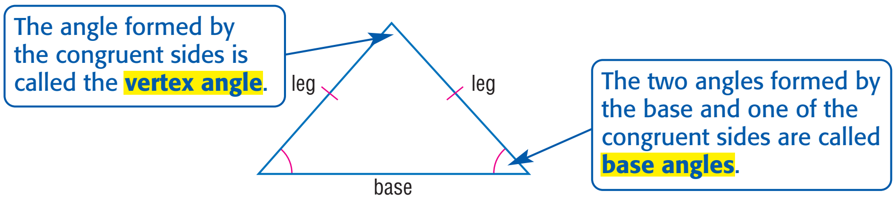
New Vocabulary

vertex angle
base angles

GET READY for the Lesson

The art of Lois Mailou Jones, a twentieth-century artist, includes paintings and textile design, as well as book illustration. Notice the isosceles triangles in this painting, *Damballah*.

Properties of Isosceles Triangles In Lesson 4-1, you learned that isosceles triangles have two congruent sides. Like the right triangle, the parts of an isosceles triangle have special names.

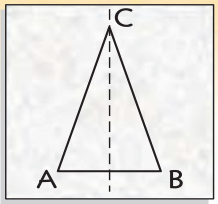


GEOMETRY LAB

Isosceles Triangles

MODEL

- Draw an acute triangle on patty paper with $\overline{AC} \cong \overline{BC}$.
- Fold the triangle through C so that A and B coincide.



ANALYZE

1. What do you observe about $\angle A$ and $\angle B$?

2. Draw an obtuse isosceles triangle. Compare the base angles.

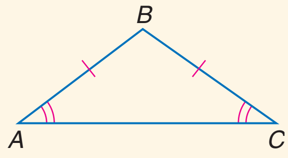
3. Draw a right isosceles triangle. Compare the base angles.

The results of the Geometry Lab suggest Theorem 4.9.

THEOREM 4.9 — Isosceles Triangle

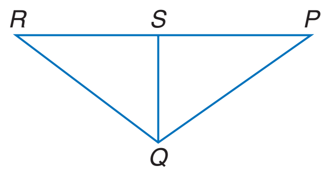
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Example: If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.



EXAMPLE 1 Proof of Theorem

Write a two-column proof of the Isosceles Triangle Theorem.



Given: $\angle PQR$, $\overline{PQ} \cong \overline{RQ}$

Prove: $\angle P \cong \angle R$

Proof:

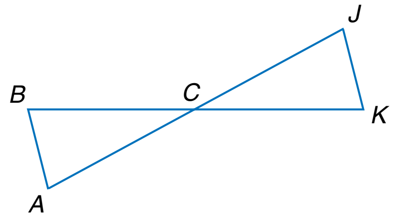
Statements	Reasons
1. Let S be the midpoint of $\overline{PR}$.	**1.** Every segment has exactly one midpoint.
2. Draw an auxiliary segment $\overline{QS}$	**2.** Two points determine a line.
3. $\overline{PS} \cong \overline{RS}$	**3.** Midpoint Theorem
4. $\overline{QS} \cong \overline{QS}$	**4.** Congruence of segments is reflexive.
5. $\overline{PQ} \cong \overline{RQ}$	**5.** Given
6. $\triangle PQS \cong \triangle RQS$	**6.** SSS
7. $\angle P \cong \angle R$	**7.** CPCTC

CHECK Your Progress

1. Write a two-column proof.

Given: $\overline{CA} \cong \overline{BC}$; $\overline{KC} \cong \overline{CJ}$
C is the midpoint of $\overline{BK}$.

Prove: $\triangle ABC \cong \triangle JKC$



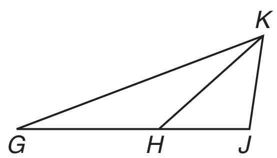
Test-Taking Tip

Diagrams Label the diagram with the given information. Use your drawing to plan the next step in solving the problem.

STANDARDIZED TEST EXAMPLE 2 Find a Missing Angle Measure

If $\overline{GH} \cong \overline{HK}$, $\overline{HJ} \cong \overline{JK}$, and $m\angle GJK = 100$, what is $m\angle HGK$?

A 10 **B** 15 **C** 20 **D** 25



Read the Test Item

$\triangle GHK$ is isosceles with base $\overline{GK}$. Likewise, $\triangle HJK$ is isosceles with base $\overline{HK}$.

(continued on the next page)

Solve the Test Item

Step 1 The base angles of $\triangle HJK$ are congruent. Let $x = m\angle KHJ = m\angle HKJ$.

$m\angle KHJ + m\angle HKJ + m\angle HJK = 180$ Angle Sum Theorem

$x + x + 100 = 180$ Substitution

$2x + 100 = 180$ Add.

$2x = 80$ Subtract 100 from each side.

$x = 40$ So, $m\angle KHJ = m\angle HKJ = 40$.

Step 2 $\angle GHK$ and $\angle KHJ$ form a linear pair. Solve for $m\angle GHK$.

$m\angle KHJ + m\angle GHK = 180$ Linear pairs are supplementary.

$40 + m\angle GHK = 180$ Substitution

$m\angle GHK = 140$ Subtract 40 from each side.

Step 3 The base angles of $\triangle GHK$ are congruent. Let y represent $m\angle HGK$ and $m\angle GKH$.

$m\angle GHK + m\angle HGK + m\angle GKH = 180$ Angle Sum Theorem

$140 + y + y = 180$ Substitution

$140 + 2y = 180$ Add.

$2y = 40$ Subtract 140 from each side.

$y = 20$ Divide each side by 2.

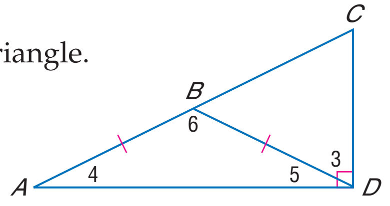
The measure of $\angle HGK$ is 20. Choice C is correct.

CHECK Your Progress

2. $\triangle ABD$ is isosceles, and $\triangle ACD$ is a right triangle. If $m\angle 6 = 136$, what is $m\angle 3$?

F 21 **H** 68

G 37 **J** 113



Online Personal Tutor at geometryonline.com

Study Tip

Look Back
You can review **converses** in Lesson 2-3.

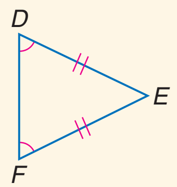
The converse of the Isosceles Triangle Theorem is also true.

THEOREM 4.10

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Abbreviation: *Conv. of Isos. △ Th.*

Example: If $\angle D \cong \angle F$, then $\overline{DE} \cong \overline{FE}$.



You will prove Theorem 4.10 in Exercise 13.

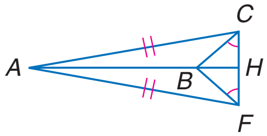
Cross-Curricular Project

Math Online You can use properties of triangles to prove Thales of Miletus' important geometric ideas. Visit geometryonline.com to continue work on your project.

EXAMPLE 3 Congruent Segments and Angles

a. Name two congruent angles.

$\angle AFC$ is opposite $\overline{AC}$ and $\angle ACF$ is opposite $\overline{AF}$, so $\angle AFC \cong \angle ACF$.



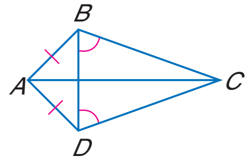
b. Name two congruent segments.

By the converse of the Isosceles Triangle Theorem, the sides opposite congruent angles are congruent. So, $\overline{BC} \cong \overline{BF}$.

CHECK Your Progress

3A. Name two congruent angles.

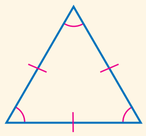
3B. Name two congruent segments.



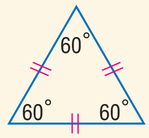
Properties of Equilateral Triangles Recall that an equilateral triangle has three congruent sides. The Isosceles Triangle Theorem leads to two corollaries about the angles of an equilateral triangle.

COROLLARIES

4.3 A triangle is equilateral if and only if it is equiangular.



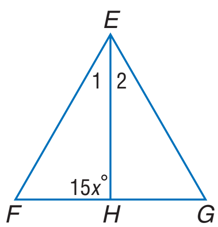
4.4 Each angle of an equilateral triangle measures 60°.



You will prove Corollaries 4.3 and 4.4 in Exercises 11 and 12.

EXAMPLE 4 Use Properties of Equilateral Triangles

$\triangle EFG$ is equilateral, and $\overline{EH}$ bisects $\angle E$.



a. Find $m\angle 1$ and $m\angle 2$.

Each angle of an equilateral triangle measures 60°. So, $m\angle 1 + m\angle 2 = 60$. Since the angle was bisected, $m\angle 1 = m\angle 2$. Thus, $m\angle 1 = m\angle 2 = 30$.

b. ALGEBRA Find x.

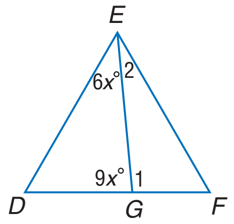
$m\angle EFH + m\angle 1 + m\angle EHF = 180$	Angle Sum Theorem
$60 + 30 + 15x = 180$	$m\angle EFH = 60$, $m\angle 1 = 30$, $m\angle EHF = 15x$
$90 + 15x = 180$	Add.
$15x = 90$	Subtract 90 from each side.
$x = 6$	Divide each side by 15.

CHECK Your Progress

△*DEF* is equilateral.

4A. Find x.

4B. Find $m\angle 1$ and $m\angle 2$.



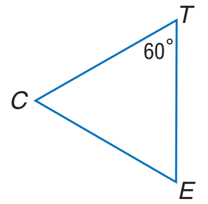
CHECK Your Understanding

Examples 1, 4 (pp. 245, 247)

PROOF **Write a two-column proof.**

1. **Given:** $\triangle CTE$ is isosceles with vertex $\angle C$.
$m\angle T = 60$

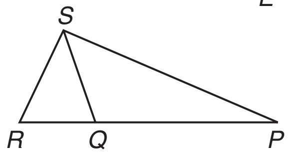
Prove: $\triangle CTE$ is equilateral.



Example 2 (p. 246)

2. **STANDARDIZED TEST PRACTICE** If $\overline{PQ} \cong \overline{QS}$, $\overline{QR} \cong \overline{RS}$, and $m\angle PRS = 72$, what is $m\angle QPS$?

A 27 **B** 54 **C** 63 **D** 72

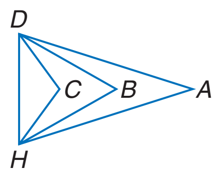


Example 3 (p. 247)

Refer to the figure.

3. If $\overline{AD} \cong \overline{AH}$, name two congruent angles.

4. If $\angle BDH \cong \angle BHD$, name two congruent segments.



Exercises

HOMEWORK HELP	
For Exercises	See Examples
5–10	3
11–13	1
14, 15	4
37, 38	2

Refer to the figure for Exercises 5–10.

5. If $\overline{LT} \cong \overline{LR}$, name two congruent angles.

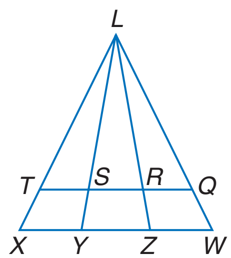
6. If $\overline{LX} \cong \overline{LW}$, name two congruent angles.

7. If $\overline{SL} \cong \overline{QL}$, name two congruent angles.

8. If $\angle LXY \cong \angle LYX$, name two congruent segments.

9. If $\angle LSR \cong \angle LRS$, name two congruent segments.

10. If $\angle LYW \cong \angle LWY$, name two congruent segments.



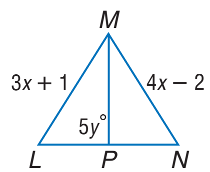
PROOF **Write a two-column proof.**

11. Corollary 4.3 **12.** Corollary 4.4 **13.** Theorem 4.10

Triangle *LMN* is equilateral, and $\overline{MP}$ bisects $\overline{LN}$.

14. Find x and y.

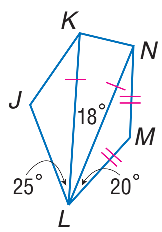
15. Find the measure of each side.



$\triangle KLN$ and $\triangle LMN$ are isosceles and $m\angle JKN = 130$. Find each measure.

16. $m\angle LNM$ **17.** $m\angle M$

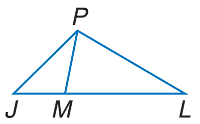
18. $m\angle LKN$ **19.** $m\angle J$



In the figure, $\overline{JM} \cong \overline{PM}$ and $\overline{ML} \cong \overline{PL}$.

20. If $m\angle PLJ = 34$, find $m\angle JPM$.

21. If $m\angle PLJ = 58$, find $m\angle PJL$.



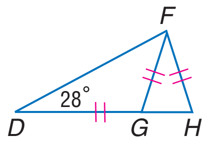
$\triangle DFG$ and $\triangle FGH$ are isosceles, $m\angle FDH = 28$, and $\overline{DG} \cong \overline{FG} \cong \overline{FH}$. Find each measure.

22. $m\angle DFG$

23. $m\angle DGF$

24. $m\angle FGH$

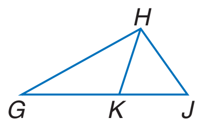
25. $m\angle GFH$

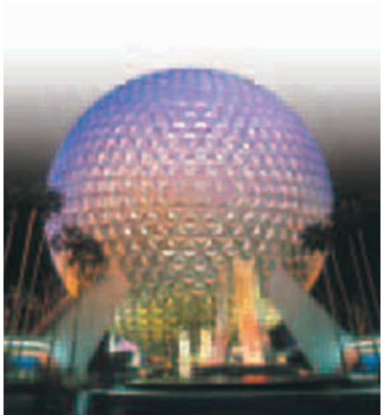


In the figure, $\overline{GK} \cong \overline{GH}$ and $\overline{HK} \cong \overline{KJ}$.

26. If $m\angle HGK = 28$, find $m\angle HJK$.

27. If $m\angle HGK = 42$, find $m\angle HKJ$.



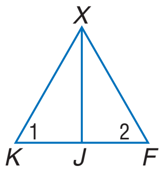


Real-World Link
Spaceship Earth is a completely spherical geodesic dome that is covered with 11,324 triangular aluminum and plastic alloy panels.

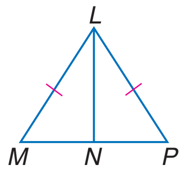
Source: disneyworld.disney.go.com

PROOF **Write a two-column proof for each of the following.**

28. **Given:** $\triangle XKF$ is equilateral.
$\overline{XJ}$ bisects $\angle X$.
Prove: J is the midpoint of $\overline{KF}$.



29. **Given:** $\triangle MLP$ is isosceles.
N is the midpoint of $\overline{MP}$.
Prove: $\overline{LN} \perp \overline{MP}$

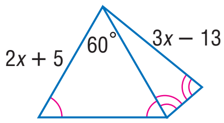


30. **DESIGN** The exterior of Spaceship Earth at Epcot Center in Orlando, Florida, is made up of triangles. Describe the minimum requirement to show that these triangles are equilateral.

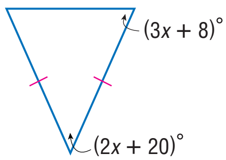
ALGEBRA **Find x.**

EXTRA PRACTICE
See pages 808, 831.
Math Online
Self-Check Quiz at
geometryonline.com

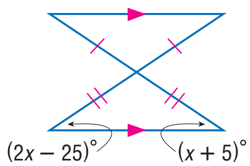
31.



32.



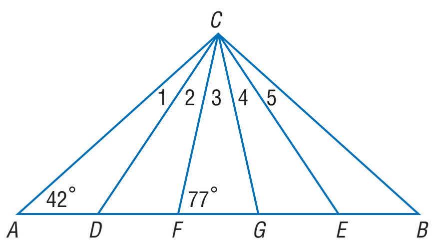
33.



H.O.T. Problems

34. **OPEN ENDED** Describe a method to construct an equilateral triangle.

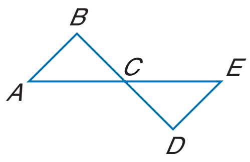
35. **CHALLENGE** In the figure, $\triangle ABC$ is isosceles, $\triangle DCE$ is equilateral, and $\triangle FCG$ is isosceles. Find the measures of the five numbered angles at vertex C.



36. ***Writing in Math*** Explain how triangles can be used in art. Describe at least three other geometric shapes and how they are used in art. Include an interpretation of how and why isosceles triangles are used in the painting shown at the beginning of the lesson.

STANDARDIZED TEST PRACTICE

37. In the figure below, $\overline{AE}$ and $\overline{BD}$ bisect each other at point C.



Which additional piece of information would be enough to prove that $\overline{CD} \cong \overline{DE}$?

A $\angle A \cong \angle C$

B $\angle B \cong \angle D$

C $\angle ACB \cong \angle EDC$

D $\angle A \cong \angle B$

38. **REVIEW** What quantity should be added to both sides of this equation to complete the square?

$$x^2 - 10x = 3$$

F -25

G -5

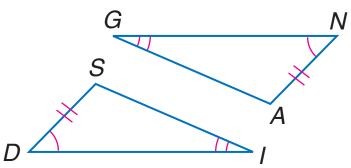
H 5

J 25

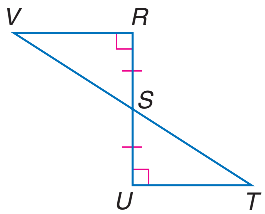
Spiral Review

PROOF **Write a paragraph proof.** (Lesson 4-5)

39. Given: $\angle N \cong \angle D$, $\angle G \cong \angle I$, $\overline{AN} \cong \overline{SD}$
Prove: $\triangle ANG \cong \triangle SDI$



40. Given: $\overline{VR} \perp \overline{RS}$, $\overline{UT} \perp \overline{SU}$, $\overline{RS} \cong \overline{US}$
Prove: $\triangle VRS \cong \triangle TUS$



Determine whether $\triangle QRS \cong \triangle EGH$ given the coordinates of the vertices. Explain. (Lesson 4-4)

41. $Q(-3, 1)$, $R(1, 2)$, $S(-1, -2)$, $E(6, -2)$, $G(2, -3)$, $H(4, 1)$

42. $Q(1, -5)$, $R(5, 1)$, $S(4, 0)$, $E(-4, -3)$, $G(-1, 2)$, $H(2, 1)$

43. **LANDSCAPING** Lucas is drawing plans for a client's backyard on graph paper. The client wants two perpendicular pathways to cross at the center of her backyard. If the center of the backyard is set at $(0, 0)$ and the first path goes from one corner of the backyard at $(-6, 12)$ to the other corner at $(6, -12)$, at what coordinates will the second path begin and end? (Lesson 3-3)

Construct a truth table for each compound statement. (Lesson 2-2)

44. a and b **45.** $\sim p$ or $\sim q$ **46.** k and $\sim m$ **47.** $\sim y$ or z

GET READY for the Next Lesson

PREREQUISITE SKILL **Find the coordinates of the midpoint of the segment with endpoints that are given.** (Lesson 1-3)

48. $A(2, 15)$, $B(7, 9)$ **49.** $C(-4, 6)$, $D(2, -12)$ **50.** $E(3, 2.5)$, $F(7.5, 4)$

4-7 Triangles and Coordinate Proof

Main Ideas

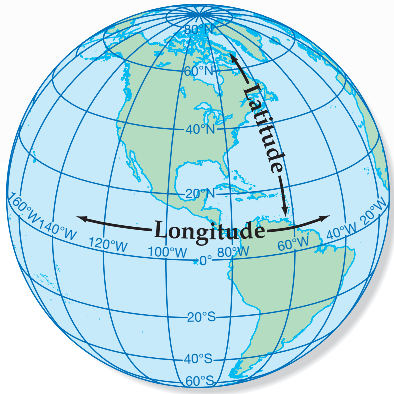
- Position and label triangles for use in coordinate proofs.
- Write coordinate proofs.

New Vocabulary

coordinate proof

GET READY for the Lesson

Navigators developed a series of circles to create a coordinate grid that allows them to determine where they are on Earth. Similar to points in coordinate geometry, locations on this grid are given two values: an east/west value (longitude) and a north/south value (latitude).



Position and Label Triangles Same as working with longitude and latitude, knowing the coordinates of points on a figure allows you to draw conclusions about it. **Coordinate proof** uses figures in the coordinate plane and algebra to prove geometric concepts. The first step in a coordinate proof is placing the figure on the coordinate plane.

Study Tip

Placement of Figures

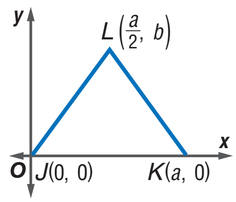
The guidelines apply to any polygon placed on the coordinate plane.

KEY CONCEPT — *Placing Figures on the Coordinate Plane*

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of a polygon on an axis.
3. Keep the figure within the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

EXAMPLE — Position and Label a Triangle

1 **Position and label isosceles triangle *JKL* on a coordinate plane so that base $\overline{JK}$ is *a* units long.**



- Use the origin as vertex *J* of the triangle.
- Place the base of the triangle along the positive *x*-axis.
- Position the triangle in the first quadrant.
- Since *K* is on the *x*-axis, its *y*-coordinate is 0. Its *x*-coordinate is *a* because the base is *a* units long.
- $\triangle JKL$ is isosceles, so the *x*-coordinate of *L* is halfway between 0 and *a* or $\frac{a}{2}$. We cannot write the *y*-coordinate in terms of *a*, so call it *b*.

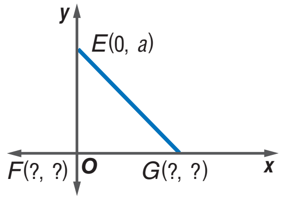
CHECK Your Progress

1. Position and label right triangle *HIJ* with legs $\overline{HI}$ and $\overline{IJ}$ on a coordinate plane so that $\overline{HI}$ is *a* units long and $\overline{IJ}$ is *b* units long.

Concepts in Motion
Animation
geometryonline.com

EXAMPLE 2 Find the Missing Coordinates

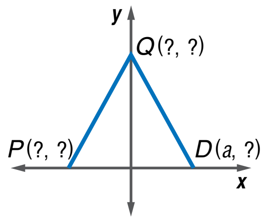
Name the missing coordinates of isosceles right triangle *EFG*.



Vertex F is positioned at the origin; its coordinates are (0, 0). Vertex E is on the y-axis, and vertex G is on the x-axis. So $\angle EFG$ is a right angle. Since $\triangle EFG$ is isosceles, $\overline{EF} \cong \overline{GF}$. EF is a units and GF must be the same. So, the coordinates of G are $(a, 0)$.

CHECK Your Progress

2. Name the missing coordinates of isosceles triangle *PDQ*.



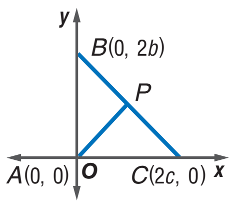
Write Proofs After a figure is placed on the coordinate plane and labeled, we can coordinate proof to verify properties and to prove theorems.

> **Study Tip**
>
> **Vertex Angle**
>
> Remember from the Geometry Lab on page 244 that an isosceles triangle can be folded in half. Thus, the x-coordinate of the vertex angle is the same as the x-coordinate of the midpoint of the base.

EXAMPLE 3 Coordinate Proof

Write a coordinate proof to prove that the measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

Place the right angle at the origin and label it A. Use coordinates that are multiples of 2 because the Midpoint Formula takes half the sum of the coordinates.



Given: right $\triangle ABC$ with right $\angle BAC$
P is the midpoint of $\overline{BC}$.

Prove: $AP = \frac{1}{2}BC$

Proof:

By the Midpoint Formula, the coordinates of P are $\left(\frac{0 + 2c}{2}, \frac{2b + 0}{2}\right)$ or (c, b). Use the Distance Formula to find AP and BC.

$$AP = \sqrt{(c - 0)^2 + (b - 0)^2} \qquad BC = \sqrt{(2c - 0)^2 + (0 - 2b)^2}$$

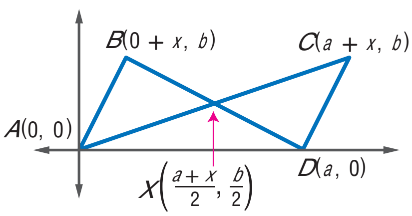
$$= \sqrt{c^2 + b^2} \qquad BC = \sqrt{4c^2 + 4b^2} \text{ or } 2\sqrt{c^2 + b^2}$$

$$\frac{1}{2}BC = \sqrt{c^2 + b^2}$$

Therefore, $AP = \frac{1}{2}BC$.

CHECK Your Progress

3. Use a coordinate proof to show that the triangles shown are congruent.



Personal Tutor at geometryonline.com

Real-World EXAMPLE Classify Triangles

4 **ARROWHEADS** **Write a coordinate proof to prove that this arrowhead is shaped like an isosceles triangle. The arrowhead is 3 inches long and 1.5 inches wide.**

The first step is to label the coordinates of each vertex. Q is at the origin, and T is at (1.5, 0). The y-coordinate of R is 3. The x-coordinate is halfway between 0 and 1.5 or 0.75. So, the coordinates of R are (0.75, 3).

If the legs of the triangle are the same length, it is isosceles. Use the Distance Formula to find QR and RT.

$$QR = \sqrt{(0.75 - 0)^2 + (3 - 0)^2}$$
$$= \sqrt{0.5625 + 9} \text{ or } \sqrt{9.5625}$$

$$RT = \sqrt{(1.5 - 0.75)^2 + (0 - 3)^2}$$
$$= \sqrt{0.5625 + 9} \text{ or } \sqrt{9.5625}$$

Since each leg is the same length, $\triangle QRT$ is isosceles. The arrowhead is shaped like an isosceles triangle.

CHECK Your Progress

4. Use coordinate geometry to classify a triangle with vertices located at the following coordinates $A(0, 0)$, $B(0, 6)$, and $C(3, 3)$.

CHECK Your Understanding

Example 1 (p. 251)

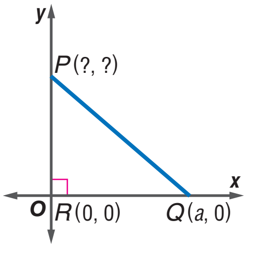
Position and label each triangle on the coordinate plane.

1. isosceles $\triangle FGH$ with base $\overline{FH}$ that is $2b$ units long
2. equilateral $\triangle CDE$ with sides a units long

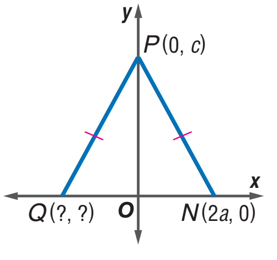
Example 2 (p. 252)

Name the missing coordinates of each triangle.

3.



4.

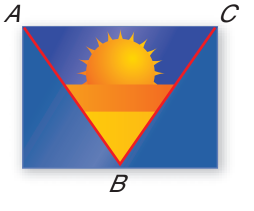


Example 3 (p. 252)

5. Write a coordinate proof for the following statement. *The midpoint of the hypotenuse of a right triangle is equidistant from each of the vertices.*

Example 4 (p. 253)

6. **FLAGS** Write a coordinate proof to prove that the large triangle in the center of the flag is isosceles. The dimensions of the flag are 4 feet by 6 feet, and point B of the triangle bisects the bottom of the flag.



Exercises

HOMEWORK HELP	
For Exercises	**See Examples**
7–12	1
13–18	2
19–22	3
23–26	4

Position and label each triangle on the coordinate plane.

7. isosceles $\triangle QRT$ with base $\overline{QR}$ that is b units long

8. equilateral $\triangle MNP$ with sides $2a$ units long

9. isosceles right $\triangle JML$ with hypotenuse $\overline{JM}$ and legs c units long

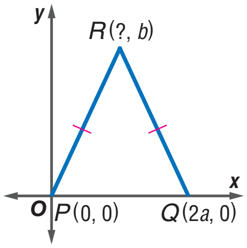
10. equilateral $\triangle WXZ$ with sides $\frac{1}{2}b$ units long

11. isosceles $\triangle PWY$ with base $\overline{PW}(a + b)$ units long

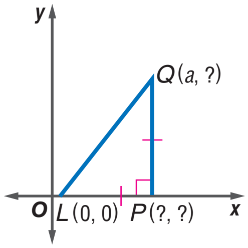
12. right $\triangle XYZ$ with hypotenuse $\overline{XZ}$, the length of $\overline{ZY}$ is twice XY, and $\overline{XY}$ is b units long

Name the missing coordinates of each triangle.

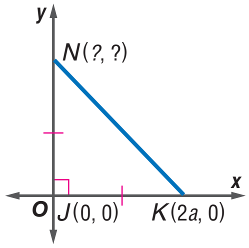
13.



14.



15.



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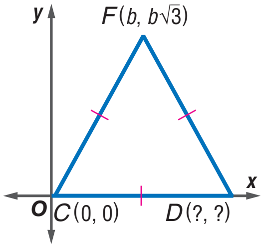

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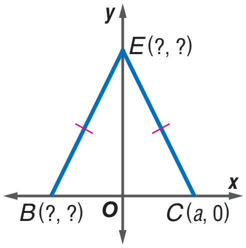
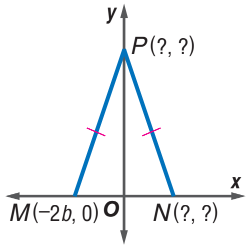


18.



Write a coordinate proof for each statement.

19. The segments joining the vertices of the base angles to the midpoints of the legs of an isosceles triangle are congruent.

20. The three segments joining the midpoints of the sides of an isosceles triangle form another isosceles triangle.

21. If a line segment joins the midpoints of two sides of a triangle, then it is parallel to the third side.

22. If a line segment joins the midpoints of two sides of a triangle, then its length is equal to one-half the length of the third side.

NAVIGATION **For Exercises 23 and 24, use the following information.**
A motor boat is located 800 yards from the port. There is a ship 800 yards to the east and another ship 800 yards to the north of the motor boat.

23. Write a coordinate proof to prove that the port, motor boat, and the ship to the north form an isosceles right triangle.

24. Write a coordinate proof to prove that the distance between the two ships is the same as the distance from the port to the northern ship.

Real-World Link

The Appalachian Trail is a 2175-mile hiking trail that stretches from Maine to Georgia. Up to 4 million people visit the trail per year.

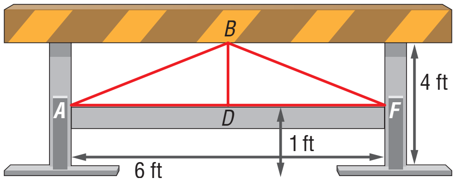
Source: appalachiantrail.org

HIKING **For Exercises 25 and 26, use the following information.**
Tami and Juan are hiking. Tami hikes 300 feet east of the camp and then hikes 500 feet north. Juan hikes 500 feet west of the camp and then 300 feet north.

25. Prove that Juan, Tami, and the camp form a right triangle.

26. Find the distance between Tami and Juan.

27. **STEEPLECHASE** Write a coordinate proof to prove that the triangles ABD and FBD are congruent. Suppose the hurdle is 6 feet wide and 4 feet tall, with the lower bar 1 foot off the ground.



EXTRA PRACTICE
See pages 809, 831.
Math Online
Self-Check Quiz at
geometryonline.com

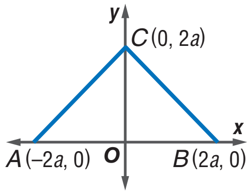
Find the coordinates of point C so $\triangle ABC$ is the indicated type of triangle. Point A has coordinates (0, 0) and B has coordinates (a, b).

28. right triangle
29. isosceles triangle
30. scalene triangle

H.O.T. Problems

31. **OPEN ENDED** Draw a scalene right triangle on the coordinate plane so it simplifies a coordinate proof. Label the coordinates of each vertex. Explain why you placed the triangle this way.

32. **CHALLENGE** Classify $\triangle ABC$ by its angles and its sides. Explain.



33. ***Writing in Math*** Use the information about the coordinate plane given on page 251 to explain how the coordinate plane can be used in proofs. Include a list of the different types of proof and a theorem from the chapter that could be proved using a coordinate proof.

STANDARDIZED TEST PRACTICE

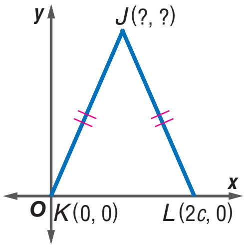
34. What are the coordinates of point J in the triangle below?

A $\left(\frac{c}{2}, c\right)$

B (c, b)

C $\left(\frac{b}{2}, c\right)$

D $\left(\frac{b}{2}, \frac{c}{2}\right)$



35. **REVIEW** What is the x-coordinate of the solution to the system of equations shown below?

$$\begin{cases} 2x - 3y = 3 \\ -4x + 2y = -18 \end{cases}$$

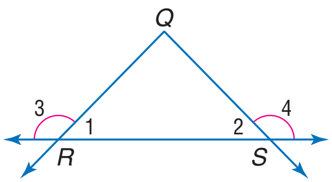
F −6 H 3

G −3 J 6

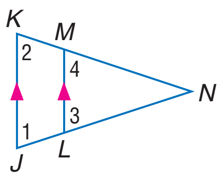
Spiral Review

Write a two-column proof. (Lessons 4-5 and 4-6)

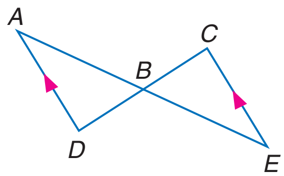
36. **Given:** $\angle 3 \cong \angle 4$
Prove: $\overline{QR} \cong \overline{QS}$



37. **Given:** isosceles triangle JKN with vertex $\angle N$, $\overline{JK} \parallel \overline{LM}$
Prove: $\triangle NML$ is isosceles.



38. **Given:** $\overline{AD} \cong \overline{CE}$; $\overline{AD} \parallel \overline{CE}$
Prove: $\triangle ABD \cong \triangle EBC$



39. **JOBS** A studio engineer charges a flat fee of \$450 for equipment rental and \$42 an hour for recording and mixing time. Write the equation that shows the cost to hire the studio engineer as a function of time. How much would it cost to hire the studio engineer for 17 hours? (Lesson 3-4)

CHAPTER 4

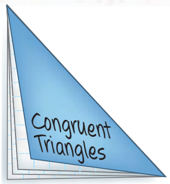
Study Guide and Review

Download Vocabulary Review from geometryonline.com

FOLDABLES Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Classifying Triangles (Lesson 4-1)

- Triangles can be classified by their angles as acute, obtuse, or right.
- Triangles can be classified by their sides as scalene, isosceles, or equilateral.

Angles of Triangles (Lesson 4-2)

- The sum of the measures of the angles of a triangle is 180°.
- The measures of an exterior angle is equal to the sum of the measures of the two remote interior angles.

Congruent Triangles (Lessons 4-3 through 4-5)

- If all of the corresponding sides of two triangles are congruent, then the triangles are congruent (SSS).
- If two corresponding sides of two triangles and the included angle are congruent, then the triangles are congruent (SAS).
- If two pairs of corresponding angles and the included sides of two triangles are congruent, then the triangles are congruent (ASA).
- If two pairs of corresponding angles and a pair of corresponding, nonincluded sides of two triangles are congruent, then the triangles are congruent (AAS).

Isosceles Triangles (Lesson 4-6)

- A triangle is equilateral if and only if it is equiangular.

Triangles and Coordinate Proof (Lesson 4-7)

- Coordinate proofs use algebra to prove geometric concepts.
- The Distance Formula, Slope Formula, and Midpoint Formula are often used in coordinate proof.

Key Vocabulary

acute triangle (p. 202)
base angles (p. 244)
congruence transformation (p. 219)
congruent triangles (p. 217)
coordinate proof (p. 251)
corollary (p. 213)
equiangular triangle (p. 202)
equilateral triangle (p. 203)
exterior angle (p. 211)
flow proof (p. 212)
included side (p. 234)
isosceles triangle (p. 203)
obtuse triangle (p. 202)
remote interior angles (p. 211)
right triangle (p. 202)
scalene triangle (p. 203)
vertex angle (p. 244)

Vocabulary Check

Select the word from the list above that best completes the following statements.

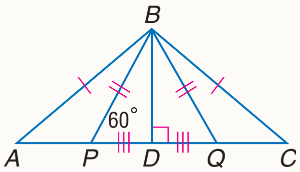
1. A triangle with an angle measure greater than 90 is a(n) __?__.
2. A triangle with exactly two congruent sides is a(n) __?__.
3. A triangle that has an angle with a measure of exactly 90° is a(n) __?__.
4. An equiangular triangle is a form of a(n) __?__.
5. A(n) __?__ uses figures in the coordinate plane and algebra to prove geometric concepts.
6. A(n) __?__ preserves a geometric figure's size and shape.
7. If all corresponding sides and angles of two triangles are congruent, those triangles are __?__.

Vocabulary Review at geometryonline.com

Lesson-by-Lesson Review

4-1 Classifying Triangles (pp. 202–208)

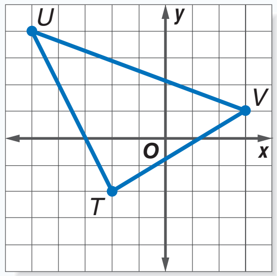
Classify each triangle by its angles and by its sides if $m\angle ABC = 100$.



8. $\triangle ABC$ **9.** $\triangle BDP$ **10.** $\triangle BPQ$

11. DISTANCE The total distance from Sufjan's to Carol's to Steven's house is 18.77 miles. The distance from Sufjan's to Steven's house is 0.81 miles longer than the distance from Sufjan's to Carol's. The distance from Sufjan's to Steven's house is 2.25 time the distance from Carol's to Steven's. Find the distance between each house. Use these lengths to classify the triangle formed by the three houses.

Example 1 **Find the measures of the sides of $\triangle TUV$. Classify the triangle by sides.**



Use the Distance Formula to find the measure of each side.

$$TU = \sqrt{[-5-(-2)]^2 + [4-(-2)]^2}$$
$$= \sqrt{9+36} \text{ or } \sqrt{45}$$
$$UV = \sqrt{[3-(-5)]^2 + (1-4)^2}$$
$$= \sqrt{64+9} \text{ or } \sqrt{73}$$
$$VT = \sqrt{(-2-3)^2 + (-2-1)^2}$$
$$= \sqrt{25+9} \text{ or } \sqrt{34}$$

Since the measures of the sides are all different, the triangle is scalene.

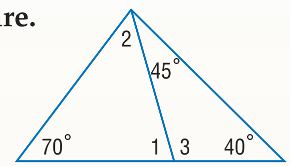
4-2 Angles of Triangles (pp. 210–216)

Find each measure.

12. $m\angle 1$

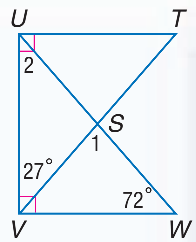
13. $m\angle 2$

14. $m\angle 3$



15. CONSTRUCTION The apex of the truss being built for Tamara's new house measures 72 degrees. If the truss is shaped like an isosceles triangle what are the measures of the other two angles?

Example 2 **If $\overline{TU} \perp \overline{UV}$ and $\overline{UV} \perp \overline{VW}$, find $m\angle 1$.**



Use the Angle Sum Theorem to write an equation.

$$m\angle 1 + 72 + m\angle TVW = 180$$
$$m\angle 1 + 72 + (90 - 27) = 180$$
$$m\angle 1 + 135 = 180$$
$$m\angle 1 = 45$$

4-3 Congruent Triangles (pp. 217–223)

Name the corresponding angles and sides for each pair of congruent triangles.

16. $\triangle EFG \cong \triangle DCB$ **17.** $\triangle NCK \cong \triangle KER$

18. QUILTING Meghan's mom is going to enter a quilt at the state fair. Name the congruent triangles found in the quilt block.

Example 3 **If $\triangle EFG \cong \triangle JKL$, name the corresponding congruent angles and sides.**

The letters of the triangles correspond to the congruent angles and sides. $\angle E \cong \angle J$, $\angle F \cong \angle K$, $\angle G \cong \angle L$, $\overline{EF} \cong \overline{JK}$, $\overline{FG} \cong \overline{KL}$, and $\overline{EG} \cong \overline{JL}$.

4-4 Proving Congruence–SSS, SAS (pp. 225–232)

Determine whether $\triangle MNP \cong \triangle QRS$ given the coordinates of the vertices. Explain.

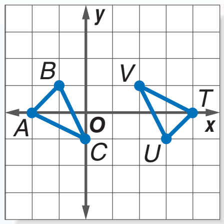
19. $M(0, 3)$, $N(-4, 3)$, $P(-4, 6)$, $Q(5, 6)$, $R(2, 6)$, $S(2, 2)$

20. $M(3, 2)$, $N(7, 4)$, $P(6, 6)$, $Q(-2, 3)$, $R(-4, 7)$, $S(-6, 6)$

21. GAMES In a game, Lupe's boats are placed at coordinates (3, 2), (0, −4), and (6, −4). Do her ships form an equilateral triangle?

22. Triangle ABC is an isosceles triangle with $\overline{AB} \cong \overline{BC}$. If there exists a line $\overline{BD}$ that bisects $\angle ABC$, show that $\triangle ABD \cong \triangle CBD$.

Example 4
Determine whether $\triangle ABC \cong \triangle TUV$. Explain.



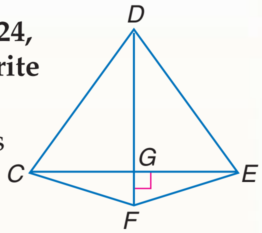
$$AB = \sqrt{[-1-(-2)]^2 + (1-0)^2}$$
$$= \sqrt{1+1} \text{ or } \sqrt{2}$$
$$BC = \sqrt{[0-(-1)]^2 + (-1-1)^2}$$
$$= \sqrt{1+4} \text{ or } \sqrt{5}$$
$$CA = \sqrt{(-2-0)^2 + [0-(-1)]^2}$$
$$= \sqrt{4+1} \text{ or } \sqrt{5}$$
$$TU = \sqrt{(3-4)^2 + (-1-0)^2}$$
$$= \sqrt{1+1} \text{ or } \sqrt{2}$$
$$UV = \sqrt{(2-3)^2 + [1-(-1)]^2}$$
$$= \sqrt{1+4} \text{ or } \sqrt{5}$$
$$VT = \sqrt{(4-2)^2 + (0-1)^2}$$
$$= \sqrt{4+1} \text{ or } \sqrt{5}$$

Therefore, $\triangle ABC \cong \triangle TUV$ by SSS.

Mixed Problem Solving
For mixed problem-solving practice, see page 831.

4-5 Proving Congruence–ASA, AAS (pp. 234–241)

For Exercises 23 and 24, use the figure and write a two-column proof.



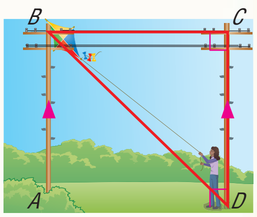
23. Given: DF bisects $\angle CDE$.
$\overline{CE} \perp \overline{DF}$

Prove: $\triangle DGC \cong \triangle DGE$

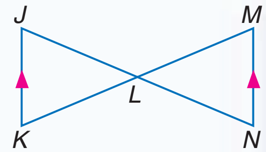
24. Given: $\triangle DGC \cong \triangle DGE$
$\triangle GCF \cong \triangle GEF$

Prove: $\triangle DFC \cong \triangle DFE$

25. KITES Kyra's kite is stuck in a set of power lines. If the power lines are stretched so that they are parallel with the ground, prove that $\triangle ABD \cong \triangle CDB$.



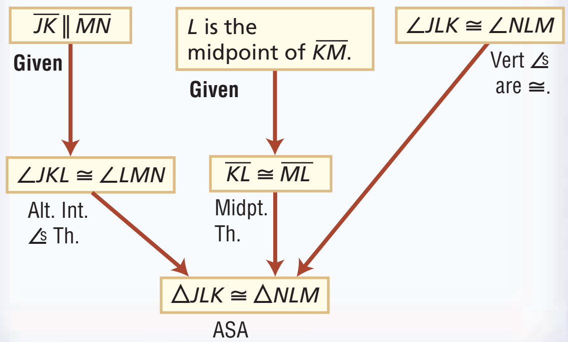
Example 5 Write a proof.



Given: $\overline{JK} \parallel \overline{MN}$
L is the midpoint of $\overline{KM}$.

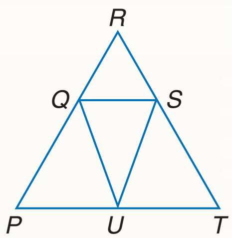
Prove: $\triangle JLK \cong \triangle NLM$

Flow Proof:



4-6 Isosceles Triangles (pp. 244–250)

For Exercises 26–28, refer to the figure.

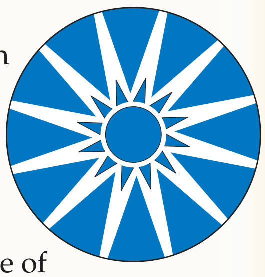


26. If $\overline{PQ} \cong \overline{UQ}$ and $m\angle P = 32$, find $m\angle PUQ$.

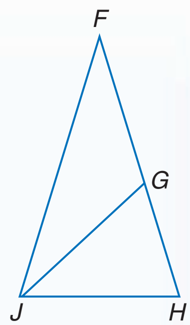
27. If $\overline{RQ} \cong \overline{RS}$ and $m\angle RQS = 75$, find $m\angle R$

28. If $\overline{RQ} \cong \overline{RS}$, $\overline{RP} \cong \overline{RT}$, and $m\angle RQS = 80$, find $m\angle P$.

29. ART This geometric design from Western Cameroon uses approximations of isosceles triangles. Trace the figure. Identify and draw one isosceles triangle of each type from the design. Describe the similarities between the different triangles.



Example 6 **If $\overline{FG} \cong \overline{GJ}$, $\overline{GJ} \cong \overline{JH}$, $\overline{FJ} \cong \overline{FH}$, and $m\angle GJH = 40$, find $m\angle H$.**



$\triangle GHJ$ is isosceles with base $\overline{GH}$, so $\angle JGH \cong \angle H$ by the Isosceles Triangle Theorem. Thus, $m\angle JGH = m\angle H$.

$$m\angle GJH + m\angle JGH + m\angle H = 180$$
$$40 + 2(m\angle H) = 180$$
$$2 \cdot m\angle H = 140$$
$$m\angle H = 70$$

4-7 Triangle and Coordinate Proof (pp. 251–255)

Position and label each triangle on the coordinate plane.

30. isosceles $\triangle TRI$ with base $\overline{TI}$ $4a$ units long

31. equilateral $\triangle BCD$ with side length $6m$ units long

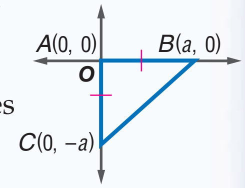
32. right $\triangle JKL$ with leg lengths of a units and b units

33. **BOATS** A sailboat is located 400 meters to the east and 250 meters to the north of a dock. A canoe is located 400 meters to the west and 250 meters to the north of the same dock. Show that the sailboat, the canoe, and the dock all form an isosceles triangle.

Position and label isosceles right triangle $\triangle ABC$ with bases of length a units on the coordinate plane.

- Use the origin as the vertex of $\triangle ABC$ that has the right angle.
- Place each of the bases along an axis, one on the x-axis and the other on the y-axis.
- Since B is on the x-axis, its y-coordinate is 0. Its x-coordinate is a because the leg of the triangle is a units long.

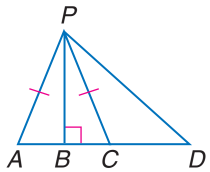
Since $\triangle ABC$ is isosceles, C should also be a distance of a units from the origin. Its coordinates should be $(0, -a)$, since it is on the negative y-axis.



CHAPTER 4 Practice Test

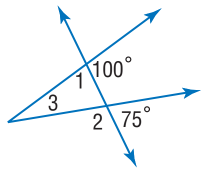
Identify the indicated triangles in the figure if $\overline{PB} \perp \overline{AD}$ and $\overline{PA} \cong \overline{PC}$.

1. obtuse
2. isosceles
3. right

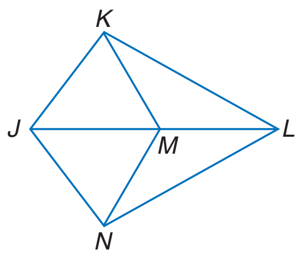


Find the measure of each angle in the figure.

4. $m\angle 1$
5. $m\angle 2$
6. $m\angle 3$



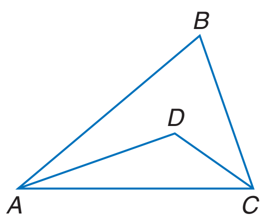
7. Write a flow proof.
   Given: $\triangle JKM \cong \triangle JNM$
   Prove: $\triangle JKL \cong \triangle JNL$



Name the corresponding angles and sides for each pair of congruent triangles.

8. $\triangle DEF \cong \triangle PQR$
9. $\triangle FMG \cong \triangle HNJ$
10. $\triangle XYZ \cong \triangle ZYX$

11. **MULTIPLE CHOICE** In $\triangle ABC$, $\overline{AD}$ and $\overline{DC}$ are angle bisectors and $m\angle B = 76$.



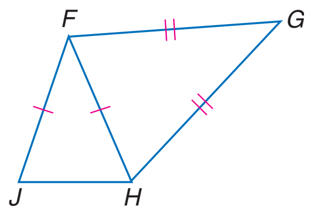
What is $m\angle ADC$?

A 26 **C** 76

B 52 **D** 128

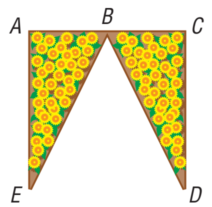
12. Determine whether $\triangle JKL \cong \triangle MNP$ given $J(-1, -2)$, $K(2, -3)$, $L(3, 1)$, $M(-6, -7)$, $N(-2, 1)$, and $P(5, 3)$. Explain.

In the figure, $\overline{FJ} \cong \overline{FH}$ and $\overline{GF} \cong \overline{GH}$.

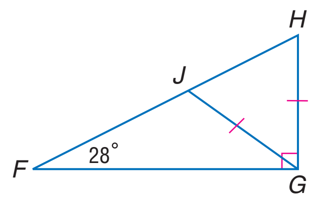


13. If $m\angle JFH = 34$, find $m\angle J$.
14. If $m\angle GHJ = 152$ and $m\angle G = 32$, find $m\angle JFH$.

15. **LANDSCAPING** A landscaper designed a garden shaped as shown in the figure. The landscaper has decided to place point B 22 feet east of point A, point C 44 feet east of point A, point E 36 feet south of point A, and point D 36 feet south of point C. The angles at points A and C are right angles. Prove that $\triangle ABE \cong \triangle CBD$.



16. **MULTIPLE CHOICE** In the figure, $\triangle FGH$ is a right triangle with hypotenuse $\overline{FH}$ and $GJ = GH$.



What is $m\angle JGH$?

F 104 **H** 56

G 62 **J** 28

Chapter Test at geometryonline.com

Standardized Test Practice

CHAPTER 4

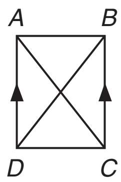
Cumulative, Chapters 1–4

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Use the proof to answer the question below.

Given: $\overline{AD} \parallel \overline{BC}$

Prove: $\triangle ABD \cong \triangle CDB$

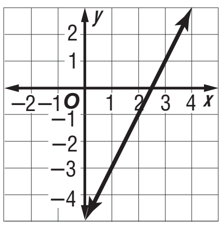


Statements	Reasons
1. $\overline{AD} \parallel \overline{BC}$	1. Given
2. $\angle ABD \cong \angle CDB$, $\angle ADB \cong \angle CBD$	2. Alternate Interior Angles Theorem
3. $\overline{BD} \cong \overline{DB}$	3. Reflexive Property
4. $\triangle ABD \cong \triangle CDB$	4. ?

What reason can be used to prove the triangles are congruent?

A AAS
B ASA
C SAS
D SSS

2. The graph of $y = 2x - 5$ is shown at the right. How would the graph be different if the number 2 in the equation was replaced with a 4?



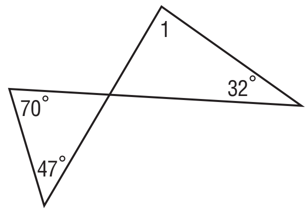
F parallel to the line shown, but shifted two units higher

G parallel to the line shown, but shifted two units lower

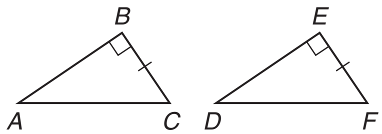
H have a steeper slope, but intercept the y-axis at the same point

J have a less steep slope, but intercept the y-axis at the same point

3. GRIDDABLE What is $m\angle 1$ in degrees?



4. In the figure below, $\overline{BC} \cong \overline{EF}$ and $\angle B \cong \angle E$.

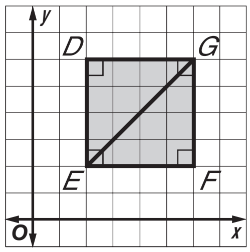


Which additional information would be enough to prove $\triangle ABC \cong \triangle DEF$?

A $\angle A \cong \angle D$ **C** $\overline{AC} \cong \overline{DF}$

B $\overline{AC} \cong \overline{BC}$ **D** $\overline{DE} \perp \overline{EF}$

5. The diagram shows square $DEFG$. Which statement could *not* be used to prove $\triangle DEG$ is a right triangle?



F $(EG)^2 = (DG)^2 + (DE)^2$

G Definition of a Square

H (slope DE)(slope DG) $= 1$

J (slope DE)(slope DG) $= -1$

6. ALGEBRA Which equation is equivalent to $4(y - 2) - 3(2y - 4) = 9$?

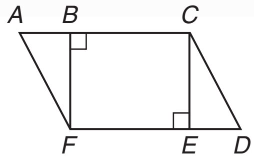
A $2y - 4 = 9$ **C** $10y - 20 = 9$

B $-2y + 4 = 9$ **D** $-2y - 4 = 9$

Preparing for Standardized Tests

For test-taking strategies and more practice, see pages 846–856.

7. In the quadrilateral, which pair of segments can be established to be congruent to prove that $\overline{AC} \parallel \overline{FD}$?



F $\overline{AC} \cong \overline{FD}$ **H** $\overline{BC} \cong \overline{FE}$

G $\overline{AF} \cong \overline{CD}$ **J** $\overline{BF} \cong \overline{CE}$

8. Which of the following is the inverse of the statement *If it is raining, then Kamika carries an umbrella?*

A If Kamika carries an umbrella, then it is raining.

B If Kamika does not carry an umbrella, then it is not raining.

C If it is not raining, then Kamika carries an umbrella.

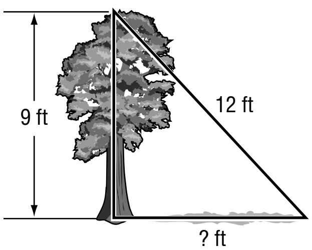
D If it is not raining, then Kamika does not carry an umbrella.

9. ALGEBRA Which of the following describes the line containing the points (2, 4) and (0, –2)?

F $y = -3x + 2$ **H** $y = \frac{1}{3}x - 2$

G $y = -\frac{1}{3}x - 4$ **J** $y = -3x + 2$

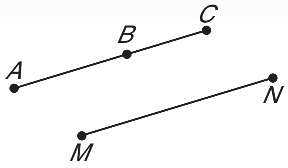
10. A 9-foot tree casts a shadow on the ground. The distance from the top of the tree to the end of the shadow is 12 feet. To the nearest foot, how long is the shadow?



A 7 ft **C** 10 ft

B 8 ft **D** 12 ft

11. In the following proof, what property justifies statement 3?



Given: $\overline{AC} \cong \overline{MN}$

Prove: $AB + BC = MN$

Statements	Reasons
1. $\overline{AC} \cong \overline{MN}$	1. Given
2. $AC = MN$	2. Def. of ≅ segments
3. $AC = AB + BC$	3. ?
4. $AC + BC = MN$	4. Substitution

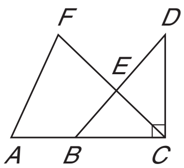
F Definition of Midpoint

G Transitive Property

H Segment Addition Postulate

J Commutative Property

12. If $\angle ACD$ is a right angle, what is the relationship between $\angle ACF$ and $\angle DCF$?



A complementary angles

B congruent angles

C supplementary angles

D vertical angles

TEST-TAKING TIP

Question 12 When you have multiple pieces of information about a figure, make a sketch of the figure so that you can mark the information that you know.

Pre-AP

Record your answer on a sheet of paper. Show your work.

13. The measures of $\triangle ABC$ are $5x$, $4x - 1$, and $3x + 13$.

a. Draw a figure to illustrate $\triangle ABC$ and find the measure of each angle.

b. Prove $\triangle ABC$ is an isosceles triangle.

NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	4-5	3-4	4-2	4-5	3-3	782	3-6	2-2	786	1-2	2-7	1-6	4-6