CHAPTER 6

# Quadrilaterals

## BIG Ideas

- Investigate interior and exterior angles of polygons.
- Recognize and apply the properties of parallelograms, rectangles, rhombi, squares, and trapezoids.
- Position quadrilaterals for use in coordinate proof.

## Key Vocabulary

parallelogram (p. 325)
rectangle (p. 340)
rhombus (p. 348)
square (p. 349)
trapezoid (p. 356)

## Real-World Link

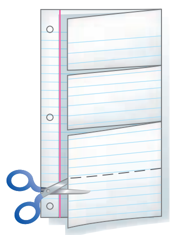
**Tennis** A tennis court is made up of rectangles. The boundaries of these rectangles are significant in the game.

## FOLDABLES Study Organizer

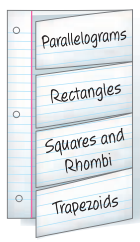
**Quadrilaterals** Make this Foldable to help you organize your notes. Begin with a sheet of notebook paper.

1 **Fold** lengthwise to the left margin.

2 **Cut** 4 tabs.



3 **Label** the tabs using the lesson concepts.





Michael Newman/PhotoEdit

# GET READY for Chapter 6

**Diagnose Readiness** You have two options for checking Prerequisite Skills.

## Option 2

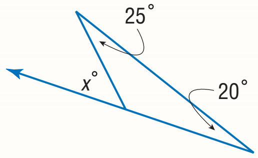
Math Online Take the Online Readiness Quiz at geometryonline.com.

## Option 1

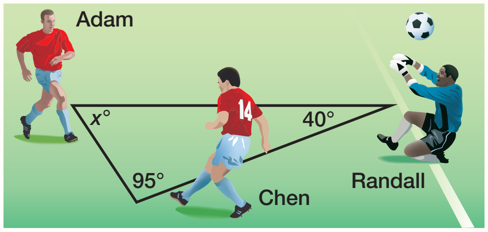
Take the Quick Check below. Refer to the Quick Review for help.

### QUICKCheck

**1.** Find $x$. (Lesson 4-2)




**2.** **SOCCER** During a soccer game, Chen passed the ball to Adam who scored a goal. What is the angle formed by Chen, Adam, and Randall? (Lesson 4-2)

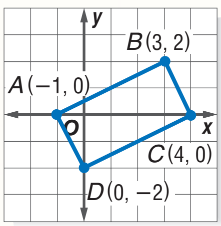



**Find the slopes of $\overline{RS}$ and $\overline{TS}$ for the given points, $R$, $T$, and $S$. Determine whether $\overline{RS}$ and $\overline{TS}$ are *perpendicular* or *not perpendicular*.** (Lesson 3-3)

**3.** $R(4, 3)$, $S(-1, 10)$, $T(13, 20)$

**4.** $R(-9, 6)$, $S(3, 8)$, $T(1, 20)$

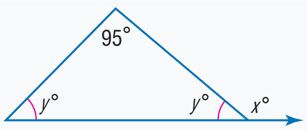
**5.** **FRAMES** Determine whether the corners of the frame are right angles. (Lesson 3-3)




### QUICKReview

**EXAMPLE 1**

**Find $x$.**




| | |
|---|---|
| $95 + y + y = 180$ | Angle Sum Theorem |
| $95 + 2y = 180$ | Combine like terms. |
| $2y = 85$ | Subtract 95 from each side. |
| $y = 42.5$ | Divide each side by 2. |
| $x + y = 180$ | Supplement Theorem |
| $x + 42.5 = 180$ | Substitution |
| $x = 137.5$ | Subtract 42.5 from each side. |

**EXAMPLE 2**

**Find the slopes of $\overline{RS}$ and $\overline{TS}$ for the given points, $R$, $T$, and $S$ with coordinates $R(0, 0)$, $S(2, 3)$, $T(-1, 5)$. Determine whether $\overline{RS}$ and $\overline{TS}$ are *perpendicular* or *not perpendicular*.**

First, find the slope $\overline{RS}$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 - 0}{2 - 0} \quad (x_1, y_1) = (0, 0), (x_2, y_2) = (2, 3)$$

$$= \frac{3}{2} \quad \text{Simplify.}$$

Next, find the slope of $\overline{TS}$. Let $(x_1, y_1) = (-1, 5)$ and $(x_2, y_2) = (2, 3)$.

$$\text{slope} = \frac{3 - 5}{2 - (-1)} \text{ or } \frac{-2}{3}$$

Since the product of the slopes is $-1$, $\overline{RS} \perp \overline{TS}$.

# 6-1 Angles of Polygons

## Main Ideas

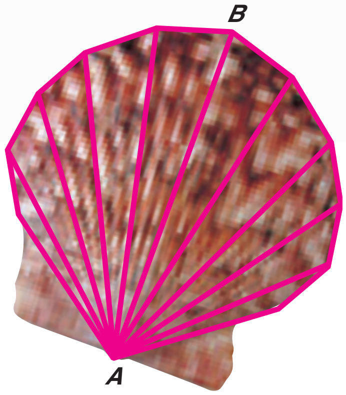
- Find the sum of the measures of the interior angles of a polygon to classify figures and solve problems.
- Find the sum of the measures of the exterior angles of a polygon to classify figures and solve problems.

## New Vocabulary

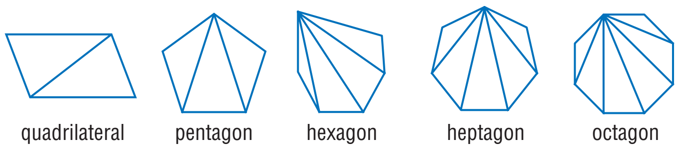
diagonal

## GET READY for the Lesson

This scallop shell resembles a 12-sided polygon with diagonals drawn from one of the vertices. A **diagonal** of a polygon is a segment that connects any two nonconsecutive vertices. For example, $\overline{AB}$ is one of the diagonals of this polygon.




**Sum of Measures of Interior Angles** Polygons with more than three sides have diagonals. The polygons below show all of the possible diagonals drawn from one vertex.




In each case, the polygon is separated into triangles. The sum of the angle measures of each polygon is the sum of the angle measures of the triangles. Since the sum of the angle measures of a triangle is 180, we can make a table to find the sum of the angle measures for several convex polygons.

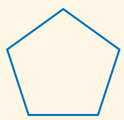
| Convex Polygon | Number of Sides | Number of Triangles | Sum of Angle Measures |
|---|---|---|---|
| triangle | 3 | 1 | (1 · 180) or 180 |
| quadrilateral | 4 | 2 | (2 · 180) or 360 |
| pentagon | 5 | 3 | (3 · 180) or 540 |
| hexagon | 6 | 4 | (4 · 180) or 720 |
| heptagon | 7 | 5 | (5 · 180) or 900 |
| octagon | 8 | 6 | (6 · 180) or 1080 |

Look for a pattern in the sum of the angle measures.

### THEOREM 6.1 — *Interior Angle Sum*

If a convex polygon has $n$ sides and $S$ is the sum of the measures of its interior angles, then $S = 180(n - 2)$.

**Example:**



$n = 5$
$S = 180(n - 2)$
$= 180(5 - 2)$ or 540

## Real-World EXAMPLE Interior Angles of Regular Polygons

**1** **CONSTRUCTION** **The Paddington family is assembling a hexagonal sandbox. What is the sum of the measures of the interior angles of the hexagon?**

$S = 180(n - 2)$ Interior Angle Sum Theorem

$= 180(6 - 2)$ $n = 6$

$= 180(4)$ or 720 The sum of the measures of the interior angles is 720.

**CHECK Your Progress**

**1.** Find the sum of the measures of the interior angles of a nonagon.

## EXAMPLE Sides of a Polygon

**Review Vocabulary**

A **regular polygon** is a convex polygon in which all of the sides are congruent and all of the angles are congruent.
(Lesson 1-6)

**2** **The measure of an interior angle of a regular polygon is 108. Find the number of sides in the polygon.**

$S = 180(n - 2)$ Interior Angle Sum Theorem

$(108)n = 180(n - 2)$ $S = 108n$

$108n = 180n - 360$ Distributive Property

$0 = 72n - 360$ Subtract $108n$ from each side.

$360 = 72n$ Add 360 to each side.

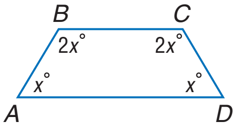
$5 = n$ Divide each side by 72. The polygon has 5 sides.

**CHECK Your Progress**

**2.** The measure of an interior angle of a regular polygon is 135. Find the number of sides in the polygon.

## EXAMPLE Interior Angles of Nonregular Polygons

**3** **ALGEBRA** **Find the measure of each interior angle.**




Since $n = 4$, the sum of the measures of the interior angles is $180(4 - 2)$ or 360.

$360 = m\angle A + m\angle B + m\angle C + m\angle D$ Sum of measures of interior angles

$360 = x + 2x + 2x + x$ Substitution

$360 = 6x$ Combine like terms.

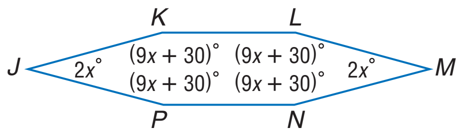
$60 = x$ Divide each side by 6.

Use the value of $x$ to find the measure of each angle.

$m\angle A = 60$, $m\angle B = 2 \cdot 60$ or 120, $m\angle C = 2 \cdot 60$ or 120, and $m\angle D = 60$.

**CHECK Your Progress**

**3.**




Personal Tutor at geometryonline.com

**Review Vocabulary**

An **exterior angle** is an angle formed by one side of a polygon and the extension of another side. (Lesson 4-2)

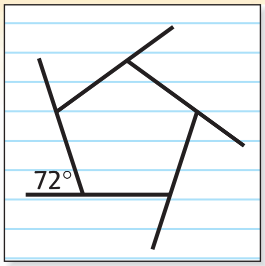
**Sum of Measures of Exterior Angles** Is there a relationship among the exterior angles of a convex polygon?

## GEOMETRY LAB

### Sum of the Exterior Angles of a Polygon

**COLLECT DATA**

- Draw a triangle, a convex quadrilateral, a convex pentagon, a convex hexagon, and a convex heptagon.
- Extend the sides of each polygon to form exactly one exterior angle at each vertex.
- Use a protractor to measure each exterior angle of each polygon and record it on your drawing.




**ANALYZE THE DATA**

**1.** Copy and complete the table.

| Polygon | triangle | quadrilateral | pentagon | hexagon | heptagon |
|---|---|---|---|---|---|
| Number of Exterior Angles | | | | | |
| Sum of Measures of Exterior Angles | | | | | |

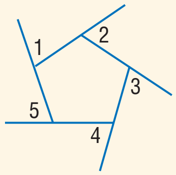
**2.** What conjecture can you make?

The Geometry Lab suggests Theorem 6.2.

**THEOREM 6.2** *Exterior Angle Sum*

If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360.

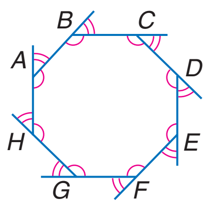
**Example:** $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360$




You will prove Theorem 6.2 in Exercise 30.

## EXAMPLE Exterior Angles

**4** **Find the measures of an exterior angle and an interior angle of convex regular octagon *ABCDEFGH*.**




$8n = 360$    $n$ = measure of each exterior angle

$n = 45$    Divide each side by 8.

The measure of each exterior angle is 45. Since each exterior angle and its corresponding interior angle form a linear pair, the measure of the interior angle is $180 - 45$ or 135.

**CHECK Your Progress**

**4.** Find the measures of an exterior angle and an interior angle of a convex regular dodecagon.

## CHECK Your Understanding

Example 1 (p. 319)

**1. AQUARIUMS** The regular polygon at the right is the base of a fish tank. Find the sum of the measures of the interior angles of the pentagon.

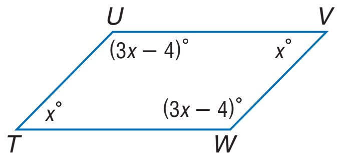
Example 2 (p. 319)

**The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.**

**2.** 60 **3.** 90

Example 3 (p. 319)

**4. ALGEBRA** Find the measure of each interior angle.




**Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon.**

Example 4 (p. 320)

**5.** 6 **6.** 18

## Exercises

| HOMEWORK HELP | |
|---|---|
| For Exercises | See Examples |
| 7–14 | 1 |
| 15–18 | 2 |
| 19–22 | 3 |
| 23–26 | 4 |

**Find the sum of the measures of the interior angles of each convex polygon.**

**7.** 32-gon **8.** 18-gon **9.** 19-gon
**10.** 27-gon **11.** $4y$-gon **12.** $2x$-gon

**13. GARDENING** Carlotta is designing a garden for her backyard. She wants a flower bed shaped like a regular octagon. Find the sum of the measures of the interior angles of the octagon.

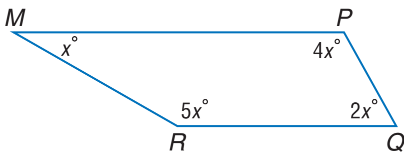
**14. GAZEBOS** A company is building regular hexagonal gazebos. Find the sum of the measures of the interior angles of the hexagon.

**The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.**

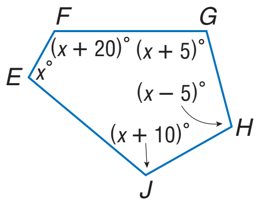
**15.** 140 **16.** 170 **17.** 160 **18.** 176.4

**ALGEBRA Find the measure of each interior angle.**

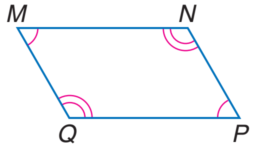
**19.**



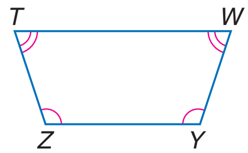

**20.**




**21.** parallelogram $MNPQ$ with $m\angle M = 10x$ and $m\angle N = 20x$




**22.** isosceles trapezoid $TWYZ$ with $\angle Z \cong \angle Y$, $m\angle Z = 30x$, $\angle T \cong \angle W$, and $m\angle T = 20x$



**Find the measures of each exterior angle and each interior angle for each regular polygon.**

**23.** decagon **24.** hexagon **25.** nonagon **26.** octagon

**Find the measures of an interior angle and an exterior angle given the number of sides of each regular polygon. Round to the nearest tenth if necessary.**

**27.** 11 **28.** 7 **29.** 12

**Real-World Link**

Thomas Jefferson's home, Monticello, features a dome on an octagonal base. The architectural elements on either side of the dome were based on a regular octagon.

**Source:** www.monticello.org

**30. PROOF** Use algebra to prove the Exterior Angle Sum Theorem.

**31. ARCHITECTURE** The Pentagon building in Washington, D.C., was designed to resemble a regular pentagon. Find the measure of an interior angle and an exterior angle of the courtyard.

**32. ARCHITECTURE** Use the information at the left to compare the dome to the architectural elements on each side of the dome. Are the interior and exterior angles the same? Find the measures of the interior and exterior angles.

**EXTRA PRACTICE** See pages 811, 833.

Math Online Self-Check Quiz at geometryonline.com

**ALGEBRA Find the measure of each interior angle using the given information.**

| Angle | Measure (°) |
|---|---|
| $A$ | $6x$ |
| $B$ | $4x + 13$ |
| $C$ | $x + 9$ |
| $D$ | $2x - 8$ |
| $E$ | $4x - 1$ |

**33.** decagon in which the measures of the interior angles are $x + 5$, $x + 10$, $x + 20$, $x + 30$, $x + 35$, $x + 40$, $x + 60$, $x + 70$, $x + 80$, and $x + 90$

**34.** polygon *ABCDE* with the interior angle measures shown in the table

**H.O.T. Problems**

**35. REASONING** Explain why the Interior Angle Sum Theorem and the Exterior Angle Sum Theorem apply only to convex polygons.

**36. OPEN ENDED** Draw a regular convex polygon and a convex polygon that is not regular with the same number of sides. Compare the sum of the interior angles for each.

**37. CHALLENGE** Two formulas can be used to find the measure of an interior angle of a regular polygon: $s = \frac{180(n-2)}{n}$ and $s = 180 - \frac{360}{n}$. Show that these are equivalent.

**38.** ***Writing in Math*** Explain how triangles are related to the Interior Angle Sum Theorem.

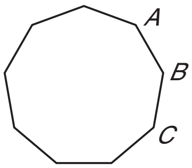
## STANDARDIZED TEST PRACTICE

**39.** The sum of the interior angles of a polygon is twice the sum of its exterior angles. What type of polygon is it?

**A** pentagon  **C** octagon

**B** hexagon  **D** decagon

**40.** If the polygon shown is regular, what is $m\angle ABC$?




**F** 140°

**G** 144°

**H** 162°

**J** 180°

**41.** **REVIEW** If $x$ is subtracted from $x^2$, the difference is 72. Which of the following could be a value of $x$?

**A** −36

**B** −9

**C** −8

**D** 72

**42.** **REVIEW** $\dfrac{3^2 \cdot 4^5 \cdot 5^3}{5^3 \cdot 3^3 \cdot 4^6} =$

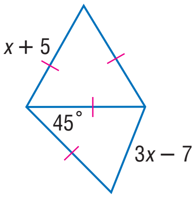
**F** $\frac{1}{60}$  **H** $\frac{3}{4}$

**G** $\frac{1}{12}$  **J** 12

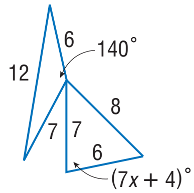
## Spiral Review

**Write an inequality to describe the possible values of $x$.** (Lesson 5-5)

**43.**




**44.**




**Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.** (Lesson 5-4)

**45.** 5, 17, 9

**46.** 17, 30, 30

**47.** 8.4, 7.2, 3.5

**48.** 4, 0.9, 4.1

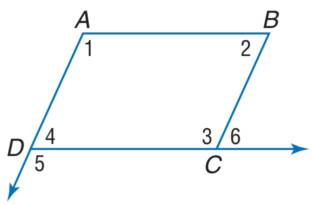
**49.** 14.3, 12, 2.2

**50.** 0.18, 0.21, 0.52

**51.** **GARDENING** A landscape designer is putting black plastic edging around a rectangular flower garden that has length 5.7 meters and width 3.8 meters. The edging is sold in 5-meter lengths. Find the perimeter of the garden and determine how much edging the designer should buy. (Lesson 1-6)


## GET READY for the Next Lesson

**PREREQUISITE SKILL** **In the figure, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Name all pairs of angles for each type indicated.** (Lesson 3-1)




**52.** consecutive interior angles

**53.** alternate interior angles

EXTEND 6-1

## Spreadsheet Lab

# Angles of Polygons

It is possible to find the interior and exterior measurements along with the sum of the interior angles of any regular polygon with $n$ number of sides using a spreadsheet.

### ACTIVITY

**Design a spreadsheet using the following steps.**

- Label the columns as shown in the spreadsheet below.
- Enter the digits 3–10 in the first column.
- The number of triangles formed by diagonals from the same vertex in a polygon is 2 less than the number of sides. Write a formula for Cell B2 to subtract 2 from each number in Cell A2.
- Enter a formula for Cell C2 so the spreadsheet will find the sum of the measures of the interior angles. Remember that the formula is $S = (n - 2)180$.
- Continue to enter formulas so that the indicated computation is performed. Then, copy each formula through Row 9. The final spreadsheet will appear as below.

Polygons and Angles.xls

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | Number of Sides | Number of Triangles | Sum of Measures of Interior Angles | Measure of Each Interior Angle | Measure of Each Exterior Angle | Sum of Measures of Exterior Angles |
| 2 | 3 | 1 | 180 | 60 | 120 | 360 |
| 3 | 4 | 2 | 360 | 90 | 90 | 360 |
| 4 | 5 | 3 | 540 | 108 | 72 | 360 |
| 5 | 6 | 4 | 720 | 120 | 60 | 360 |
| 6 | 7 | 5 | 900 | 128.57 | 51.43 | 360 |
| 7 | 8 | 6 | 1080 | 135 | 45 | 360 |
| 8 | 9 | 7 | 1260 | 140 | 40 | 360 |
| 9 | 10 | 8 | 1440 | 144 | 36 | 360 |
| 10 | | | | | | |

Sheet 1 | Sheet 2 | Sheet 3

### ANALYZE THE RESULTS

1. Write the formula to find the measure of each interior angle in the polygon.
2. Write the formula to find the sum of the measures of the exterior angles.
3. What is the measure of each interior angle if the number of sides is 1? 2?
4. Is it possible to have values of 1 and 2 for the number of sides? Explain.

**For Exercises 5–7, use the spreadsheet.**

5. How many triangles are in a polygon with 15 sides?
6. Find the measure of an exterior angle of a polygon with 15 sides.
7. Find the measure of an interior angle of a polygon with 110 sides.

# 6-2 Parallelograms

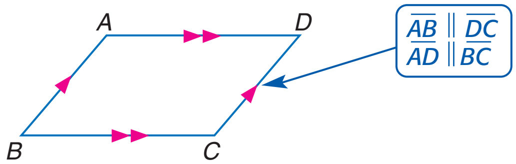
## Main Ideas

- Recognize and apply properties of the sides and angles of parallelograms.
- Recognize and apply properties of the diagonals of parallelograms.

## GET READY for the Lesson

To chart a course, sailors use a parallel ruler. One edge of the ruler is placed at the starting position. Then the other ruler is moved until its edge reaches the compass rose printed on the chart. Reading the compass determines which direction to travel. Each pair of opposite sides of the ruler are parallel.

**Sides and Angles of Parallelograms** A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.




▱*ABCD*

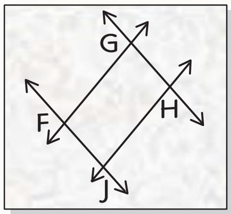
This lab will help you make conjectures about the sides and angles of a parallelogram.

## GEOMETRY LAB

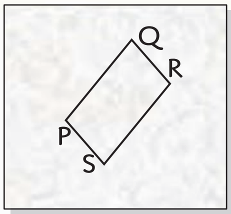
### Properties of Parallelograms

**MAKE A MODEL**

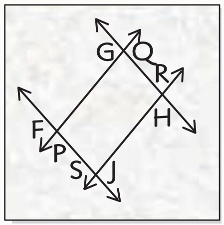
**Step 1** Draw two sets of intersecting parallel lines on patty paper. Label the vertices *FGHJ*.




**Step 2** Trace *FGHJ*. Label the second parallelogram *PQRS* so ∠*F* and ∠*P* are congruent.




**Step 3** Rotate *PQRS* on *FGHJ* to compare sides and angles.



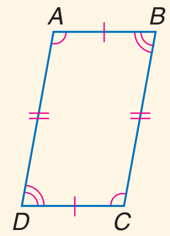

**ANALYZE**

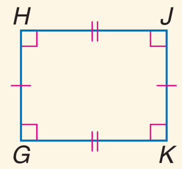
1. List all of the segments that are congruent.
2. List all of the angles that are congruent.
3. **Make a conjecture** about the angle relationships you observed.
4. Test your conjecture.

The Geometry Lab leads to four properties of parallelograms.

## THEOREMS

| Theorem | Examples |
|---|---|
| **6.3** Opposite sides of a parallelogram are congruent. **Abbreviation:** *Opp. sides of ▱ are ≅.* | $\overline{AB} \cong \overline{DC}$<br>$\overline{AD} \cong \overline{BC}$ |
| **6.4** Opposite angles in a parallelogram are congruent. **Abbreviation:** *Opp. ∡ of ▱ are ≅.* | $\angle A \cong \angle C$<br>$\angle B \cong \angle D$ |
| **6.5** Consecutive angles in a parallelogram are supplementary. **Abbreviation:** *Cons. ∡ in ▱ are suppl.* | $m\angle A + m\angle B = 180$<br>$m\angle B + m\angle C = 180$<br>$m\angle C + m\angle D = 180$<br>$m\angle D + m\angle A = 180$ |
| **6.6** If a parallelogram has one right angle, it has four right angles. **Abbreviation:** *If ▱ has 1 rt. ∠, it has 4 rt. ∡.* | $m\angle G = 90$<br>$m\angle H = 90$<br>$m\angle J = 90$<br>$m\angle K = 90$ |

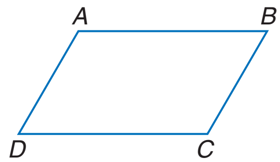



You will prove Theorems 6.3 and 6.5 in Exercises 34 and 35, respectively.

## EXAMPLE Proof of Theorem 6.4

**1** **Write a two-column proof of Theorem 6.4.**




**Given:** ▱$ABCD$

**Prove:** $\angle A \cong \angle C$, $\angle D \cong \angle B$

**Proof:**

| Statements | Reasons |
|---|---|
| **1.** ▱$ABCD$ | **1.** Given |
| **2.** $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$ | **2.** Definition of parallelogram |
| **3.** $\angle A$ and $\angle D$ are supplementary.<br>$\angle D$ and $\angle C$ are supplementary.<br>$\angle C$ and $\angle B$ are supplementary. | **3.** If parallel lines are cut by a transversal, consecutive interior angles are supplementary. |
| **4.** $\angle A \cong \angle C$<br>$\angle D \cong \angle B$ | **4.** Supplements of the same angles are congruent. |

**Study Tip**

**Including a Figure**
Theorems are presented in general terms. In a proof, you must include a drawing so that you can refer to segments and angles specifically.

**CHECK Your Progress**

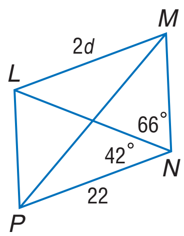
**1. PROOF** Write a paragraph proof of Theorem 6.6.


**Given:** ▱$MNPQ$
$\angle M$ is a right angle.

**Prove:** $\angle N$, $\angle P$, and $\angle Q$ are right angles.

## Real-World EXAMPLE Properties of Parallelograms

**2** **ADVERTISING** **Quadrilateral *LMNP* is a parallelogram designed to be part of a new company logo. Find $m\angle PLM$, $m\angle LMN$, and *d*.**




$m\angle MNP = 66 + 42$ or $108$ Angle Addition Theorem

$$\angle PLM \cong \angle MNP \quad \text{Opp. } \angle\text{s of } \square \text{ are } \cong.$$
$$m\angle PLM = m\angle MNP \quad \text{Definition of congruent angles}$$
$$m\angle PLM = 108 \quad \text{Substitution}$$

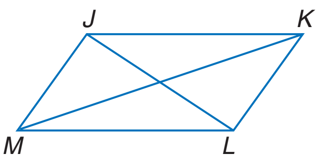
$$m\angle PLM + m\angle LMN = 180 \quad \text{Cons. } \angle\text{s of } \square \text{ are suppl.}$$
$$108 + m\angle LMN = 180 \quad \text{Substitution}$$
$$m\angle LMN = 72 \quad \text{Subtract 108 from each side.}$$

$$\overline{LM} \cong \overline{PN} \quad \text{Opp. sides of } \square \text{ are } \cong.$$
$$LM = PN \quad \text{Definition of congruent segments}$$
$$2d = 22 \quad \text{Substitution}$$
$$d = 11 \quad \text{Divide each side by 2.}$$

**CHECK Your Progress**

**2.** Refer to ▱$LMNP$. If the perimeter of the parallelogram is 74 units, find $MN$.

**Diagonals of Parallelograms** In parallelogram $JKLM$, $\overline{JL}$ and $\overline{KM}$ are diagonals. Theorem 6.7 states the relationship between diagonals of a parallelogram.

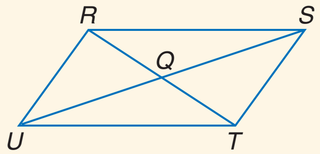



**THEOREM 6.7**

The diagonals of a parallelogram bisect each other.

**Abbreviation:** *Diag. of ▱ bisect each other.*

**Example:** $\overline{RQ} \cong \overline{QT}$ and $\overline{SQ} \cong \overline{QU}$




You will prove Theorem 6.7 in Exercise 36.

**STANDARDIZED TEST EXAMPLE** **Diagonals of a Parallelogram**

**5** What are the coordinates of the intersection of the diagonals of parallelogram $ABCD$ with vertices $A(2, 5)$, $B(6, 6)$, $C(4, 0)$, and $D(0, -1)$?

**A** (4, 2) **B** (4.5, 2) **C** $\left(\frac{7}{6}, -\frac{5}{2}\right)$ **D** (3, 2.5)

### Read the Test Item

Since the diagonals of a parallelogram bisect each other, the intersection point is the midpoint of $\overline{AC}$ and $\overline{BD}$.

### Solve the Test Item

Find the midpoint of $\overline{AC}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 4}{2}, \frac{5 + 0}{2}\right) \quad \text{Midpoint Formula}$$
$$= (3, 2.5) \quad \text{Simplify.}$$

The coordinates of the intersection of the diagonals of parallelogram $ABCD$ are (3, 2.5). The answer is D.

**Test-Taking Tip**

**Check Answers** Always check your answer. To check the answer to this problem, find the coordinates of the midpoint of $\overline{BD}$.

**CHECK Your Progress**

3. **COORDINATE GEOMETRY** Determine the coordinates of the intersection of the diagonals of ▱$RSTU$ with vertices $R(-8, -2)$, $S(-6, 7)$, $T(6, 7)$, and $U(4, -2)$.

**F** $(-1, 2.5)$ **G** $(1, -4)$ **H** $(5, 4.5)$ **J** $(-1.5, -2, 5)$

**Personal Tutor at** geometryonline.com

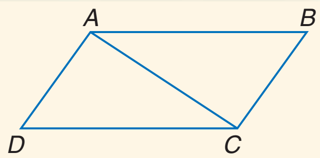
Theorem 6.8 describes another characteristic of the diagonals of a parallelogram.

**THEOREM 6.8**

Each diagonal of a parallelogram separates the parallelogram into two congruent triangles.

**Abbreviation:** *Diag. separates ▱ into 2 ≅ △s.*

**Example:** $\triangle ACD \cong \triangle CAB$




You will prove Theorem 6.8 in Exercise 37.

## CHECK Your Understanding

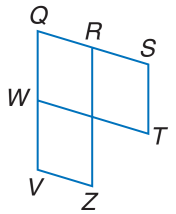
**Example 1** (p. 326)

**PROOF** **Write the indicated type of proof.**

1. two-column

**Given:** ▱$VZRQ$ and ▱$WQST$

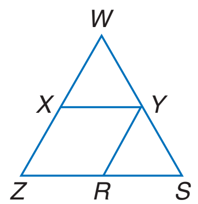
**Prove:** $\angle Z \cong \angle T$




2. paragraph

**Given:** ▱$XYRZ$, $\overline{WZ} \cong \overline{WS}$

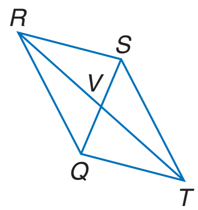
**Prove:** $\angle XYR \cong \angle S$




**Example 2** (p. 326)

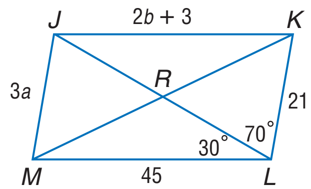
**Complete each statement about ▱$QRST$. Justify your answer.**

3. $\overline{SV} \cong$ __?__.
4. $\triangle VRS \cong$ __?__.
5. $\angle TSR$ is supplementary to __?__.




**Use ▱$JKLM$ to find each measure or value.**

6. $m\angle MJK$
7. $m\angle JML$
8. $m\angle JKL$
9. $m\angle KJL$
10. $a$
11. $b$




**Example 3** (p. 327)

12. **STANDARDIZED TEST PRACTICE** Parallelogram $GHJK$ has vertices $G(-3, 4)$, $H(1, 1)$, and $J(3, -5)$. Which are possible coordinates for vertex $K$?

**A** $(-1, 1)$ **B** $(-2, 0)$ **C** $(-1, -2)$ **D** $(-2, -1)$

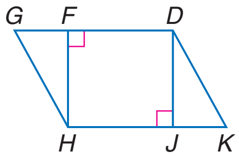
# Exercises

**HOMEWORK HELP**

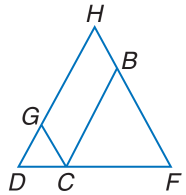
| For Exercises | See Examples |
|---|---|
| 13, 14, 34–37 | 1 |
| 15–30 | 2 |
| 31–33 | 3 |

**PROOF** Write a two-column proof.

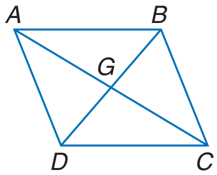
**13. Given:** $\square DGHK$, $\overline{FH} \perp \overline{GD}$, $\overline{DJ} \perp \overline{HK}$
**Prove:** $\triangle DJK \cong \triangle HFG$



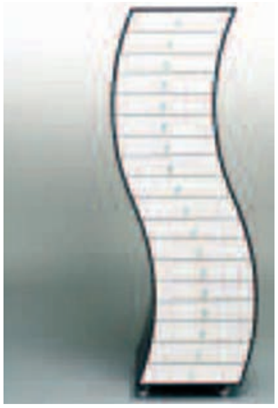

**14. Given:** $\square BCGH$, $\overline{HD} \cong \overline{FD}$
**Prove:** $\angle F \cong \angle GCB$




Complete each statement about $\square ABCD$. Justify your answer.




**15.** $\angle DAB \cong$ __?__.
**16.** $\angle ABD \cong$ __?__.
**17.** $\overline{AB} \parallel$ __?__.
**18.** $\overline{BG} \cong$ __?__.
**19.** $\triangle ABD \cong$ __?__.
**20.** $\angle ACD \cong$ __?__.

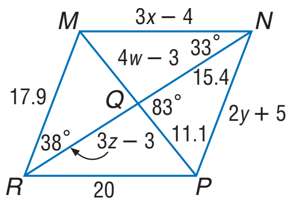


**Real-World Link**

Shiro Kuramata designed furniture that was functional and aesthetically pleasing. His style is surreal and minimalist.

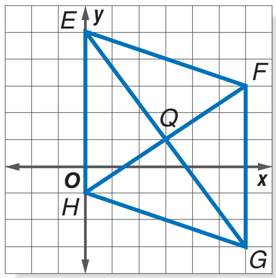
**Source:** designboom.com

**ALGEBRA** Use $\square MNPR$ to find each measure or value. Round to the nearest tenth if necessary.




**21.** $m\angle MNP$
**22.** $m\angle NRP$
**23.** $m\angle RNP$
**24.** $m\angle RMN$
**25.** $m\angle MQN$
**26.** $m\angle MQR$
**27.** $x$
**28.** $y$
**29.** $w$
**30.** $z$

**COORDINATE GEOMETRY** For Exercises 31–33, refer to $\square EFGH$.




**31.** Use the Distance Formula to verify that the diagonals bisect each other.

**32.** Determine whether the diagonals of this parallelogram are congruent.

**33.** Find the slopes of $\overline{EH}$ and $\overline{EF}$. Are the consecutive sides perpendicular? Explain.

Write the indicated type of proof.

**34.** two-column proof of Theorem 6.3
**35.** two-column proof of Theorem 6.5
**36.** paragraph proof of Theorem 6.7
**37.** two-column proof of Theorem 6.8

**38. DESIGN** The chest of drawers shown at the left is called *Side 2*. It was designed by Shiro Kuramata. Describe the properties of parallelograms the artist may have used to place each drawer pull.

**39. ALGEBRA** Parallelogram $ABCD$ has diagonals $\overline{AC}$ and $\overline{DB}$ that intersect at $P$. If $AP = 3a + 18$, $AC = 12a$, $PB = a + 2b$, and $PD = 3b + 1$, find $a$, $b$, and $DB$.

**40. ALGEBRA** In parallelogram $ABCD$, $AB = 2x + 5$, $m\angle BAC = 2y$, $m\angle B = 120$, $m\angle CAD = 21$, and $CD = 21$. Find $x$ and $y$.

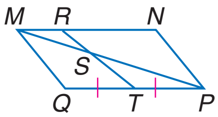
**EXTRA PRACTICE**
See pages 813, 835.

**Math Online**
Self-Check Quiz at geometryonline.com

**H.O.T. Problems**

**41. OPEN ENDED** Draw a parallelogram with one side twice as long as another side.

**42. CHALLENGE** Compare the corresponding angles of $\triangle MSR$ and $\triangle PST$, given that $MNPQ$ is a parallelogram with $MR = \frac{1}{4}MN$. What can you conclude about these triangles?




**43. *Writing in Math*** Describe the characteristics of the sides and angles of a parallelogram and the properties of the diagonals of a parallelogram.

## STANDARDIZED TEST PRACTICE

**44.** Two consecutive angles of a parallelogram measure $(3x + 42)°$ and $(9x - 18)°$. What are the measures of the angles?

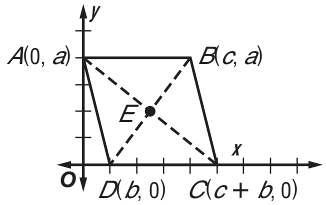
**A** 13, 167

**B** 58.5, 31.5

**C** 39, 141

**D** 81, 99

**45.** Figure $ABCD$ is a parallelogram.




What are the coordinates of point $E$?

**F** $\left(\frac{a}{c}, \frac{c}{2}\right)$

**G** $\left(\frac{c+b}{2}, \frac{a+b}{2}\right)$

**H** $\left(\frac{a+c}{2}, \frac{b}{2}\right)$

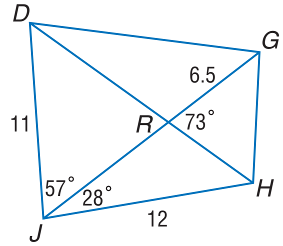
**J** $\left(\frac{c+b}{2}, \frac{a}{2}\right)$

## Spiral Review

**Find the sum of the measures of the interior angles of each convex polygon.** (Lesson 6-1)

**46.** 14-gon **47.** 22-gon **48.** 17-gon **49.** 36-gon

**Write an inequality relating the given pair of angles or segment measures.** (Lesson 5-5)




**50.** $m\angle DRJ, m\angle HRJ$

**51.** $DG, GH$

**52.** $m\angle JDH, m\angle DHJ$

**53. JOBS** Jamie works at a gift shop after school. She is paid \$10 per hour plus a 15% commission on merchandise that she sells. Write an equation that represents her earnings in a week if she sold \$550 worth of merchandise. (Lesson 3-4)

## GET READY for the Next Lesson

**PREREQUISITE SKILL The vertices of a quadrilateral are $A(-5, -2)$ $B(-2, 5)$, $C(2, -2)$, and $D(-1, -9)$. Determine whether each segment is a side or a diagonal of the quadrilateral, and find the slope of each segment.** (Lesson 3-3)

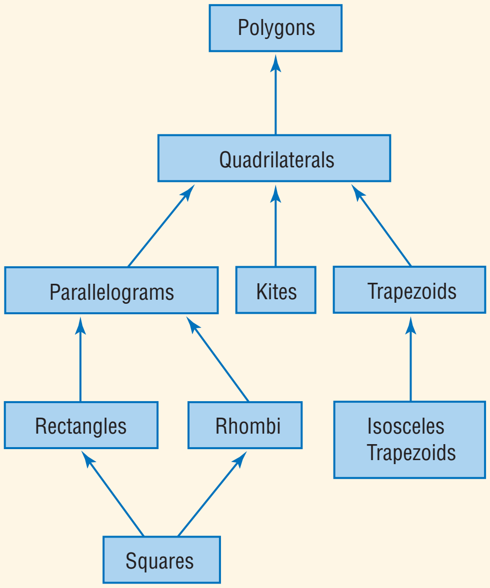
**54.** $\overline{AB}$ **55.** $\overline{BD}$ **56.** $\overline{CD}$

# READING MATH

## Hierarchy of Polygons

A *hierarchy* is a ranking of classes or sets of things. Examples of some classes of polygons are rectangles, rhombi, trapezoids, parallelograms, squares, and quadrilaterals. These classes are arranged in the hierarchy at the right.

You will study rectangles, squares, rhombi, trapezoids, and kites in the remaining lessons of Chapter 6.




Use the following information to help read the hierarchy diagram.

- The class that is the broadest is listed first, followed by the other classes in order. For example, *polygons* is the broadest class in the hierarchy diagram above, and *squares* is a very specific class.
- Each class is contained within any class linked above it in the hierarchy. For example, *all* squares are also rhombi, rectangles, parallelograms, quadrilaterals, and polygons. However, an isosceles trapezoid is not a square or a kite.
- Some, but not all, elements of each class are contained within lower classes in the hierarchy. For example, some trapezoids are isosceles trapezoids, and some rectangles are squares.

## Reading to Learn

**Refer to the hierarchy diagram at the right. Write *true, false,* or *not enough information* for each statement.**

**1.** All mogs are jums.

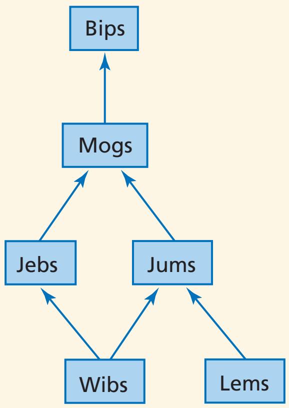
**2.** Some jebs are jums.

**3.** All lems are jums.

**4.** Some wibs are jums.

**5.** All mogs are bips.

**6.** Draw a hierarchy diagram to show these classes: equilateral triangles, polygons, isosceles triangles, triangles, and scalene triangles.



EXPLORE 6-3

## Graphing Calculator Lab

# Parallelograms

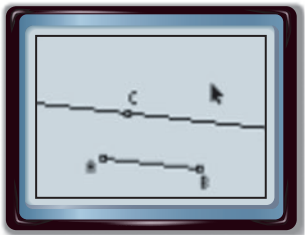
You can use the Cabri Junior application on a TI-83/84 Plus graphing calculator to discover properties of parallelograms.

### ACTIVITY

**Construct a quadrilateral with one pair of sides that are both parallel and congruent.**

**Step 1** Construct a segment using the Segment tool on the F2 menu. Label the segment $\overline{AB}$. This is one side of the quadrilateral.

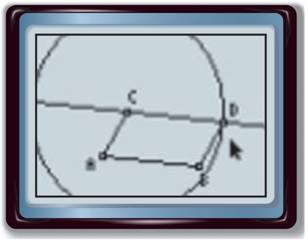
**Step 2** Use the Parallel tool on the F3 menu to construct a line parallel to the segment. Pressing ENTER will draw the line and a point on the line. Label the point *C*.



Steps 1 and 2

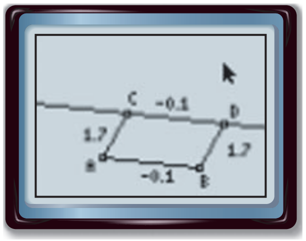
**Step 3** Access the Compass tool on the F3 menu. Set the compass to the length of $\overline{AB}$ by selecting one endpoint of the segment and then the other. Construct a circle centered at *C*.

**Step 4** Use the Point Intersection tool on the F2 menu to draw a point at the intersection of the line and the circle. Label the point *D*. Then use the Segment tool on the F2 menu to draw $\overline{AC}$ and $\overline{BD}$.



Steps 3 and 4

**Step 5** Use the Hide/Show tool on the F5 menu to hide the circle. Then access the Slope tool under Measure on the F5 menu. Display the slopes of $\overline{AB}$, $\overline{BD}$, $\overline{CD}$, and $\overline{AC}$.




Step 5

### ANALYZE THE RESULTS

1. What is the relationship between sides $\overline{AB}$ and $\overline{CD}$? Explain how you know.
2. What do you observe about the slopes of opposite sides of the quadrilateral? What type of quadrilateral is *ABDC*? Explain.
3. Click on point *A* and drag it to change the shape of *ABDC*. What do you observe?
4. **Make a conjecture** about a quadrilateral with a pair of opposite sides that are both congruent and parallel.
5. Use the graphing calculator to construct a quadrilateral with both pairs of opposite sides congruent. Then analyze the slopes of the sides of the quadrilateral. **Make a conjecture** based on your observations.



Other Calculator Keystrokes at geometryonline.com

# 6-3 Tests for Parallelograms

## Main Ideas

- Recognize the conditions that ensure a quadrilateral is a parallelogram.
- Prove that a set of points forms a parallelogram in the coordinate plane.

## GET READY for the Lesson

The roof of the covered bridge appears to be a parallelogram. Each pair of opposite sides looks as if they are the same length. How can we know for sure if this shape is really a parallelogram?

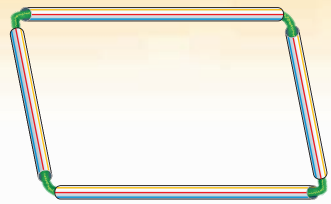
**Conditions for a Parallelogram** By definition, the opposite sides of a parallelogram are parallel. So, if a quadrilateral has each pair of opposite sides parallel, then it is a parallelogram. Other tests can be used to determine if a quadrilateral is a parallelogram.

## GEOMETRY LAB

### Testing for a Parallelogram

**MODEL**

- Cut two straws to one length and two other straws to a different length.
- Connect the straws by inserting a pipe cleaner in one end of each size of straw to form a quadrilateral like the one shown at the right.



- Shift the sides to form quadrilaterals of different shapes.

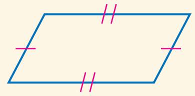
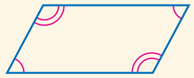
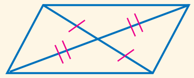
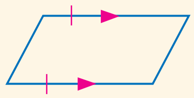
**ANALYZE**

1. Measure the distance between the opposite sides of the quadrilateral in at least three places. Repeat this process for several figures. What can you conclude about opposite sides?
2. Classify the quadrilaterals that you formed.
3. Compare the measures of pairs of opposite sides.
4. Measure the four angles in several of the quadrilaterals. What relationships do you find?

**MAKE A CONJECTURE**

5. What conditions are necessary to verify that a quadrilateral is a parallelogram?

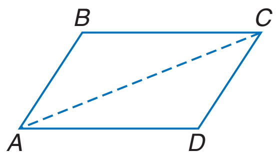
## THEOREMS — *Proving Parallelograms*

| | | Example |
|---|---|---|
| **6.9** | If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. **Abbreviation:** *If both pairs of opp. sides are ≅, then quad. is ▱.* |  |
| **6.10** | If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. **Abbreviation:** *If both pairs of opp. ∠s are ≅, then quad. is ▱.* |  |
| **6.11** | If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. **Abbreviation:** *If diag. bisect each other, then quad. is ▱.* |  |
| **6.12** | If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. **Abbreviation:** *If one pair of opp. sides is ∥ and ≅, then the quad. is a ▱.* |  |

You will prove Theorems 6.9 and 6.11 in Exercises 18 and 19, respectively.

## EXAMPLE 1 Write a Proof

**Proof** **Write a paragraph proof of Theorem 6.10.**




**Given:** $\angle A \cong \angle C$, $\angle B \cong \angle D$

**Prove:** $ABCD$ is a parallelogram.

**Paragraph Proof:**

Because two points determine a line, we can draw $\overline{AC}$. We now have two triangles. We know the sum of the angle measures of a triangle is 180, so the sum of the angle measures of two triangles is 360. Therefore, $m\angle A + m\angle B + m\angle C + m\angle D = 360$.

Since $\angle A \cong \angle C$ and $\angle B \cong \angle D$, $m\angle A = m\angle C$ and $m\angle B = m\angle D$. Substitute to find that $m\angle A + m\angle A + m\angle B + m\angle B = 360$, or $2(m\angle A) + 2(m\angle B) = 360$. Dividing each side of the equation by 2 yields $m\angle A + m\angle B = 180$. This means that consecutive angles are supplementary and $\overline{AD} \parallel \overline{BC}$.

Likewise, $2m\angle A + 2m\angle D = 360$, or $m\angle A + m\angle D = 180$. These consecutive supplementary angles verify that $\overline{AB} \parallel \overline{DC}$. Opposite sides are parallel, so $ABCD$ is a parallelogram.

**CHECK Your Progress**

1. **PROOF** Write a two-column proof of Theorem 6.12.

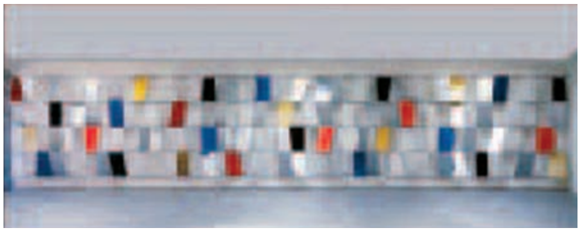
**Real-World Link**

Ellsworth Kelly created *Sculpture for a Large Wall* in 1957. The sculpture is made of 104 aluminum panels. The piece is over 65 feet long, 11 feet high, and 2 feet deep.

**Source:** www.moma.org

## Real-World EXAMPLE Properties of Parallelograms

**2** **ART** **Some panels in the sculpture appear to be parallelograms. Describe the information needed to determine whether these panels are parallelograms.**



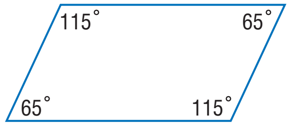
A panel is a parallelogram if both pairs of opposite sides are congruent, or if one pair of opposite sides is congruent and parallel. If the diagonals bisect each other, or if both pairs of opposite angles are congruent, then the panel is a parallelogram.

**CHECK Your Progress**

**2.** **ART** Tiffany has several pieces of tile that she is planning to make into a mosaic. How can she tell if the quadrilaterals are parallelograms?

## EXAMPLE Properties of Parallelograms

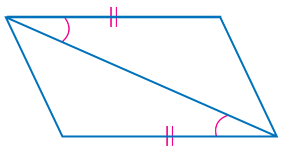
**3** **Determine whether the quadrilateral is a parallelogram. Justify your answer.**




Each pair of opposite angles has the same measure. Therefore, they are congruent. If both pairs of opposite angles are congruent, the quadrilateral is a parallelogram.

**CHECK Your Progress**

**3.**



A quadrilateral is a parallelogram if any one of the following is true.

### CONCEPT SUMMARY — *Tests for a Parallelogram*

1. Both pairs of opposite sides are parallel. (Definition)
2. Both pairs of opposite sides are congruent. (Theorem 6.9)
3. Both pairs of opposite angles are congruent. (Theorem 6.10)
4. Diagonals bisect each other. (Theorem 6.11)
5. A pair of opposite sides is both parallel and congruent. (Theorem 6.12)

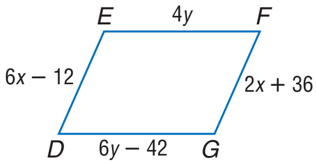
**Study Tip**

**Common Misconceptions**

If a quadrilateral meets one of the five tests, it is a parallelogram. All of the properties of parallelograms need not be shown.

## EXAMPLE 4 Find Measures

**ALGEBRA** **Find $x$ and $y$ so that the quadrilateral is a parallelogram.**



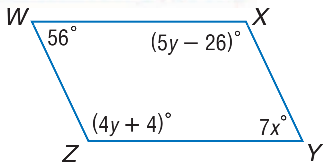

Opposite sides of a parallelogram are congruent.

| | | | |
|---|---|---|---|
| $\overline{EF} \cong \overline{DG}$ | Opp. sides of ▱ are ≅. | $\overline{DE} \cong \overline{FG}$ | Opp. sides of ▱ are ≅. |
| $EF = DG$ | Def. of ≅ segments | $DE = FG$ | Def. of ≅ segments |
| $4y = 6y - 42$ | Substitution | $6x - 12 = 2x + 36$ | Substitution |
| $-2y = -42$ | Subtract $6y$. | $4x = 48$ | Subtract $2x$ and add 12. |
| $y = 21$ | Divide by $-2$. | $x = 12$ | Divide by 4. |

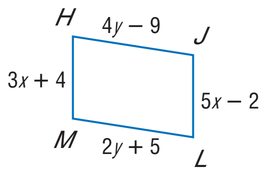
So, when $x$ is 12 and $y$ is 21, $DEFG$ is a parallelogram.

**CHECK Your Progress**

**4A.**




**4B.**




Online **Personal Tutor at** geometryonline.com

**Parallelograms on the Coordinate Plane** We can use the Distance Formula and the Slope Formula to determine if a quadrilateral is a parallelogram in the coordinate plane.

## EXAMPLE 5 Use Slope and Distance

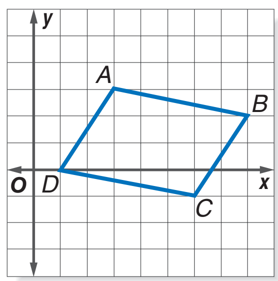
**Study Tip**

**Coordinate Geometry**

The Midpoint Formula can also be used to show that a quadrilateral is a parallelogram by Theorem 6.11.

**COORDINATE GEOMETRY** **Determine whether the figure with vertices $A(3, 3)$, $B(8, 2)$, $C(6, -1)$, $D(1, 0)$ is a parallelogram. Use the Slope Formula.**

If the opposite sides of a quadrilateral are parallel, then it is a parallelogram.




slope of $\overline{AB} = \frac{2-3}{8-3}$ or $\frac{-1}{5}$

slope of $\overline{DC} = \frac{-1-0}{6-1}$ or $\frac{-1}{5}$

slope of $\overline{AD} = \frac{3-0}{3-1}$ or $\frac{3}{2}$

slope of $\overline{BC} = \frac{-1-2}{6-8}$ or $\frac{3}{2}$

Since opposite sides have the same slope, $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. Therefore, $ABCD$ is a parallelogram by definition.

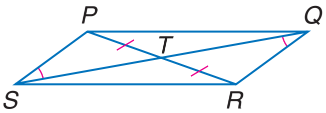
**CHECK Your Progress**

**5.** $F(-2, 4)$, $G(4, 2)$, $H(4, -2)$, $J(-2, -1)$; Midpoint Formula

## CHECK Your Understanding

**Example 1** (p. 334)

**1. PROOF** Write a two-column proof to prove that $PQRS$ is a parallelogram given that $\overline{PT} \cong \overline{TR}$ and $\angle TSP \cong \angle TQR$.




**Example 2** (p. 335)

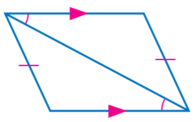
**2. ART** Texas artist Robert Rauschenberg created *Trophy II (for Teeny and Marcel Duchamp)* in 1960. The piece is a combination of several canvases. Describe one method to determine if the panels are parallelograms.

Robert Rauschenberg. *Trophy II (for Teeny and Marcel Duchamp)*, 1960. Oil, charcoal, paper, fabric, metal on canvas, drinking glass, metal chain, spoon, necktie. Collection Walker Art Center, Minneapolis. Gift of the T.B. Walker Foundation, 1970. Art © Robert Rauschenberg/Licensed by VAGA, New York, NY

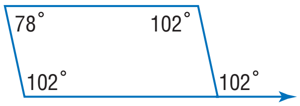
**Example 3** (p. 335)

**Determine whether each quadrilateral is a parallelogram. Justify your answer.**

**3.**



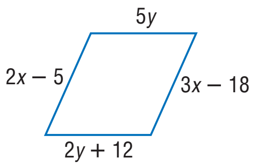
**4.**



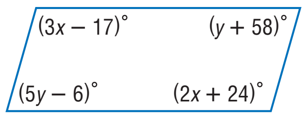

**Example 4** (p. 336)

**ALGEBRA Find $x$ and $y$ so that each quadrilateral is a parallelogram.**

**5.**




**6.**




**Example 5** (p. 336)

**COORDINATE GEOMETRY Determine whether the figure with the given vertices is a parallelogram. Use the method indicated.**

**7.** $B(0, 0)$, $C(4, 1)$, $D(6, 5)$, $E(2, 4)$; Slope Formula

**8.** $E(-4, -3)$, $F(4, -1)$, $G(2, 3)$, $H(-6, 2)$; Midpoint Formula

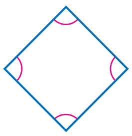
## Exercises

**HOMEWORK HELP**

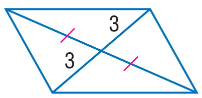
| For Exercises | See Examples |
|---|---|
| 9–14 | 3 |
| 15–17 | 2 |
| 18, 19 | 1 |
| 20–25 | 4 |
| 26–29 | 5 |

**Determine whether each quadrilateral is a parallelogram. Justify your answer.**

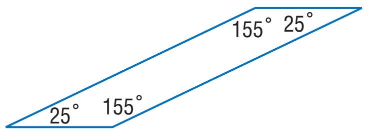
**9.**



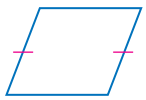
**10.**



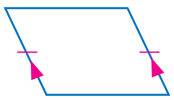

**11.**



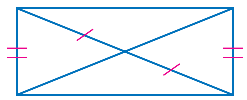

**12.**



**13.**

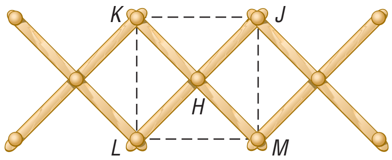


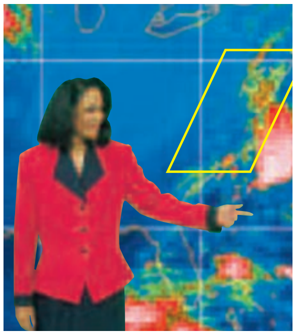
**14.**



**15. TANGRAMS** A tangram set consists of seven pieces: a small square, two small congruent right triangles, two large congruent right triangles, a medium-sized right triangle, and a quadrilateral. How can you determine the shape of the quadrilateral? Explain.

**16. STORAGE** Songan purchased an expandable hat rack that has 11 pegs. In the figure, $H$ is the midpoint of $\overline{KM}$ and $\overline{JL}$. What type of figure is $JKLM$? Explain.



**17. METEOROLOGY** To show the center of a storm, television stations superimpose a "watchbox" over the weather map. Describe how you can tell whether the watchbox is a parallelogram.

**Real-World Career**

**Atmospheric Scientist**

An atmospheric scientist, or meteorologist, uses math to study weather patterns. They can work for private companies, the Federal Government, or television stations.

Math Online

For more information, go to geometryonline.com.

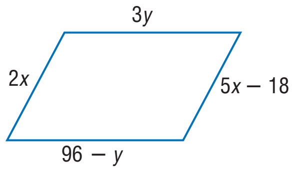
**PROOF** **Write a two-column proof of each theorem.**

**18.** Theorem 6.9

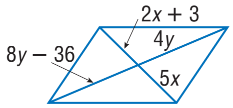
**19.** Theorem 6.11

**ALGEBRA** **Find $x$ and $y$ so that each quadrilateral is a parallelogram.**

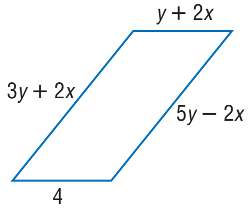
**20.**



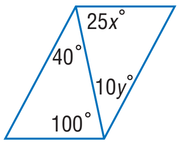

**21.**



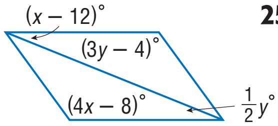

**22.**



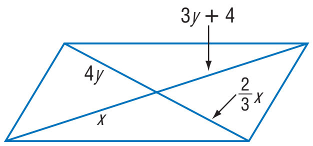

**23.**




**24.**




**25.**




**COORDINATE GEOMETRY** **Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.**

**26.** $B(-6, -3)$, $C(2, -3)$, $E(4, 4)$, $G(-4, 4)$; Midpoint Formula

**27.** $H(5, 6)$, $J(9, 0)$, $K(8, -5)$, $L(3, -2)$; Distance Formula

**28.** $C(-7, 3)$, $D(-3, 2)$, $F(0, -4)$, $G(-4, -3)$; Distance and Slope Formulas

**29.** $G(-2, 8)$, $H(4, 4)$, $J(6, -3)$, $K(-1, -7)$; Distance and Slope Formulas

**30.** Quadrilateral $MNPR$ has vertices $M(-6, 6)$, $N(-1, -1)$, $P(-2, -4)$, and $R(-5, -2)$. Determine how to move one vertex to make $MNPR$ a parallelogram.

**31.** Quadrilateral $QSTW$ has vertices $Q(-3, 3)$, $S(4, 1)$, $T(-1, -2)$, and $W(-5, -1)$. Determine how to move one vertex to make $QSTW$ a parallelogram.

**EXTRA PRACTICE**

See pages 811, 833.

Math Online

Self-Check Quiz at geometryonline.com

**COORDINATE GEOMETRY** **The coordinates of three of the vertices of a parallelogram are given. Find the possible coordinates for the fourth vertex.**

**32.** $A(1, 4)$, $B(7, 5)$, and $C(4, -1)$

**33.** $Q(-2, 2)$, $R(1, 1)$, and $S(-1, -1)$

**H.O.T. Problems**

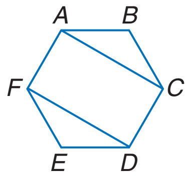
**34. REASONING** Felisha claims she discovered a new geometry theorem: a diagonal of a parallelogram bisects its angles. Determine whether this theorem is true. Find an example or counterexample.

**35. OPEN ENDED** Draw a parallelogram. Label the congruent angles. Explain how you determined it was a parallelogram.

36. **FIND THE ERROR** Carter and Shaniqua are describing ways to show that a quadrilateral is a parallelogram. Who is correct? Explain your reasoning.

| Carter | Shaniqua |
|---|---|
| A quadrilateral is a parallelogram if one pair of sides is congruent and one pair of opposite sides is parallel. | A quadrilateral is a parallelogram if one pair of opposite sides is congruent and parallel. |

37. **CHALLENGE** Write a proof to prove that *FDCA* is a parallelogram if *ABCDEF* is a regular hexagon.




38. ***Writing in Math*** Describe the information needed to prove that a quadrilateral is a parallelogram.

## STANDARDIZED TEST PRACTICE

39. If sides $\overline{AB}$ and $\overline{DC}$ of quadrilateral *ABCD* are parallel, which additional information would be sufficient to prove that quadrilateral *ABCD* is a parallelogram?

**A** $\overline{AB} \cong \overline{AC}$ **C** $\overline{AC} \cong \overline{BD}$

**B** $\overline{AB} \cong \overline{DC}$ **D** $\overline{AD} \cong \overline{BC}$

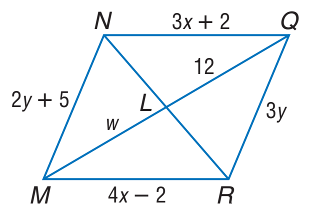
40. **REVIEW** Jarod's average driving speed for a 5-hour trip was 58 miles per hour. During the first 3 hours, he drove 50 miles per hour. What was his average speed in miles per hour for the last 2 hours of his trip?

**F** 70 **H** 60

**G** 66 **J** 54

## Spiral Review

**Use ▱*NQRM* to find each measure or value.** (Lesson 6–2)




41. *w* 42. *x*

43. *NQ* 44. *QR*

**The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.** (Lesson 6–1)

45. 135 46. 144 47. 168 48. 162

49. **ATHLETICS** Maddox was at the gym for just over two hours. He swam laps in the pool and lifted weights. Prove that he did one of these activities for more than an hour. (Lesson 5–3)

## GET READY for the Next Lesson

**PREREQUISITE SKILL** **Use slope to determine whether $\overline{AB}$ and $\overline{BC}$ are *perpendicular* or *not perpendicular*.** (Lesson 3–3)

50. $A(2, 5)$, $B(6, 3)$, $C(8, 7)$ 51. $A(-1, 2)$, $B(0, 7)$, $C(4, 1)$

# 6-4 Rectangles

## Main Ideas

- Recognize and apply properties of rectangles.
- Determine whether parallelograms are rectangles.

## New Vocabulary

rectangle

**GET READY for the Lesson**

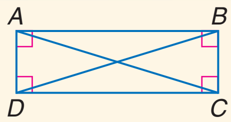
Many sports are played on fields marked by parallel lines. A tennis court has parallel serving lines for each player. Parallel lines divide the court for singles and doubles play. The service box is marked by perpendicular lines.

**Properties of Rectangles** A **rectangle** is a quadrilateral with four right angles. Since both pairs of opposite angles are congruent, it follows that it is a special type of parallelogram. Thus, a rectangle has all the properties of a parallelogram. In addition, the diagonals of a rectangle are also congruent.

**THEOREM 6.13**

If a parallelogram is a rectangle, then the diagonals are congruent.

**Abbreviation:** *If ▭ is rectangle, diag. are ≅.*




$\overline{AC} \cong \overline{BD}$

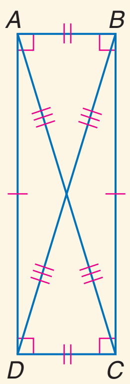
You will prove Theorem 6.13 in Exercise 33.

If a quadrilateral is a rectangle, then the following properties are true.

**KEY CONCEPT** — *Rectangle*

**Words** A rectangle is a quadrilateral with four right angles.

| Properties | Examples |
|---|---|
| 1. Opposite sides are congruent and parallel. | $\overline{AB} \cong \overline{DC}$  $\overline{AB} \parallel \overline{DC}$<br>$\overline{BC} \cong \overline{AD}$  $\overline{BC} \parallel \overline{AD}$ |
| 2. Opposite angles are congruent. | $\angle A \cong \angle C$<br>$\angle B \cong \angle D$ |
| 3. Consecutive angles are supplementary. | $m\angle A + m\angle B = 180$<br>$m\angle B + m\angle C = 180$<br>$m\angle C + m\angle D = 180$<br>$m\angle D + m\angle A = 180$ |
| 4. Diagonals are congruent and bisect each other. | $\overline{AC} \cong \overline{BD}$<br>$\overline{AC}$ and $\overline{BD}$ bisect each other. |
| 5. All four angles are right angles. | $m\angle DAB = m\angle BCD =$<br>$m\angle ABC = m\angle ADC = 90$ |

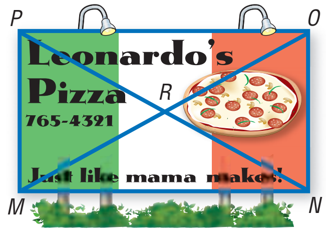





Simon Bruty/Getty Images

## Real-World EXAMPLE — Diagonals of a Rectangle

**1** **ALGEBRA** **Quadrilateral *MNOP* is a billboard in the shape of a rectangle. If $MO = 6x + 14$ and $PN = 9x + 5$, find $x$. Then find $NR$.**




$$\overline{MO} \cong \overline{PN}$$ Diagonals of a rectangle are ≅.

$$MO = PN$$ Definition of congruent segments

$$6x + 14 = 9x + 5$$ Substitution

$$14 = 3x + 5$$ Subtract $6x$ from each side.

$$9 = 3x$$ Subtract 5 from each side.

$$3 = x$$ Divide each side by 3.

$$NR = \frac{1}{2}PN$$ Diagonals bisect each other.

$$= \frac{1}{2}(9x + 5)$$ Substitution

$$= \frac{1}{2}(9 \cdot 3 + 5)$$ Substitute 3 for $x$.

$$= \frac{1}{2}(27 + 5)$$

$$= \frac{1}{2}(32)$$

$$= 16$$

**CHECK Your Progress**

**1.** Refer to rectangle *MNOP*. If $MO = 4y + 12$ and $PR = 3y - 5$, find $y$.

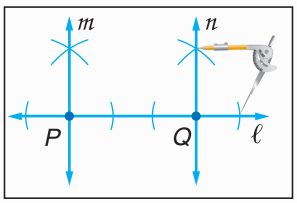
Concepts in Motion
**Animation**
geometryonline.com

Rectangles can be constructed using perpendicular lines.

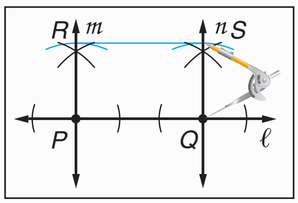
## CONSTRUCTION

### Rectangle

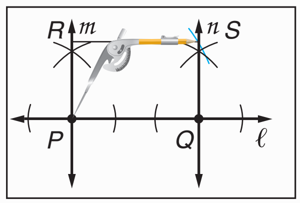
**Step 1** Use a straightedge to draw line $\ell$. Label points $P$ and $Q$ on $\ell$. Now construct lines perpendicular to $\ell$ through $P$ and through $Q$. Label them $m$ and $n$.




**Step 2** Place the compass point at $P$ and mark off a segment on $m$. Using the same compass setting, place the compass at $Q$ and mark a segment on $n$. Label these points $R$ and $S$. Draw $\overline{RS}$.

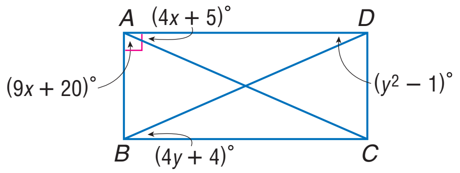



**Step 3** Locate the compass setting that represents $PS$ and compare to the setting for $QR$. The measures should be the same.



## EXAMPLE 2 Angles of a Rectangle

**ALGEBRA Quadrilateral *ABCD* is a rectangle. Find *y*.**




Since a rectangle is a parallelogram, opposite sides are parallel. So, alternate interior angles are congruent.

| | |
|---|---|
| $\angle ADB \cong \angle CBD$ | Alternate Interior Angles Theorem |
| $m\angle ADB = m\angle CBD$ | Definition of $\cong$ angles |
| $y^2 - 1 = 4y + 4$ | Substitution |
| $y^2 - 4y - 5 = 0$ | Subtract 4*y* and 4 from each side. |
| $(y - 5)(y + 1) = 0$ | Factor. |
| $y - 5 = 0 \qquad y + 1 = 0$ | |
| $y = 5 \qquad y = -1$ | Disregard $y = -1$ because it yields angle measures of 0. |

**CHECK Your Progress**

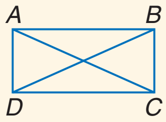
**2.** Refer to rectangle *ABCD*. Find *x*.

**Prove That Parallelograms Are Rectangles** The converse of Theorem 6.13 is also true.

### THEOREM 6.14

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

**Abbreviation:** *If diagonals of ▱ are ≅, ▱ is a rectangle.*




$\overline{AC} \cong \overline{BD}$

You will prove Theorem 6.14 in Exercise 34.

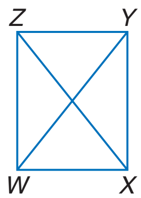
**Real-World Link**

It is important to square a window frame because over time the opening may have become "out-of-square." If the window is not properly situated in the framed opening, air and moisture can leak through cracks.

**Source:** www.supersealwindows.com/guide/measurement

## Real-World EXAMPLE 3 Diagonals of a Parallelogram

**WINDOWS Trent is building a tree house for his younger brother. He has measured the window opening to be sure that the opposite sides are congruent. He measures the diagonals to make sure that they are congruent. This is called *squaring* the frame. How does he know that the corners are 90° angles?**




First draw a diagram and label the vertices. We know that $\overline{WX} \cong \overline{ZY}$, $\overline{XY} \cong \overline{WZ}$, and $\overline{WY} \cong \overline{XZ}$.

Because $\overline{WX} \cong \overline{ZY}$ and $\overline{XY} \cong \overline{WZ}$, *WXYZ* is a parallelogram.

$\overline{XZ}$ and $\overline{WY}$ are diagonals and they are congruent. A parallelogram with congruent diagonals is a rectangle. So, the corners are 90° angles.

**CHECK Your Progress**

**3. CRAFTS** Antonia is making her own picture frame. How can she determine if the measure of each corner is 90°?

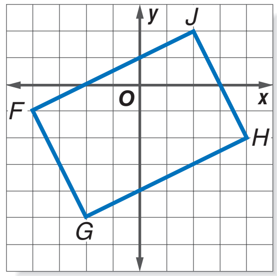
**Study Tip**

**Rectangles and Parallelograms**

A rectangle is a parallelogram, but a parallelogram is not necessarily a rectangle.

## EXAMPLE 4 Rectangle on a Coordinate Plane

**COORDINATE GEOMETRY Quadrilateral $FGHJ$ has vertices $F(-4, -1)$, $G(-2, -5)$, $H(4, -2)$, and $J(2, 2)$. Determine whether $FGHJ$ is a rectangle.**




**Method 1** Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to see if consecutive sides are perpendicular.

slope of $\overline{FJ} = \frac{2-(-1)}{2-(-4)}$ or $\frac{1}{2}$

slope of $\overline{GH} = \frac{-2-(-5)}{4-(-2)}$ or $\frac{1}{2}$

slope of $\overline{FG} = \frac{-5-(-1)}{-2-(-4)}$ or $-2$

slope of $\overline{JH} = \frac{-2-2}{4-2}$ or $-2$

Because $\overline{FJ} \parallel \overline{GH}$ and $\overline{FG} \parallel \overline{JH}$, quadrilateral $FGHJ$ is a parallelogram.

The product of the slopes of consecutive sides is $-1$. This means that $\overline{FJ} \perp \overline{FG}$, $\overline{FJ} \perp \overline{JH}$, $\overline{JH} \perp \overline{GH}$, and $\overline{FG} \perp \overline{GH}$. The perpendicular segments create four right angles. Therefore, by definition $FGHJ$ is a rectangle.

**Cross-Curricular Project**

Math Online You can use a rectangle with special dimensions to discover the golden mean. Visit geometryonline.com.

**Method 2** Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, to determine whether opposite sides are congruent.

First, we must show that quadrilateral $FGHJ$ is a parallelogram.

$$FJ = \sqrt{(-4-2)^2 + (-1-2)^2} = \sqrt{36+9} = \sqrt{45} \text{ or } 3\sqrt{5}$$

$$GH = \sqrt{(-2-4)^2 + [-5-(-2)]^2} = \sqrt{36+9} = \sqrt{45} \text{ or } 3\sqrt{5}$$

$$FG = \sqrt{[-4-(-2)]^2 + [-1-(-5)]^2} = \sqrt{4+16} = \sqrt{20} \text{ or } 2\sqrt{5}$$

$$JH = \sqrt{(2-4)^2 + [2-(-2)]^2} = \sqrt{4+16} = \sqrt{20} \text{ or } 2\sqrt{5}$$

Since each pair of opposite sides of the quadrilateral have the same measure, they are congruent. Quadrilateral $FGHJ$ is a parallelogram.

$$FH = \sqrt{(-4-4)^2 + [-1-(-2)]^2} = \sqrt{64+1} = \sqrt{65}$$

$$GJ = \sqrt{(-2-2)^2 + (-5-2)^2} = \sqrt{16+49} = \sqrt{65}$$

The length of each diagonal is $\sqrt{65}$. Since the diagonals are congruent, $FGHJ$ is a rectangle by Theorem 6.14.

**CHECK Your Progress**

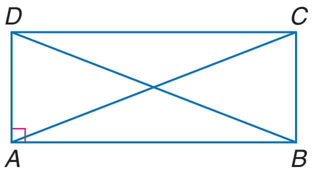
4. **COORDINATE GEOMETRY** Quadrilateral $JKLM$ has vertices $J(-10, 2)$, $K(-8, -6)$, $L(5, -3)$, and $M(2, 5)$. Determine whether $JKLM$ is a rectangle. Justify your answer.

Online **Personal Tutor at** geometryonline.com

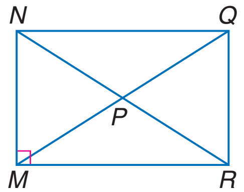
## CHECK Your Understanding

**Example 1** (p. 341)

**1. ALGEBRA** *ABCD* is a rectangle. If $AC = 30 - x$ and $BD = 4x - 60$, find $x$.

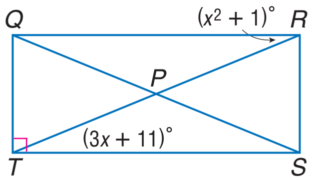



**2. ALGEBRA** *MNQR* is a rectangle. If $NR = 2x + 10$ and $NP = 2x - 30$, find *MP*.




**Example 2** (p. 342)

**ALGEBRA Quadrilateral *QRST* is a rectangle. Find each value or measure.**




**3.** $x$

**4.** $m\angle RPS$

**Example 3** (p. 342)

**5. FRAMING** Mrs. Walker has a rectangular picture that is 12 inches by 48 inches. Because this is not a standard size, a special frame must be built. What can the framer do to guarantee that the frame is a rectangle? Justify your reasoning.

**Example 4** (p. 343)

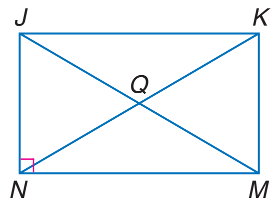
**6. COORDINATE GEOMETRY** Quadrilateral *EFGH* has vertices $E(-4, -3)$, $F(3, -1)$, $G(2, 3)$, and $H(-5, 1)$. Determine whether *EFGH* is a rectangle.

## Exercises

**HOMEWORK HELP**

| For Exercises | See Examples |
|---|---|
| 7–12 | 1 |
| 13–21 | 2 |
| 22, 23 | 3 |
| 24–31 | 4 |

**ALGEBRA Quadrilateral *JKMN* is a rectangle.**




**7.** If $NQ = 5x - 3$ and $QM = 4x + 6$, find *NK*.

**8.** If $NQ = 2x + 3$ and $QK = 5x - 9$, find *JQ*.

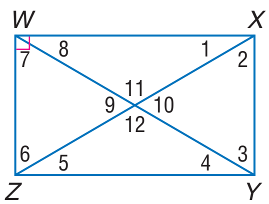
**9.** If $NM = 8x - 14$ and $JK = x^2 + 1$, find *JK*.

**10.** If $m\angle NJM = 2x - 3$ and $m\angle KJM = x + 5$, find $x$.

**11.** If $m\angle NKM = x^2 + 4$ and $m\angle KNM = x + 30$, find $m\angle JKN$.

**12.** If $m\angle JKN = 2x^2 + 2$ and $m\angle NKM = 14x$, find $x$.

***WXYZ* is a rectangle. Find each measure if $m\angle 1 = 30$.**



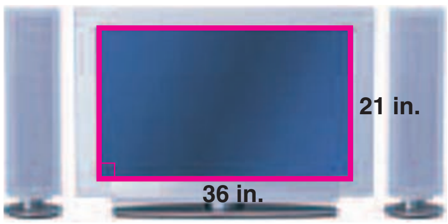

**13.** $m\angle 2$ **14.** $m\angle 3$ **15.** $m\angle 4$

**16.** $m\angle 5$ **17.** $m\angle 6$ **18.** $m\angle 7$

**19.** $m\angle 8$ **20.** $m\angle 9$ **21.** $m\angle 12$

**22. PATIOS** A contractor has been hired to pour a rectangular concrete patio. How can he be sure that the frame in which he will pour the concrete is rectangular?

**23. TELEVISION** Television screens are measured on the diagonal. What is the measure of the diagonal of this screen?



**Real-World Link**

Myrtle Beach, South Carolina, has 45 miniature golf courses within 20 miles of the Grand Strand, the region that is home to Myrtle Beach and several other towns.

**Source:** U.S. ProMini Golf Association

**COORDINATE GEOMETRY** **Determine whether *DFGH* is a rectangle given each set of vertices. Justify your answer.**

**24.** $D(9, -1)$, $F(9, 5)$, $G(-6, 5)$, $H(-6, 1)$

**25.** $D(6, 2)$, $F(8, -1)$, $G(10, 6)$, $H(12, 3)$

**26.** $D(-4, -3)$, $F(-5, 8)$, $G(6, 9)$, $H(7, -2)$

**COORDINATE GEOMETRY** **The vertices of *WXYZ* are $W(2, 4)$, $X(-2, 0)$, $Y(-1, -7)$, and $Z(9, 3)$.**

**27.** Find $WY$ and $XZ$.

**28.** Find the coordinates of the midpoints of $\overline{WY}$ and $\overline{XZ}$.

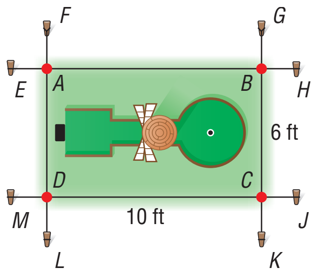
**29.** Is *WXYZ* a rectangle? Explain.

**COORDINATE GEOMETRY** **The vertices of parallelogram *ABCD* are $A(-4, -4)$, $B(2, -1)$, $C(0, 3)$, and $D(-6, 0)$.**

**30.** Determine whether *ABCD* is a rectangle.

**31.** If *ABCD* is a rectangle and *E*, *F*, *G*, and *H* are midpoints of its sides, what can you conclude about *EFGH*?

**32.** **MINIATURE GOLF** The windmill section of a miniature golf course will be a rectangle 10 feet long and 6 feet wide. Suppose the contractor placed stakes and strings to mark the boundaries with the corners at *A*, *B*, *C*, and *D*. The contractor measured *BD* and *AC* and found that $AC > BD$. Describe where to move the stakes *L* and *K* to make *ABCD* a rectangle. Explain.

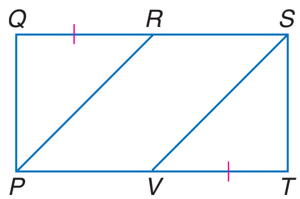



**PROOF** **Write a two-column proof.**

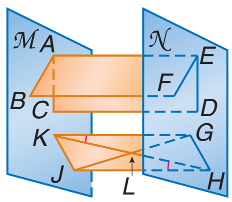
**33.** Theorem 6.13

**34.** Theorem 6.14

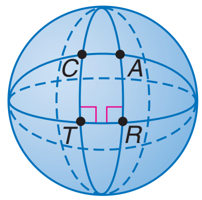
**35.** **Given:** *PQST* is a rectangle.
$\overline{QR} \cong \overline{VT}$
**Prove:** $\overline{PR} \cong \overline{VS}$




**36.** **Given:** *DEAC* and *FEAB* are rectangles.
$\angle GKH \cong \angle JHK$
$\overline{GJ}$ and $\overline{HK}$ intersect at *L*.
**Prove:** *GHJK* is a parallelogram.




**NON-EUCLIDEAN GEOMETRY** **The figure shows a *Saccheri quadrilateral* on a sphere. Note that it has four sides with $\overline{CT} \perp \overline{TR}$, $\overline{AR} \perp \overline{TR}$, and $\overline{CT} \cong \overline{AR}$.**




**37.** Is $\overline{CT}$ parallel to $\overline{AR}$? Explain.

**38.** How does *AC* compare to *TR*?

**39.** Can a rectangle exist in non-Euclidean geometry? Explain.

**40.** **RESEARCH** Use the Internet or another source to investigate the similarities and differences between non-Euclidean geometry and Euclidean geometry.

**EXTRA PRACTICE**
See pages 812, 833.

**Math Online**
Self-Check Quiz at
geometryonline.com

**Study Tip**

**Look Back**
To review **Non-Euclidean geometry**, refer to Extend Lesson 3-6.

## H.O.T. Problems

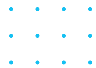
**41. REASONING** Draw a counterexample to the statement *If the diagonals are congruent, the quadrilateral is a rectangle.*

**42. OPEN ENDED** Draw two congruent right triangles with a common hypotenuse. Do the legs form a rectangle? Justify your answer.

**43. FIND THE ERROR** McKenna and Consuelo are defining a rectangle for an assignment. Who is correct? Explain.

| McKenna | Consuelo |
|---|---|
| A rectangle is a parallelogram with one right angle. | A rectangle has a pair of parallel opposite sides and a right angle. |

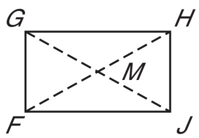
**44. CHALLENGE** Using four of the twelve points as corners, how many rectangles can be drawn?



**45. *Writing in Math*** How can you determine whether a parallelogram is a rectangle? Explain your reasoning.

## STANDARDIZED TEST PRACTICE

**46.** If $FJ = -3x + 5y$, $FM = 3x + y$, $GH = 11$, and $GM = 13$, what values of $x$ and $y$ make parallelogram $FGHJ$ a rectangle?




**A** $x = 3, y = 4$ **C** $x = 7, y = 8$

**B** $x = 4, y = 3$ **D** $x = 8, y = 7$

**47. REVIEW** A rectangular playground is surrounded by an 80-foot fence. One side of the playground is 10 feet longer than the other. Which of the following equations could be used to find $s$, the shorter side of the playground?

**F** $10s + s = 80$

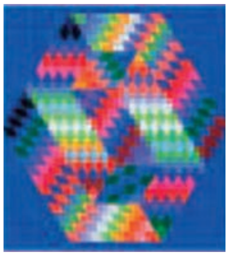
**G** $4s + 10 = 80$

**H** $s(s + 10) = 80$

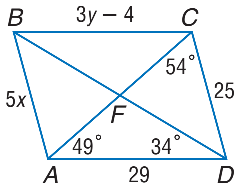
**J** $2(s + 10) + 2s = 80$

## Spiral Review

**48. OPTIC ART** Victor Vasarely created art in the op art style. This piece *AMBIGU-B*, consists of multi-colored parallelograms. Describe one method to ensure that the shapes are parallelograms. (Lesson 6-3)



**For Exercises 49–54, use ▱*ABCD*. Find each measure or value.** (Lesson 6–2)




**49.** $m\angle AFD$ **50.** $m\angle CDF$

**51.** $y$ **52.** $x$

## GET READY for the Next Lesson

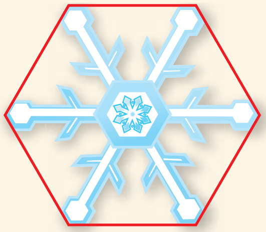
**PREREQUISITE SKILL Find the distance between each pair of points.** (Lesson 1–4)

**53.** $(1, -2), (-3, 1)$ **54.** $(-5, 9), (5, 12)$ **55.** $(1, 4), (22, 24)$

# CHAPTER 6 Mid-Chapter Quiz

**Lessons 6-1 through 6-4**

**1.** **SNOW** The snowflake pictured is a regular hexagon. Find the sum of the measures of the interior angles of the hexagon. (Lesson 6-1)



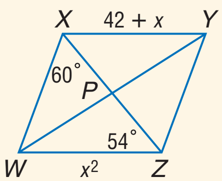
**2.** The measure of an interior angle of a regular polygon is $147\frac{3}{11}$. Find the number of sides in the polygon. (Lesson 6-1)

**3.** How many degrees are there in the sum of the exterior angles of a dodecagon? (Lesson 6-1)

**4.** Find the measure of each exterior angle of a regular pentagon. (Lesson 6-1)

**5.** If each exterior angle of a regular polygon measures 40°, how many sides does the polygon have? (Lesson 6-1)

**Use ▱*WXYZ* to find each measure.** (Lesson 6-2)




**6.** $WZ =$ ___?___

**7.** $m\angle XYZ =$ ___?___

**8.** **MULTIPLE CHOICE** Two opposite angles of a parallelogram measure $(5x - 25)°$ and $(3x + 5)°$. Find the measures of the angles. (Lesson 6-2)

**A** 50, 50

**B** 55, 125

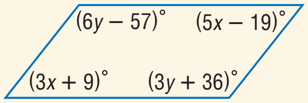
**C** 90, 90

**D** 109, 71

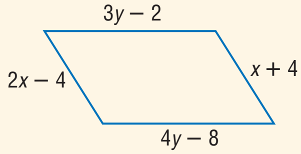
**9.** Parallelogram *JKLM* has vertices $J(0, 7)$, $K(9, 7)$, and $L(6, 0)$. Find the coordinates of *M*. (Lesson 6-2)

**ALGEBRA Find *x* and *y* so that each quadrilateral is a parallelogram.** (Lesson 6-3)

**10.**




**11.**

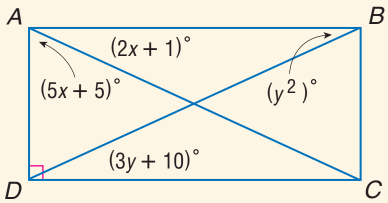



**COORDINATE GEOMETRY Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.** (Lesson 6-3)

**12.** $Q(-3, -6)$, $R(2, 2)$, $S(-1, 6)$, $T(-5, 2)$; Distance and Slope formulas

**13.** $W(-6, -5)$, $X(-1, -4)$, $Y(0, -1)$, $Z(-5, -2)$; Midpoint formula

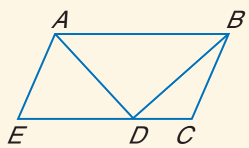
**Quadrilateral *ABCD* is a rectangle.** (Lesson 6-4)




**14.** Find *x*.

**15.** Find *y*.

**16.** **MULTIPLE CHOICE** In the figure, quadrilateral *ABCE* is a parallelogram. If $\angle ADE \cong \angle BDC$, which of the following *must* be true? (Lesson 6-4)




**F** $\overline{AD} \cong \overline{DB}$

**G** $\overline{ED} \cong \overline{AD}$

**H** $\overline{ED} \cong \overline{DC}$

**J** $\overline{AE} \cong \overline{DC}$

# 6-5 Rhombi and Squares

## Main Ideas

- Recognize and apply the properties of rhombi.
- Recognize and apply the properties of squares.

## New Vocabulary

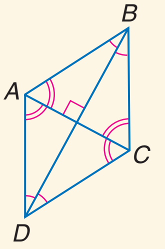
rhombus
square

## GET READY for the Lesson

Professor Stan Wagon at Macalester College in St. Paul, Minnesota, developed a bicycle with square wheels. There are two front wheels so the rider can balance without turning the handlebars. Riding over a specially curved road ensures a smooth ride.

**Properties of Rhombi** A square is a special type of parallelogram called a rhombus. A **rhombus** is a quadrilateral with all four sides congruent. All of the properties of parallelograms can be applied to rhombi. There are three other characteristics of rhombi described in the following theorems.

**THEOREMS** *Rhombus*

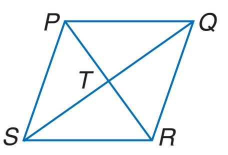
| | Theorem | Examples | |
|---|---|---|---|
| **6.15** | The diagonals of a rhombus are perpendicular. | $\overline{AC} \perp \overline{BD}$ |   |
| **6.16** | If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem 6.15) | If $\overline{BD} \perp \overline{AC}$ then $\square ABCD$ is a rhombus. | |
| **6.17** | Each diagonal of a rhombus bisects a pair of opposite angles. | $\angle DAC \cong \angle BAC \cong \angle DCA \cong \angle BCA$<br>$\angle ABD \cong \angle CBD \cong \angle ADB \cong \angle CDB$ | |

You will prove Theorems 6.16 and 6.17 in Exercises 9 and 10, respectively.

### EXAMPLE 1 Proof of Theorem 6.15

**Given:** $PQRS$ is a rhombus.

**Prove:** $\overline{PR} \perp \overline{SQ}$




**Paragraph Proof:**

By the definition of a rhombus, $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{PS}$.



Courtesy Professor Stan Wagon/Photo by Deanna Haunsperger

**Study Tip**

**Proof**

Since a rhombus has four congruent sides, one diagonal separates the rhombus into two congruent isosceles triangles. Drawing two diagonals separates the rhombus into four congruent right triangles.

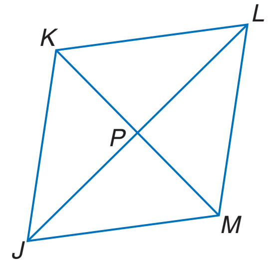
A rhombus is a parallelogram and the diagonals of a parallelogram bisect each other, so $\overline{QS}$ bisects $\overline{PR}$ at $T$. Thus, $\overline{PT} \cong \overline{RT}$. $\overline{QT} \cong \overline{QT}$ because congruence of segments is reflexive. Thus, $\triangle PQT \cong \triangle RQT$ by SSS. $\angle QTP \cong \angle QTR$ by CPCTC. $\angle QTP$ and $\angle QTR$ also form a linear pair. Two congruent angles that form a linear pair are right angles. $\angle QTP$ is a right angle, so $\overline{PR} \perp \overline{SQ}$ by the definition of perpendicular lines.

**CHECK Your Progress**

**1. PROOF** Write a paragraph proof.

**Given:** $JKLM$ is a parallelogram. $\triangle JKL$ is isosceles.

**Prove:** $JKLM$ is a rhombus.

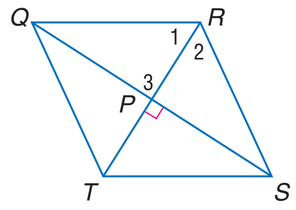



**Reading Math**

**Rhombi** The plural form of rhombus is *rhombi,* pronounced ROM-bye.

## EXAMPLE Measures of a Rhombus

**2 ALGEBRA Use rhombus $QRST$ and the given information to find the value of each variable.**




**a. Find $y$ if $m\angle 3 = y^2 - 31$.**

$m\angle 3 = 90$ The diagonals of a rhombus are perpendicular.

$y^2 - 31 = 90$ Substitution

$y^2 = 121$ Add 31 to each side.

$y = \pm 11$ Take the square root of each side.

The value of $y$ can be 11 or $-11$.

**b. Find $m\angle TQS$ if $m\angle RST = 56$.**

$m\angle TQR = m\angle RST$ Opposite angles are congruent.

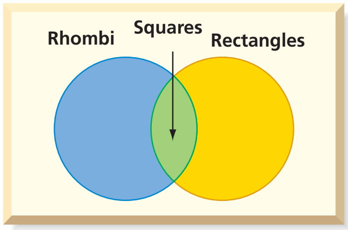
$m\angle TQR = 56$ Substitution

The diagonals of a rhombus bisect the angles. So, $m\angle TQS$ is $\frac{1}{2}(56)$ or 28.

**CHECK Your Progress**

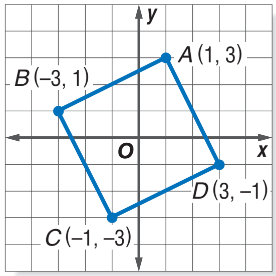
**2. ALGEBRA** Use rhombus $QRST$ to find $m\angle QTS$ if $m\angle 2 = 57$.

**Properties of Squares** If a quadrilateral is both a rhombus and a rectangle, then it is a **square**. All of the properties of parallelograms and rectangles can be applied to squares.



## EXAMPLE Squares

**3 COORDINATE GEOMETRY Determine whether parallelogram *ABCD* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.**




**Explore** Plot the vertices on a coordinate plane.

**Plan** If the diagonals are perpendicular, then *ABCD* is either a rhombus or a square. The diagonals of a rectangle are congruent. If the diagonals are congruent and perpendicular, then *ABCD* is a square.

**Solve** Use the Distance Formula to compare the lengths of the diagonals.

$$DB = \sqrt{[3-(-3)]^2 + (-1-1)^2} = \sqrt{36+4} = \sqrt{40} \text{ or } 2\sqrt{10}$$

$$AC = \sqrt{[1-(-1)]^2 + [3-(-3)]^2} = \sqrt{4+36} = \sqrt{40} \text{ or } 2\sqrt{10}$$

Use slope to determine whether the diagonals are perpendicular.

$$\text{slope of } \overline{DB} = \frac{1-(-1)}{-3-3} \text{ or } -\frac{1}{3} \qquad \text{slope of } \overline{AC} = \frac{-3-3}{-1-1} \text{ or } 3$$

Since the slope of $\overline{AC}$ is the negative reciprocal of the slope of $\overline{DB}$, the diagonals are perpendicular. $\overline{DB}$ and $\overline{AC}$ have the same measure, so the diagonals are congruent. *ABCD* is a rhombus, a rectangle, and a square.

**Check** You can verify that *ABCD* is a square by finding the measure and slope of each side. All four sides are congruent and consecutive sides are perpendicular.

### CHECK Your Progress

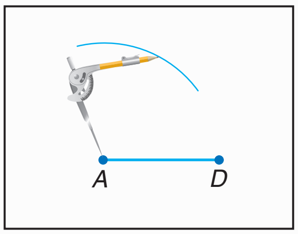
**3. COORDINATE GEOMETRY** Given the vertices *J*(5, 0), *K*(8, −11), *L*(−3, −14), *M*(−6, −3), determine whether parallelogram *JKLM* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

Concepts in Motion
Animation
geometryonline.com

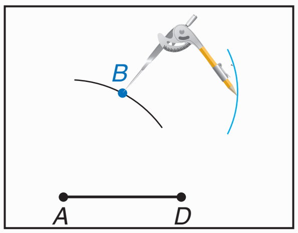
## CONSTRUCTION

### Rhombus

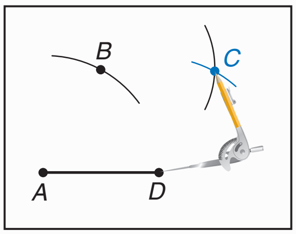
**Step 1** Draw any segment $\overline{AD}$. Place the compass point at *A*, open to the width of *AD*, and draw an arc above $\overline{AD}$.



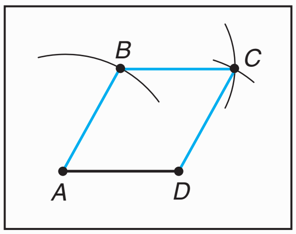

**Step 2** Label any point on the arc as *B*. Using the same setting, place the compass at *B*, and draw an arc to the right of *B*.




**Step 3** Place the compass at *D*, and draw an arc to intersect the arc drawn from *B*. Label the point of intersection *C*.



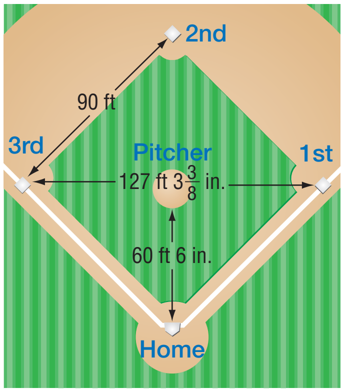

**Step 4** Use a straightedge to draw $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$.




**Conclusion:** Since all of the sides are congruent, quadrilateral *ABCD* is a rhombus.

## Real-World EXAMPLE — Diagonals of a Square

**4** **BASEBALL** **The infield of a baseball diamond is a square, as shown at the right. Is the pitcher's mound located in the center of the infield? Explain.**




Since a square is a parallelogram, the diagonals bisect each other. Since a square is a rhombus, the diagonals are congruent. Therefore, the distance from first base to third base is equal to the distance between home plate and second base.

Thus, the distance from home plate to the center of the infield is 127 feet $3\frac{3}{8}$ inches divided by 2 or 63 feet $7\frac{11}{16}$ inches. This distance is longer than the distance from home plate to the pitcher's mound so the pitcher's mound is not located in the center of the field. It is about 3 feet closer to home.

### CHECK Your Progress

4. **STAINED GLASS** Kathey is designing a stained glass piece with rhombus-shaped tiles. Describe how she can determine if the tiles are rhombi.

Online Personal Tutor at geometryonline.com

If a quadrilateral is a rhombus or a square, then the following properties are true.

### Study Tip

**Square and Rhombus**

A square is a rhombus, but a rhombus is not necessarily a square.

### CONCEPT SUMMARY — *Properties of Rhombi and Squares*

| Rhombi | Squares |
|---|---|
| 1. A rhombus has all the properties of a parallelogram. | 1. A square has all the properties of a parallelogram. |
| 2. All sides are congruent. | 2. A square has all the properties of a rectangle. |
| 3. Diagonals are perpendicular. | 3. A square has all the properties of a rhombus. |
| 4. Diagonals bisect the angles of the rhombus. | |

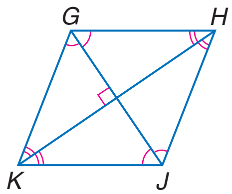
## CHECK Your Understanding

Example 1 (p. 349)

1. **PROOF** Write a two-column proof.

**Given:** $\triangle KGH$, $\triangle HJK$, $\triangle GHJ$, and $\triangle JKG$ are isosceles.

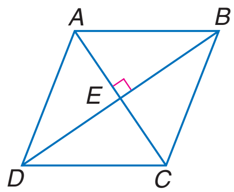
**Prove:** $GHJK$ is a rhombus.




Example 2 (p. 349)

**ALGEBRA** **In rhombus $ABCD$, $AB = 2x + 3$ and $BC = 5x$.**

2. Find $x$.
3. Find $AD$.
4. Find $m\angle AEB$.
5. Find $m\angle BCD$ if $m\angle ABC = 83.2$.



Example 3 (p. 350)

**COORDINATE GEOMETRY** **Given each set of vertices, determine whether ▱*MNPQ* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.**

**6.** $M(0, 3)$, $N(-3, 0)$, $P(0, -3)$, $Q(3, 0)$

**7.** $M(-4, 0)$, $N(-3, 3)$, $P(2, 2)$, $Q(1, -1)$

Example 4 (p. 351)

**8.** **REMODELING** The Steiner family is remodeling their kitchen. Each side of the floor measures 10 feet. What other measurements should be made to determine whether the floor is a square?

## Exercises

| HOMEWORK HELP | |
| --- | --- |
| For Exercises | See Examples |
| 9–14 | 1 |
| 15–18 | 2 |
| 19–22 | 3 |
| 23-24 | 4 |

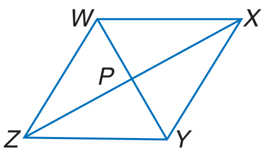
**PROOF** **Write a paragraph proof for each theorem.**

**9.** Theorem 6.16

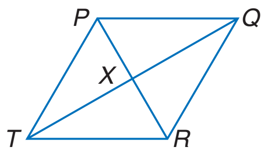
**10.** Theorem 6.17

**PROOF** **Write a two-column proof.**

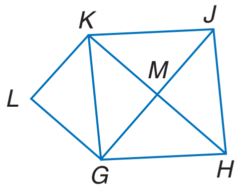
**11. Given:** $\triangle WZY \cong \triangle WXY$, $\triangle WZY$ and $\triangle XYZ$ are isosceles.
**Prove:** *WXYZ* is a rhombus.



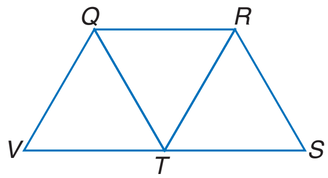

**12. Given:** $\triangle TPX \cong \triangle QPX \cong \triangle QRX \cong \triangle TRX$
**Prove:** *TPQR* is a rhombus.



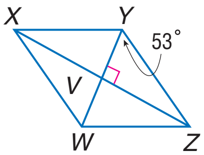

**13. Given:** $\triangle LGK \cong \triangle MJK$
*GHJK* is a parallelogram.
**Prove:** *GHJK* is a rhombus.




**14. Given:** *QRST* and *QRTV* are rhombi.
**Prove:** $\triangle QRT$ is equilateral.




**ALGEBRA** **Use rhombus *XYZW* with $m\angle WYZ = 53$, $VW = 3$, $XV = 2a - 2$, and $ZV = \frac{5a + 1}{4}$.**




**15.** Find $m\angle YZV$.

**16.** Find $m\angle XYW$.

**17.** Find $XZ$.

**18.** Find $XW$.

**COORDINATE GEOMETRY** **Given each set of vertices, determine whether ▱*EFGH* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.**

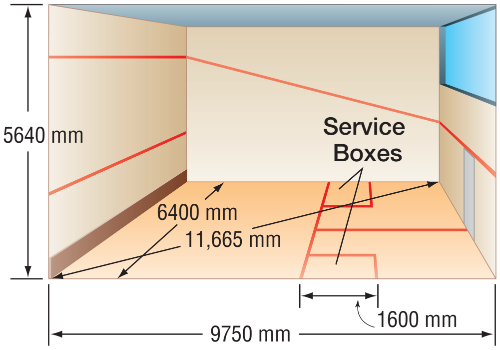
**19.** $E(1, 10)$, $F(-4, 0)$, $G(7, 2)$, $H(12, 12)$

**20.** $E(-7, 3)$, $F(-2, 3)$, $G(1, 7)$, $H(-4, 7)$

**21.** $E(1, 5)$, $F(6, 5)$, $G(6, 10)$, $H(1, 10)$

**22.** $E(-2, -1)$, $F(-4, 3)$, $G(1, 5)$, $H(3, 1)$

**SQUASH** **For Exercises 23 and 24, use the diagram of the court for squash, a game similar to racquetball and tennis.**




**23.** The diagram labels the diagonal as 11,665 millimeters. Is this correct? Explain.

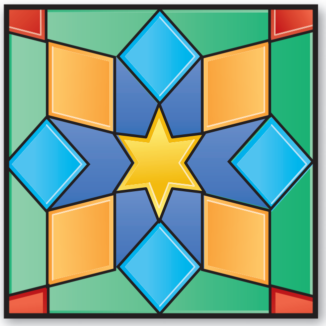
**24.** The service boxes are squares. Find the length of the diagonal.

**Construct each figure using a compass and ruler.**

**25.** a square with one side 3 centimeters long

**26.** a square with a diagonal 5 centimeters long

**27.** **MOSAIC** This pattern is composed of repeating shapes. Use a ruler or a protractor to determine which type of quadrilateral best represents the brown shapes.



**28.** **DESIGN** Otto Prutscher designed the plant stand at the left in 1903. The base is a square, and the base of each of the five boxes is also a square. Suppose each smaller box is one half as wide as the base. Use the information at the left to find the dimensions of the base of one of the smaller boxes.

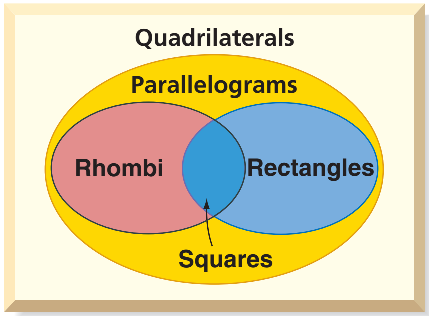
**Real-World Link**

The overall dimensions of the plant stand are $36\frac{1}{2}$ inches tall by $15\frac{3}{4}$ inches wide.

**Source:** www.metmuseum.org

**29.** **PERIMETER** The diagonals of a rhombus are 12 centimeters and 16 centimeters long. Find the perimeter of the rhombus.

**Use the Venn diagram to determine whether each statement is *always*, *sometimes*, or *never* true.**




**30.** A parallelogram is a square.

**31.** A square is a rhombus.

**32.** A rectangle is a parallelogram.

**33.** A rhombus is a rectangle but not a square.

**34.** A rhombus is a square.

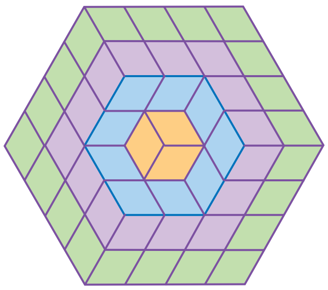
**35.** *True* or *false*? A quadrilateral is a square only if it is also a rectangle. Explain your reasoning.

**EXTRA PRACTICE**
See pages 812, 833.

Math Online
Self-Check Quiz at geometryonline.com

**H.O.T. Problems**

**36.** **CHALLENGE** The pattern at the right is a series of rhombi that continue to form hexagons that increase in size. Copy and complete the table.



| Hexagon | Number of Rhombi |
|---|---|
| 1 | 3 |
| 2 | 12 |
| 3 | 27 |
| 4 | 48 |
| 5 | |
| 6 | |
| $x$ | |

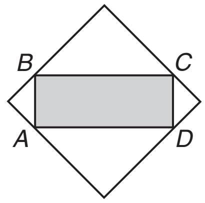
**37.** **CHALLENGE** State the converse of Theorem 6.17. Then write a paragraph proof of this converse.

**38.** **OPEN ENDED** Find the vertices of a square with diagonals that are contained in the lines $y = x$ and $y = -x + 6$. Justify your reasoning.

**39.** ***Writing in Math*** Refer to the information on page 348. Explain the difference between squares and rhombi, and describe how nonsquare rhombus-shaped wheels would work with the curved road.

## STANDARDIZED TEST PRACTICE

**40.** Points $A$, $B$, $C$, and $D$ are on a square. The area of the square is 36 square units. What is the perimeter of rectangle $ABCD$?




A 24 units

B $12\sqrt{2}$ units

C 12 units

D $6\sqrt{2}$

**41.** **REVIEW** If the equation below has no real solutions, then which of the following could *not* be the value of $a$?

$$ax^2 - 6x + 2 = 0$$

F 3

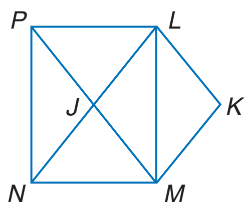
G 4

H 5

J 6

## Spiral Review

**ALGEBRA** **Use rectangle *LMNP*, parallelogram *LKMJ*, and the given information to solve each problem.** (Lesson 6-4)




**42.** If $LN = 10$, $LJ = 2x + 1$, and $PJ = 3x - 1$, find $x$.

**43.** If $m\angle PLK = 110$, find $m\angle LKM$.

**44.** If $m\angle MJN = 35$, find $m\angle MPN$.

**COORDINATE GEOMETRY** **Determine whether the points are the vertices of a parallelogram. Use the method indicated.** (Lesson 6-3)

**45.** $P(0, 2)$, $Q(6, 4)$, $R(4, 0)$, $S(-2, -2)$; Distance Formula

**46.** $K(-3, -7)$, $L(3, 2)$, $M(1, 7)$, $N(-3, 1)$; Slope Formula

**47.** **GEOGRAPHY** The distance between San Jose, California, and Las Vegas, Nevada, is about 375 miles. The distance from Las Vegas to Carlsbad, California, is about 243 miles. Use the Triangle Inequality Theorem to find the possible distance between San Jose and Carlsbad. (Lesson 5-4)

## GET READY for the Next Lesson

**PREREQUISITE SKILL** **Solve each equation.** (Pages 781 and 782)

**48.** $\frac{1}{2}(8x - 6x - 7) = 5$

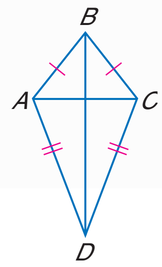
**49.** $\frac{1}{2}(7x + 3x + 1) = 12.5$

**50.** $\frac{1}{2}(4x + 6 + 2x + 13) = 15.5$

EXTEND 6-5

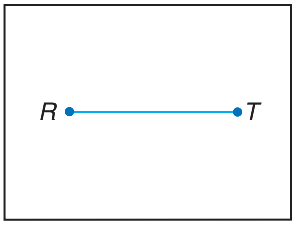
# Geometry Lab
# Kites

A **kite** is a quadrilateral with exactly two distinct pairs of adjacent congruent sides. In kite *ABCD*, diagonal $\overline{BD}$ separates the kite into two congruent triangles (SSS). Diagonal $\overline{AC}$ separates the kite into two noncongruent isosceles triangles.

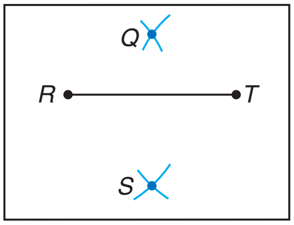



## ACTIVITY Construct a kite *QRST*.

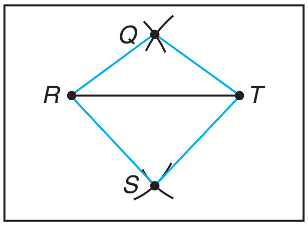
**Step 1** Draw $\overline{RT}$.




**Step 2** Choose a compass setting greater than $\frac{1}{2}RT$. Place the compass at point *R* and draw an arc above $\overline{RT}$. Then without changing the compass setting, move the compass to point *T* and draw an arc that intersects the first one. Label the intersection point *Q*. Increase the compass setting. Place the compass at *R* and draw an arc below $\overline{RT}$. Then, without changing the compass setting, draw an arc from point *T* to intersect the other arc. Label the intersection point *S*.




**Step 3** Draw *QRST*.




## Model

1. Draw $\overline{QS}$ in kite *QRST*. Use a protractor to measure the angles formed by the intersection of $\overline{QS}$ and $\overline{RT}$.
2. Measure the interior angles of kite *QRST*. Are any congruent?
3. Label the intersection of $\overline{QS}$ and $\overline{RT}$ as point *N*. Find the lengths of $\overline{QN}$, $\overline{NS}$, $\overline{TN}$, and $\overline{NR}$. How are they related?
4. How many pairs of congruent triangles can be found in kite *QRST*?
5. Construct another kite *JKLM*. Repeat Exercises 1–4.
6. Make conjectures about angles, sides, and diagonals of kites.
7. Determine whether the lines with equations $y = 4x - 3$, $y = 7x - 60$, $x - 4y = -3$, and $x - 7y = -60$ determine the sides of a kite. Justify your reasoning.

# 6-6 Trapezoids

## Main Ideas

- Recognize and apply the properties of trapezoids.
- Solve problems involving the medians of trapezoids.

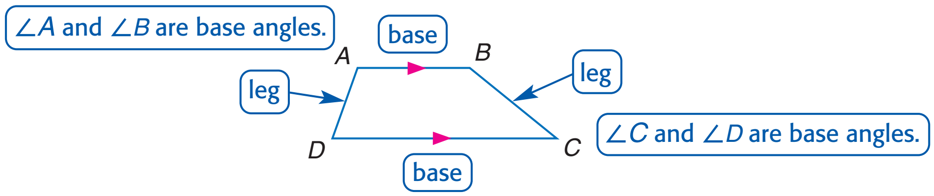
## New Vocabulary

trapezoid
isosceles trapezoid
median

## GET READY for the Lesson

*Cleopatra's Needle* in New York City's Central Park was given to the United States in the late 19th century by the Egyptian government. The width of the base is longer than the width at the top. Each face of the monument is an example of a trapezoid.

**Properties of Trapezoids** A **trapezoid** is a quadrilateral with exactly one pair of parallel sides called *bases.* There are two pairs of *base angles* formed by one base and the legs. The nonparallel sides are called *legs.* If the legs are congruent, then the trapezoid is an **isosceles trapezoid**.




### THEOREMS — Isosceles Trapezoid

**6.18** Each pair of base angles of an isosceles trapezoid are congruent.

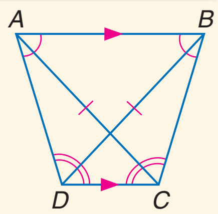
**6.19** The diagonals of an isosceles trapezoid are congruent.

**Example:**

$\angle DAB \cong \angle CBA$

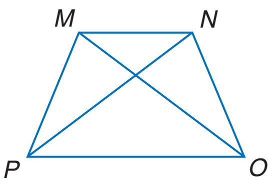
$\angle ADC \cong \angle BCD$

$\overline{AC} \cong \overline{BD}$




## EXAMPLE 1 Proof of Theorem 6.19

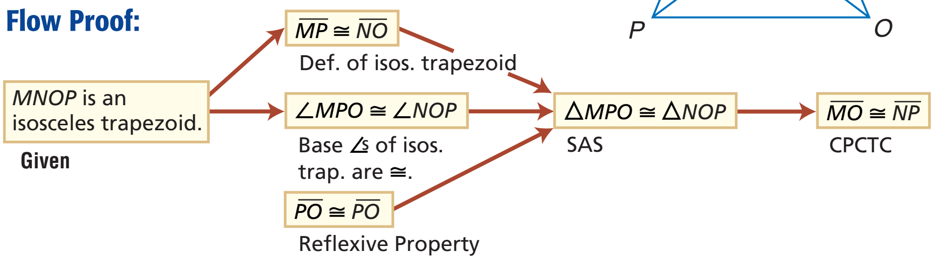
**Write a flow proof of Theorem 6.19.**




**Given:** *MNOP* is an isosceles trapezoid.

**Prove:** $\overline{MO} \cong \overline{NP}$

**Flow Proof:**



CHECK Your Progress

1. **PROOF** Write a paragraph proof of Theorem 6.18.

Online Personal Tutor at geometryonline.com

**Real-World Link**

Barnett Newman designed this sculpture to be 50% larger. This piece was designed for an exhibition in Japan but it could not be built as large as the artist wanted because of size limitations on cargo from New York to Japan.

**Source:** www.sfmoma.org

## Real-World EXAMPLE Identify Isosceles Trapezoids

**2** **ART** **The sculpture pictured is *Zim Zum I* by Barnett Newman. The walls are connected at right angles. In perspective, the rectangular panels appear to be trapezoids. Use a ruler and protractor to determine if the images of the front panels are isosceles trapezoids. Explain.**

The panel on the left is an isosceles trapezoid. The bases are parallel and are different lengths. The legs are not parallel, and they are the same length.

The panel on the right is not an isosceles trapezoid. Each side is a different length.

CHECK Your Progress

2. Use a compass and ruler to construct an equilateral triangle. Draw a segment with endpoints that are the midpoints of two sides. Use a protractor and a ruler to determine if this segment separates the triangle into an equilateral triangle and an isosceles trapezoid.

## EXAMPLE Identify Trapezoids

**3** **COORDINATE GEOMETRY** **Quadrilateral *JKLM* has vertices *J*(−18, −1), *K*(−6, 8), *L*(18, 1), and *M*(−18, −26).**

**a. Verify that *JKLM* is a trapezoid.**

A quadrilateral is a trapezoid if exactly one pair of opposite sides are parallel. Use the Slope Formula.

$$\text{slope of } \overline{JK} = \frac{8-(-1)}{-6-(-18)} = \frac{9}{12} \text{ or } \frac{3}{4}$$

$$\text{slope of } \overline{ML} = \frac{1-(-26)}{18-(-18)} = \frac{27}{36} \text{ or } \frac{3}{4}$$

$$\text{slope of } \overline{JM} = \frac{-26-(-1)}{-18-(-18)} = \frac{-25}{0} \text{ or undefined}$$

$$\text{slope of } \overline{KL} = \frac{1-8}{18-(-6)} = \frac{-7}{24}$$

Since $\overline{JK} \parallel \overline{ML}$, *JKLM* is a trapezoid.

**b. Determine whether *JKLM* is an isosceles trapezoid. Explain.**

First use the Distance Formula to show that the legs are congruent.

$$JM = \sqrt{[-18-(-18)]^2 + [-1-(-26)]^2} = \sqrt{0+625} = \sqrt{625} \text{ or } 25$$

$$KL = \sqrt{(-6-18)^2 + (8-1)^2} = \sqrt{576+49} = \sqrt{625} \text{ or } 25$$

Since the legs are congruent, *JKLM* is an isosceles trapezoid.

## Check Your Progress

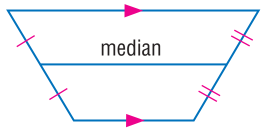
**3.** Quadrilateral *QRST* has vertices $Q(-8, -4)$, $R(0, 8)$, $S(6, 8)$, and $T(-6, -10)$. Verify that *QRST* is a trapezoid and determine whether *QRST* is an isosceles trapezoid.

### Study Tip

**Median**

The median of a trapezoid can also be called a *midsegment*.

**Medians of Trapezoids** The segment that joins the midpoints of the legs of a trapezoid is called the **median**. It is parallel to and equidistant from each base. You can construct the median of a trapezoid using a compass and a straightedge.




**Vocabulary Link**

**Median**

**Everyday Use** a strip dividing a highway

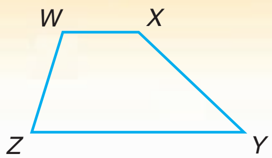
**Math Use** a segment dividing the legs of a trapezoid in half

## GEOMETRY LAB

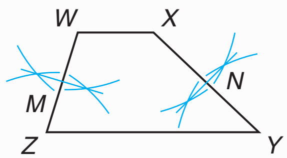
### Median of a Trapezoid

**MODEL**

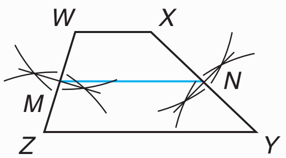
**Step 1** Draw a trapezoid *WXYZ* with legs $\overline{XY}$ and $\overline{WZ}$.




**Step 2** Construct the perpendicular bisectors of $\overline{WZ}$ and $\overline{XY}$. Label the midpoints *M* and *N*.




**Step 3** Draw $\overline{MN}$.




**ANALYZE**

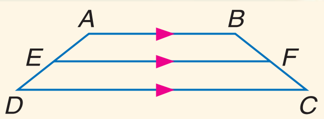
1. Measure $\overline{WX}$, $\overline{ZY}$, and $\overline{MN}$ to the nearest millimeter.
2. **Make a conjecture** based on your observations.
3. Draw an isosceles trapezoid *WXYZ*. Repeat Steps 1, 2, and 3. Is your conjecture valid? Explain.

The results of the Geometry Lab suggest Theorem 6.20.

### THEOREM 6.20

The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases.

**Example:** $EF = \frac{1}{2}(AB + DC)$




You will prove Theorem 6.20 in Exercise 26 of Lesson 6-7.

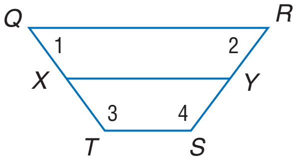
**Study Tip**

**Isosceles Trapezoid**

If you extend the legs of an isosceles trapezoid until they meet, you will have an isosceles triangle. Recall that the base angles of an isosceles triangle are congruent.

## Real-World EXAMPLE Median of a Trapezoid

**4** **ALGEBRA** **In the diagram, *QRST* represents an outdoor eating area in the shape of an isosceles trapezoid. The median $\overline{XY}$ represents the sidewalk through the area.**




**a. Find *TS* if *QR* = 22 and *XY* = 15.**

$XY = \frac{1}{2}(QR + TS)$ Theorem 6.20

$15 = \frac{1}{2}(22 + TS)$ Substitution

$30 = 22 + TS$ Multiply each side by 2.

$8 = TS$ Subtract 22 from each side.

**b. Find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$ if $m\angle 1 = 4a - 10$ and $m\angle 3 = 3a + 32.5$.**

Since $\overline{QR} \parallel \overline{TS}$, $\angle 1$ and $\angle 3$ are supplementary. Because this is an isosceles trapezoid, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$.

$m\angle 1 + m\angle 3 = 180$ Consecutive Interior Angles Theorem

$4a - 10 + 3a + 32.5 = 180$ Substitution

$7a + 22.5 = 180$ Combine like terms.

$7a = 157.5$ Subtract 22.5 from each side.

$a = 22.5$ Divide each side by 7.

If $a = 22.5$, then $m\angle 1 = 80$ and $m\angle 3 = 100$.
Because $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, $m\angle 2 = 80$ and $m\angle 4 = 100$.

### CHECK Your Progress

**4A.** **ALGEBRA** *JKLM* is an isosceles trapezoid with $\overline{JK} \parallel \overline{LM}$ and median $\overline{RP}$. Find *RP* if $JK = 2(x + 3)$, $RP = 5 + x$, and $ML = \frac{1}{2}x - 1$.

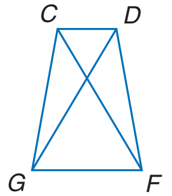
**4B.** Find the measure of each base angle of *JKLM* if $m\angle L = x$ and $m\angle J = 3x + 12$.

**Concepts in Motion**
**Interactive Lab**
geometryonline.com

## CHECK Your Understanding

Example 1 (p. 356)

**1.** **PROOF** *CDFG* is an isosceles trapezoid with bases $\overline{CD}$ and $\overline{FG}$. Write a flow proof to prove $\angle DGF \cong \angle CFG$.




Example 2 (p. 357)

**2.** **PHOTOGRAPHY** Photographs can show a building in a perspective that makes it appear to be a different shape. Identify the types of quadrilaterals in the photograph.

Example 3 (p. 357)

**COORDINATE GEOMETRY** **Quadrilateral *QRST* has vertices *Q*(−3, 2), *R*(−1, 6), *S*(4, 6), and *T*(6, 2).**

**3.** Verify that *QRST* is a trapezoid.

**4.** Determine whether *QRST* is an isosceles trapezoid. Explain.

**Example 4** (p. 359)

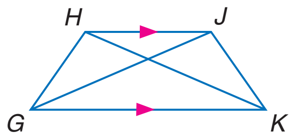
5. **ALGEBRA** *EFGH* is an isosceles trapezoid with bases $\overline{EF}$ and $\overline{GH}$ and median $\overline{YZ}$. If $EF = 3x + 8$, $GH = 4x - 10$, and $YZ = 13$, find $x$.
6. **ALGEBRA** Find the measure of each base angle of *EFGH* if $m\angle E = 7x$ and $m\angle G = 16x - 4$.

## Exercises

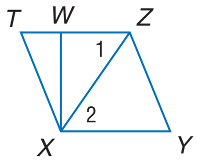
| HOMEWORK HELP | |
|---|---|
| For Exercises | See Examples |
| 7–10 | 1 |
| 11–12 | 2 |
| 13–16 | 3 |
| 17–20 | 4 |

**PROOF** **Write a flow proof.**

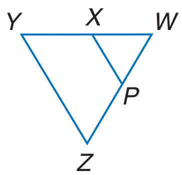
7. **Given:** $\overline{HJ} \parallel \overline{GK}$, $\triangle HGK \cong \triangle JKG$, $\overline{HG} \nparallel \overline{JK}$
**Prove:** *GHJK* is an isosceles trapezoid.



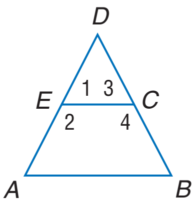

8. **Given:** $\triangle TZX \cong \triangle YXZ$, $\overline{WX} \nparallel \overline{ZY}$
**Prove:** *XYZW* is a trapezoid.




9. **Given:** *ZYXP* is an isosceles trapezoid.
**Prove:** $\triangle PWX$ is isosceles.




10. **Given:** *E* and *C* are midpoints of $\overline{AD}$ and $\overline{DB}$; $\overline{AD} \cong \overline{DB}$ and $\angle A \cong \angle 1$.
**Prove:** *ABCE* is an isosceles trapezoid.

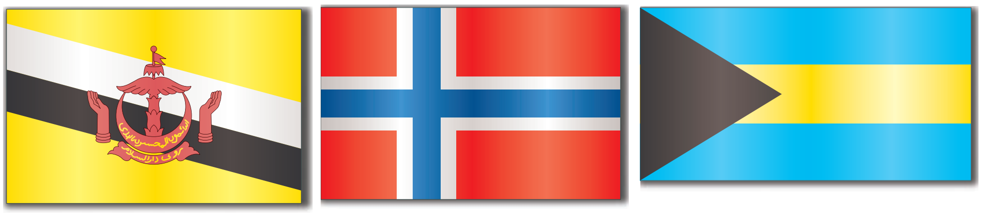



**Real-World Link**

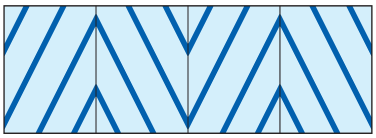
Ohio is the only state not to have a rectangular flag. The swallowtail design is properly called the Ohio burgee.

**Source:** 50states.com

11. **FLAGS** Study the flags shown below. Use a ruler and protractor to determine if any of the flags contain parallelograms, rectangles, rhombi, squares, or trapezoids.



12. **INTERIOR DESIGN** Peta is making a valance for a window treatment. She is using striped fabric cut on the bias, or diagonal, to create a chevron pattern. Identify the polygons formed in the fabric.

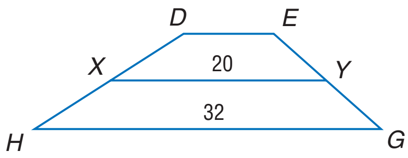


**COORDINATE GEOMETRY** **For each quadrilateral with the vertices given, a. verify that the quadrilateral is a trapezoid, and b. determine whether the figure is an isosceles trapezoid.**

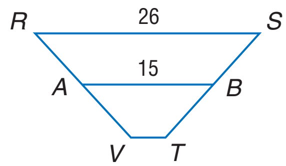
13. $A(-3, 3)$, $B(-4, -1)$, $C(5, -1)$, $D(2, 3)$
14. $G(-5, -4)$, $H(5, 4)$, $J(0, 5)$, $K(-5, 1)$
15. $C(-1, 1)$, $D(-5, -3)$, $E(-4, -10)$, $F(6, 0)$
16. $Q(-12, 1)$, $R(-9, 4)$, $S(-4, 3)$, $T(-11, -4)$

**ALGEBRA** **Find the missing value for the given trapezoid.**

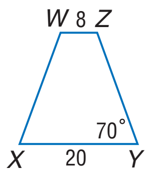
**17.** For trapezoid *DEGH*, *X* and *Y* are midpoints of the legs. Find *DE*.



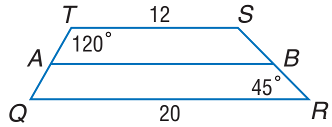

**18.** For trapezoid *RSTV*, *A* and *B* are midpoints of the legs. Find *VT*.




**19.** For isosceles trapezoid *XYZW*, find the length of the median, $m\angle W$, and $m\angle Z$.



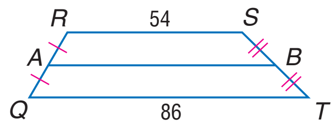

**20.** For trapezoid *QRST*, *A* and *B* are midpoints of the legs. Find *AB*, $m\angle Q$, and $m\angle S$.




**For Exercises 21 and 22, use trapezoid *QRST*.**

**21.** Let $\overline{GH}$ be the median of *RSBA*. Find *GH*.

**22.** Let $\overline{JK}$ be the median of *ABTQ*. Find *JK*.




**CONSTRUCTION** **Use a compass and ruler to construct each figure.**

**23.** an isosceles trapezoid

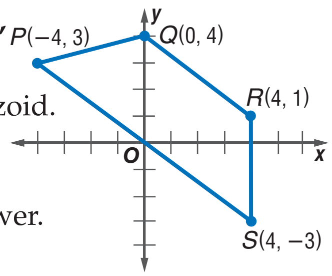
**24.** trapezoid with a median 2 centimeters long

**COORDINATE GEOMETRY** **Determine whether each figure is a *trapezoid*, a *parallelogram*, a *square*, a *rhombus*, or a *quadrilateral* given the coordinates of the vertices. Choose the most specific term. Explain.**

**25.** $B(1, 2)$, $C(4, 4)$, $D(5, 1)$, $E(2, -1)$

**26.** $G(-2, 2)$, $H(4, 2)$, $J(6, -1)$, $K(-4, -1)$

**COORDINATE GEOMETRY** **For Exercises 27–29, refer to quadrilateral *PQRS*.**




**27.** Determine whether the figure is a trapezoid. If so, is it isosceles? Explain.

**28.** Is the median contained in the line with equation $y = -\frac{3}{4}x + 1$? Justify your answer.

**29.** Find the length of the median.

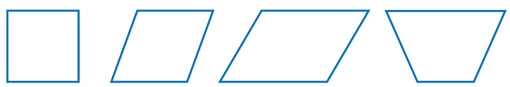
EXTRA PRACTICE
See pages 812, 833.


**H.O.T. Problems**

**30.** **OPEN ENDED** Draw an isosceles trapezoid and a trapezoid that is not isosceles. Draw the median for each. Is the median parallel to the bases in both trapezoids? Justify your answer.

**31.** **CHALLENGE** State the converse of Theorem 6.19. Then write a paragraph proof of this converse.

**32.** **Which One Doesn't Belong?** Identify the figure that does not belong with the other three. Explain.



33. *Writing in Math* Describe the characteristics of a trapezoid. List the minimum requirements to show that a quadrilateral is a trapezoid.

## STANDARDIZED TEST PRACTICE

34. Which figure can serve as a counterexample to the conjecture below?

| |
|---|
| If the diagonals of a quadrilateral are congruent, then the quadrilateral is a rectangle. |

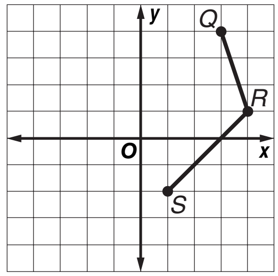
A square

B rhombus

C parallelogram

D isosceles trapezoid

35. **REVIEW** A portion of isosceles trapezoid *QRST* is shown.



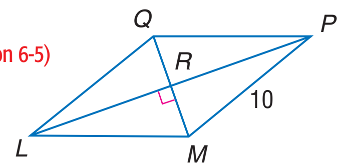

At what coordinates should vertex *T* be placed so that $\overline{TQ} \parallel \overline{SR}$ in order to complete *QRST*?

F (0, 1) H (−2, −1)

G (−1, 0) J (−2, 0)

## Spiral Review

**ALGEBRA In rhombus *LMPQ*, $m\angle QLM = 2x^2 - 10$, $m\angle QPM = 8x$, and $MP = 10$. Find the indicated measures.** (Lesson 6-5)




36. $m\angle LPQ$

37. $QL$

38. $m\angle LQP$

39. $m\angle LQM$

**COORDINATE GEOMETRY For Exercises 40–42, refer to quadrilateral *RSTV* with vertices $R(-7, -3)$, $S(0, 4)$, $T(3, 1)$, and $V(-4, -7)$.** (Lesson 6-4)

40. Find *RS* and *TV*.

41. Find the coordinates of the midpoints of $\overline{RT}$ and $\overline{SV}$.

42. Is *RSTV* a rectangle? Explain.

43. **RECREATION** The table below shows the number of visitors to areas in the United States National Park system in millions. What is the average rate of change of the number of visitors per year? (Lesson 3-3)

| Year | 1999 | 2002 |
|---|---|---|
| Visitors (millions) | 287.1 | 277.3 |

**Source:** *Statistical Abstract of the United States*

## GET READY for the Next Lesson

**PREREQUISITE SKILL Write an expression for the slope of the segment given the coordinates of the endpoints.** (Lesson 3-3)

44. $(0, a), (-a, 2a)$

45. $(-a, b), (a, b)$

46. $(c, c), (c, d)$

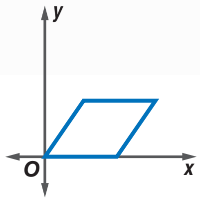
# 6-7 Coordinate Proof with Quadrilaterals

**Main Ideas**

- Position and label quadrilaterals for use in coordinate proofs.
- Prove theorems using coordinate proofs.

**GET READY for the Lesson**

In Chapter 4, you learned that variable coordinates can be assigned to the vertices of triangles. Then the Distance and Midpoint Formulas and coordinate proofs were used to prove theorems. The same can be done with quadrilaterals.




**Study Tip**

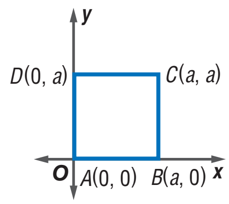
**Look Back**
To review **placing a figure on a coordinate plane,** see Lesson 4-7.

**Position Figures** The first step to using a coordinate proof is to place the figure on the coordinate plane. The placement of the figure can simplify the steps of the proof.

## EXAMPLE 1 Positioning a Square

**Position and label a square with sides *a* units long on the coordinate plane.**

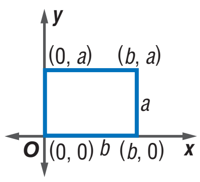
- Let *A*, *B*, *C*, and *D* be vertices of a square with sides *a* units long.
- Place the square with vertex *A* at the origin, $\overline{AB}$ along the positive *x*-axis, and $\overline{AD}$ along the *y*-axis. Label the vertices *A*, *B*, *C*, and *D*.
- The *y*-coordinate of *B* is 0 because the vertex is on the *x*-axis. Since the side length is *a*, the *x*-coordinate is *a*.
- *D* is on the *y*-axis so the *x*-coordinate is 0. The *y*-coordinate is $0 + a$ or *a*.
- The *x*-coordinate of *C* is also *a*. The *y*-coordinate is $0 + a$ or *a* because the side $\overline{BC}$ is *a* units long.



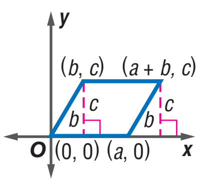

**CHECK Your Progress**

1. Position and label a rectangle with a length of 2*a* units and a width of *a* units.

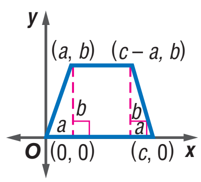
Some examples of quadrilaterals placed on the coordinate plane are given below. Notice how the figures have been placed so the coordinates of the vertices are as simple as possible.



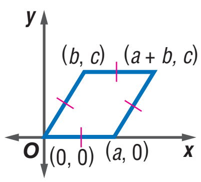
rectangle



parallelogram



isosceles trapezoid



rhombus

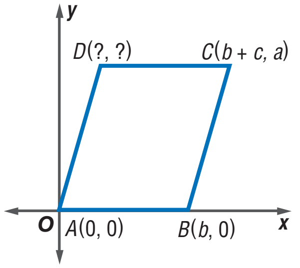
## EXAMPLE 2 Find Missing Coordinates

**Name the missing coordinates for the parallelogram.**

Opposite sides of a parallelogram are congruent and parallel. So, the $y$-coordinate of $D$ is $a$.

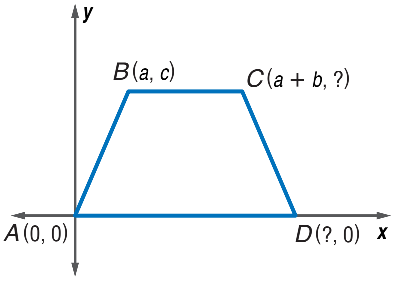
The length of $\overline{AB}$ is $b$, and the length of $\overline{DC}$ is $b$. So, the $x$-coordinate of $D$ is $(b + c) - b$ or $c$.

The coordinates of $D$ are $(c, a)$.




**CHECK Your Progress**

**2.** Name the missing coordinates for the isosceles trapezoid.




**Prove Theorems** Once a figure has been placed on the coordinate plane, we can prove theorems using the Slope, Midpoint, and Distance Formulas.

## EXAMPLE 3 Coordinate Proof

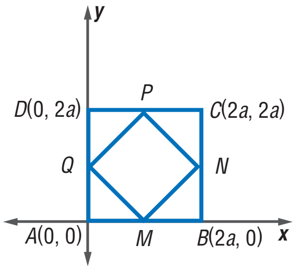
**Study Tip**

**Problem Solving**

To prove that a quadrilateral is a square, you can also show that all sides are congruent and that the diagonals bisect each other.

**Place a square on a coordinate plane. Label the midpoints of the sides, $M$, $N$, $P$, and $Q$. Write a coordinate proof to prove that $MNPQ$ is a square.**

The first step is to position a square on the coordinate plane. Label the vertices to make computations as simple as possible.




**Given:** $ABCD$ is a square.
$M$, $N$, $P$, and $Q$ are midpoints.

**Prove:** $MNPQ$ is a square.

**Coordinate Proof:**

By the Midpoint Formula, the coordinates of $M$, $N$, $P$, and $Q$ are as follows.

$$M\left(\frac{2a + 0}{2}, \frac{0 + 0}{2}\right) = (a, 0)$$

$$N\left(\frac{2a + 2a}{2}, \frac{2a + 0}{2}\right) = (2a, a)$$

$$P\left(\frac{0 + 2a}{2}, \frac{2a + 2a}{2}\right) = (a, 2a)$$

$$Q\left(\frac{0 + 0}{2}, \frac{0 + 2a}{2}\right) = (0, a)$$

Find the slopes of $\overline{QP}$, $\overline{MN}$, $\overline{QM}$, and $\overline{PN}$.

slope of $\overline{QP} = \frac{2a - a}{a - 0}$ or 1     slope of $\overline{MN} = \frac{a - 0}{2a - a}$ or 1

slope of $\overline{QM} = \frac{0 - a}{a - 0}$ or $-1$     slope of $\overline{PN} = \frac{a - 2a}{2a - a}$ or $-1$

Each pair of opposite sides have the same slope, so they are parallel. Consecutive sides form right angles because their slopes are negative reciprocals.

Use the Distance Formula to find the lengths of $\overline{QP}$ and $\overline{QM}$.

$$QP = \sqrt{(0 - a)^2 + (a - 2a)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} \text{ or } a\sqrt{2}$$

$$QM = \sqrt{(0 - a)^2 + (a - 0)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} \text{ or } a\sqrt{2}$$

*MNPQ* is a square because each pair of opposite sides is parallel, and consecutive sides form right angles and are congruent.

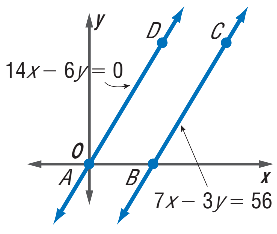
**CHECK Your Progress**

**3.** Write a coordinate proof for the statement: *If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.*

**Online Personal Tutor at** geometryonline.com

## Real-World EXAMPLE Properties of Quadrilaterals

**4 PARKING Write a coordinate proof to prove that the sides of the parking space are parallel.**




**Given:** $14x - 6y = 0$; $7x - 3y = 56$

**Prove:** $\overline{AD} \parallel \overline{BC}$

**Proof:** Rewrite both equations in slope-intercept form.

$14x - 6y = 0$

$$\frac{-6y}{-6} = \frac{-14x}{-6}$$

$$y = \frac{7}{3}x$$

$7x - 3y = 56$

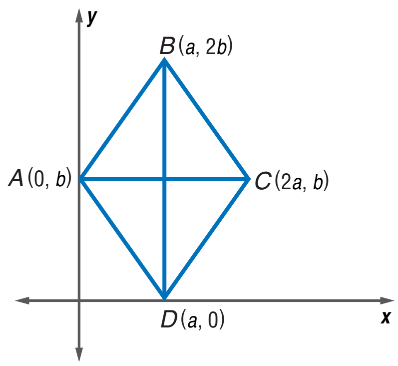
$$\frac{-3y}{-3} = \frac{-7x + 56}{-3}$$

$$y = \frac{7}{3}x - \frac{56}{3}$$

Since $\overline{AD}$ and $\overline{BC}$ have the same slope, they are parallel.

**CHECK Your Progress**

**4.** Write a coordinate proof to prove that the crossbars of a rhombus-shaped window are perpendicular.



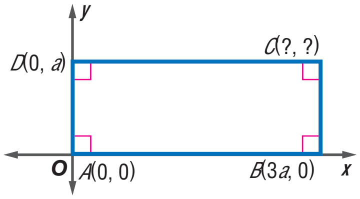
## CHECK Your Understanding

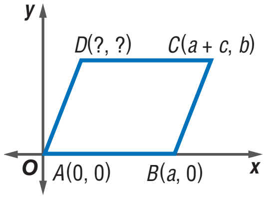
Example 1 (p. 363)

1. Position and label a rectangle with length $a$ units and height $a + b$ units on the coordinate plane.

**Name the missing coordinates for each quadrilateral.**

Example 2 (p. 364)

2.



3.


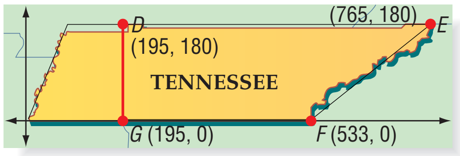

**Write a coordinate proof for each statement.**

Example 3 (p. 364)

4. The diagonals of a parallelogram bisect each other.
5. The diagonals of a square are perpendicular.

Example 4 (p. 365)

6. **STATES** The state of Tennessee can be separated into two shapes that resemble quadrilaterals. Write a coordinate proof to prove that *DEFG* is a trapezoid. All measures are approximate and given in kilometers.



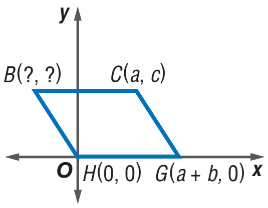

## Exercises

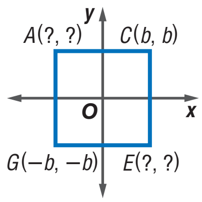
**Position and label each quadrilateral on the coordinate plane.**

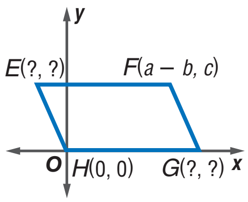
| HOMEWORK HELP | |
|---|---|
| For Exercises | See Examples |
| 7, 8 | 1 |
| 9–14 | 2 |
| 15–20 | 3 |
| 21–23 | 4 |

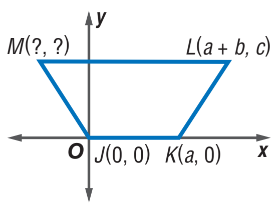
7. isosceles trapezoid with height $c$ units, bases $a$ units and $a + 2b$ units
8. parallelogram with side length $c$ units and height $b$ units

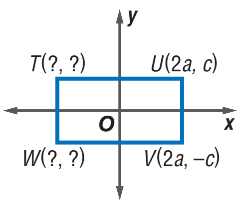
**Name the missing coordinates for each parallelogram or trapezoid.**

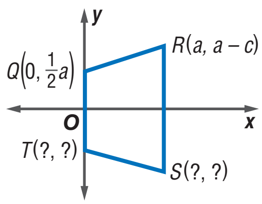
9.



10.



11.



12.



13.



14.



**EXTRA PRACTICE**
See pages 813, 833.
Math Online
Self-Check Quiz at geometryonline.com

**Real-World Link**

The Leaning Tower of Pisa is sinking. In 1838, the foundation was excavated to reveal the bases of the columns.

**Source:** torre.duomo.pisa.it

**Position and label each figure on the coordinate plane. Then write a coordinate proof for each of the following.**

**15.** The diagonals of a rectangle are congruent.

**16.** If the diagonals of a parallelogram are congruent, then it is a rectangle.

**17.** The diagonals of an isosceles trapezoid are congruent.

**18.** The median of an isosceles trapezoid is parallel to the bases.

**19.** The segments joining the midpoints of the sides of a rectangle form a rhombus.

**20.** The segments joining the midpoints of the sides of a quadrilateral form a parallelogram.

**ARCHITECTURE For Exercises 21–23, use the following information.**
The Leaning Tower of Pisa is approximately 60 meters tall, from base to belfry. The tower leans about 5.5° so the top right corner is 4.5 meters to the right of the bottom right corner.

**21.** Position and label the tower on a coordinate plane.

**22.** Is it possible to write a coordinate proof to prove that the sides of the tower are parallel? Explain.

**23.** From the given information, what conclusion can be drawn?

## H.O.T. Problems

**24. REASONING** Explain how to position a quadrilateral to simplify the steps of the proof.

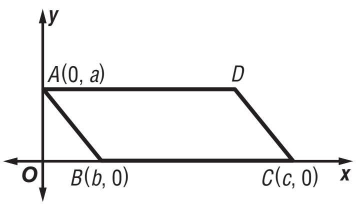
**25. OPEN ENDED** Position and label a trapezoid with two vertices on the *y*-axis.

**26. CHALLENGE** Position and label a trapezoid that is not isosceles on the coordinate plane. Then write a coordinate proof to prove Theorem 6.20 on page 358.

**27. *Writing in Math*** Describe how the coordinate plane can be used in proofs. Include guidelines for placing a figure on a coordinate grid in your answer.

## STANDARDIZED TEST PRACTICE

**28.** In the figure, *ABCD* is a parallelogram. What are the coordinates of point *D*?



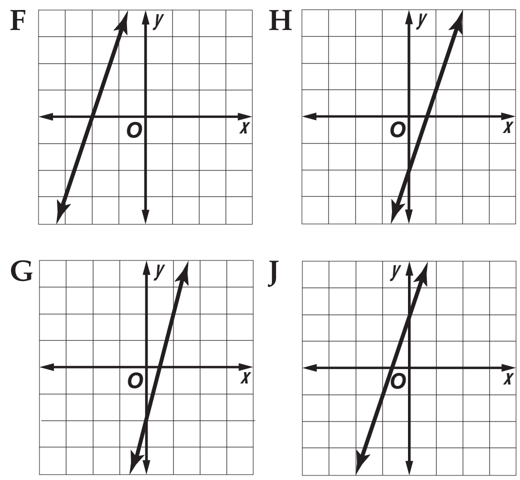

**A** $(a, c + b)$

**B** $(c + b, a)$

**C** $(b - c, a)$

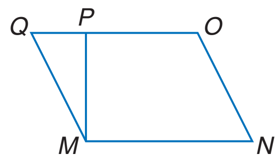
**D** $(c - b, a)$

**29. REVIEW** Which *best* represents the graph of $-3x + y = -2$?



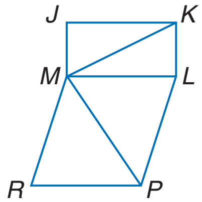
## Spiral Review

**30. PROOF** Write a two-column proof. (Lesson 6-6)




**Given:** $MNOP$ is a trapezoid with bases $\overline{MN}$ and $\overline{OP}$.
$\overline{MN} \cong \overline{QO}$

**Prove:** $MNOQ$ is a parallelogram.




**$JKLM$ is a rectangle. $MLPR$ is a rhombus. $\angle JMK \cong \angle RMP$, $m\angle JMK = 55$, and $m\angle MRP = 70$.** (Lesson 6-5)

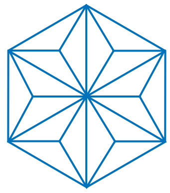
**31.** Find $m\angle MPR$.

**32.** Find $m\angle KML$.

**33.** Find $m\angle KLP$.

**34. COORDINATE GEOMETRY** Given $\triangle STU$ with vertices $S(0, 5)$, $T(0, 0)$, and $U(-2, 0)$, and $\triangle XYZ$ with vertices $X(4, 8)$, $Y(4, 3)$, and $Z(6, 3)$, show that $\triangle STU \cong \triangle XYZ$. (Lesson 4-4)

**ARCHITECTURE For Exercises 35 and 36, use the following information.**
The geodesic dome was developed by Buckminster Fuller in the 1940s as an energy-efficient building. The figure at the right shows the basic structure of one geodesic dome. (Lesson 4-1)



**35.** How many equilateral triangles are in the figure?

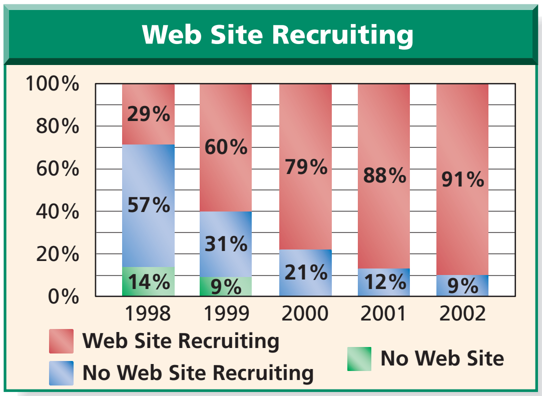
**36.** How many obtuse triangles are in the figure?

**JOBS For Exercises 37–39, refer to the graph at the right.** (Lesson 3-3)

**37.** What was the rate of change for companies that did not use Web sites to recruit employees from 1998 to 2002?

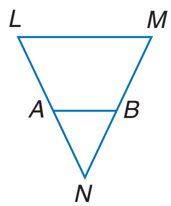
**38.** What was the rate of change for companies that did use Web sites to recruit employees from 1998 to 2002?

**39.** Predict the year in which 100% of companies will use Web sites for recruitment. Justify your answer.




**Source:** iLogos Research

**40. PROOF Write a two-column proof.** (Lesson 2-7)




**Given:** $NL = NM$
$AL = BM$

**Prove:** $NA = NB$

## Cross-Curricular Project

### Geometry and History

**Who is behind this geometry idea anyway?** It is time to complete your project. Use the information and data you have gathered about your research topic, two mathematicians, and a geometry problem to prepare a portfolio or Web page. Be sure to include illustrations and/or tables in the presentation.

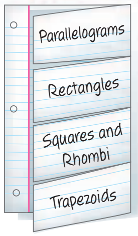
Math Online **Cross-Curricular Project at** geometryonline.com

# CHAPTER 6 Study Guide and Review

Download Vocabulary Review from geometryonline.com

## FOLDABLES Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.




## Key Concepts

### Angles of Polygons (Lesson 6-1)

- The sum of the measures of the interior angles of a polygon is given by the formula $S = 180(n - 2)$.
- The sum of the measures of the exterior angles of a convex polygon is 360.

### Properties of Parallelograms (Lesson 6-2)

- Opposite sides are congruent and parallel.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- If a parallelogram has one right angle, it has four right angles.
- Diagonals bisect each other.

### Tests for Parallelograms (Lesson 6-3)

- If a quadrilateral has the properties of a parallelogram, then it is a parallelogram.

### Properties of Rectangles, Rhombi, Squares, and Trapezoids (Lessons 6-4 to 6-6)

- A rectangle has all the properties of a parallelogram. Diagonals are congruent and bisect each other. All four angles are right angles.
- A rhombus has all the properties of a parallelogram. All sides are congruent. Diagonals are perpendicular. Each diagonal bisects a pair of opposite angles.
- A square has all the properties of a parallelogram, a rectangle, and a rhombus.
- In an isosceles trapezoid, both pairs of base angles are congruent and the diagonals are congruent.

## Key Vocabulary

diagonal (p. 318)
isosceles trapezoid (p. 356)
kite (p. 355)
median (p. 358)
parallelogram (p. 325)
rectangle (p. 340)
rhombus (p. 348)
square (p. 349)
trapezoid (p. 356)

## Vocabulary Check

**State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.**

1. The diagonals of a rhombus are perpendicular.
2. A trapezoid has all the properties of a parallelogram, a rectangle, and a rhombus.
3. If a parallelogram is a rhombus, then the diagonals are congruent.
4. Every parallelogram is a quadrilateral.
5. A(n) rhombus is a quadrilateral with exactly one pair of parallel sides.
6. Each diagonal of a rectangle bisects a pair of opposite angles.
7. If a quadrilateral is both a rhombus and a rectangle, then it is a square.
8. Both pairs of base angles in a(n) isosceles trapezoid are congruent.
9. All squares are rectangles.
10. If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a rhombus.

# Lesson-by-Lesson Review

## 6–1 Angles of Polygons (pp. 318–321)

**11. ARCHITECTURE** The schoolhouse below was built in 1924 in Essex County, New York. If its floor is in the shape of a regular polygon and the measure of an interior angle is 135, find the number of sides the schoolhouse has.

**Example 1 Find the sum of the measures of the interior angles and the measure of an interior angle of a regular decagon.**

$S = 180(n - 2)$ Interior Angle Sum Theorem

$= 180(10 - 2)$ $n = 10$

$= 180(8)$ or $1440$ Simplify.

The sum of the measures of the interior angles is 1440. The measure of each interior angle is $1440 \div 10$ or 144.

## 6–2 Parallelograms (pp. 323–329)

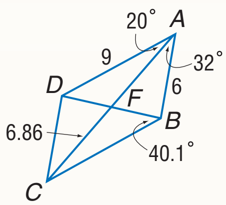
**Use ▱*ABCD* to find each measure.**

**12.** $m\angle BCD$

**13.** $AF$

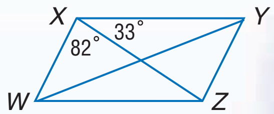
**14.** $m\angle BDC$

**15.** $BC$




**16. ART** One way to draw a cube is to draw three parallelograms. State which properties of a parallelogram an artist might use to draw a cube.

**Example 2 *WXYZ* is a parallelogram. Find $m\angle YZW$ and $m\angle XWZ$.**




$m\angle YZW = m\angle WXY$

$= 82 + 33$ or $115$

$m\angle XWZ + m\angle WXY = 180$

$m\angle XWZ + (82 + 33) = 180$

$m\angle XWZ + 115 = 180$

$m\angle XWZ = 65$

## 6–3 Tests for Parallelograms (pp. 331–337)

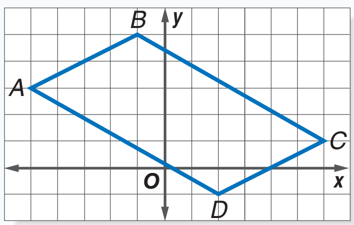
**Determine whether the figure with the given vertices is a parallelogram. Use the method indicated.**

**17.** $A(-2, 5)$, $B(4, 4)$, $C(6, -3)$, and $D(-1, -2)$; Distance Formula

**18.** $H(0, 4)$, $J(-4, 6)$, $K(5, 6)$, and $L(9, 4)$; Midpoint Formula

**19.** $S(-2, -1)$, $T(2, 5)$, $V(-10, 13)$, and $W(-14, 7)$; Slope Formula

**Example 3 Determine whether the figure below is a parallelogram. Use the Distance and Slope Formulas.**






Adirondack Architectural Heritage

**Mixed Problem Solving**
For mixed problem-solving practice, see page 833.

**20.** **GEOGRAPHY** Describe how you could tell whether a map of the state of Colorado is a parallelogram.


$AB = \sqrt{[-5 - (-1)]^2 + (3 - 5)^2}$

$= \sqrt{(-4)^2 + (-2)^2} = \sqrt{20}$ or $2\sqrt{5}$

$CD = \sqrt{(6 - 2)^2 + [1 - (-1)]^2}$

$= \sqrt{4^2 + 2^2} = \sqrt{20}$ or $2\sqrt{5}$

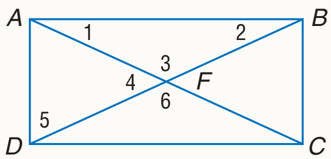
slope of $\overline{AB} = \frac{5 - 3}{-1 - (-5)}$ or $\frac{1}{2}$

slope of $\overline{CD} = \frac{-1 - 1}{2 - 6}$ or $\frac{1}{2}$

Since one pair of opposite sides is congruent and parallel, $ABCD$ is a parallelogram.

## 6–4 Rectangles (pp. 338–344)

**21.** If $m\angle 1 = 12x + 4$ and $m\angle 2 = 16x - 12$ in rectangle $ABCD$, find $m\angle 2$.




**22.** **QUILTS** Mrs. Diller is making a quilt. She has cut several possible rectangles out of fabric. If Mrs. Diller does not own a protractor, how can she be sure that the pieces she has cut are rectangles?

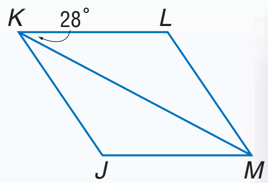
**Example 4** **Refer to rectangle *ABCD*. If $CF = 4x + 1$ and $DF = x + 13$, find $x$.**

| | |
|---|---|
| $\overline{CF} \cong \overline{DF}$ | Diag. bisect each other. |
| $CF = DF$ | Def. of $\cong$ segments |
| $4x + 1 = x + 13$ | Substitution |
| $3x + 1 = 13$ | Subtract $x$ from each side. |
| $3x = 12$ | Subtract 1 from each side. |
| $x = 4$ | Divide each side by 3. |

## 6–5 Rhombi and Squares (pp. 346–352)

**23.** **SIGNS** This sign is a parallelogram. Determine if it is also a square. Explain.


**Example 5** **Find $m\angle JMK$.**




Opposite sides of a rhombus are parallel, so $\overline{KL} \parallel \overline{JM}$.

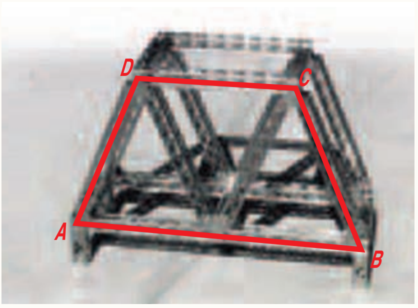
$\angle JMK \cong \angle LKM$ by the Alternate Interior Angle Theorem. By substitution, $m\angle JMK = 28$.

# Study Guide and Review

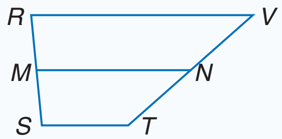
## 6–6 Trapezoids (pp. 354–361)

**24.** Trapezoid *JKLM* has median *XY*. Find $a$ if $JK = 28$, $XY = 4a - 4.5$, and $ML = 3a - 2$.

**25. ART** Artist Chris Burden created the sculpture *Trapezoid Bridge* shown below. State how you could determine whether the bridge is an isosceles trapezoid.




**Example 6** **Trapezoid *RSTV* has median $\overline{MN}$. Find $x$ if $MN = 60$, $ST = 4x - 1$, and $RV = 6x + 11$.**




$MN = \frac{1}{2}(ST + RV)$ Median of a trapezoid

$60 = \frac{1}{2}[(4x - 1) + (6x + 11)]$ Substitution

$120 = 4x - 1 + 6x + 11$ Multiply.

$120 = 10x + 10$ Simplify.

$110 = 10x$ Subtract 10 from each side.

$11 = x$ Divide each side by 10.

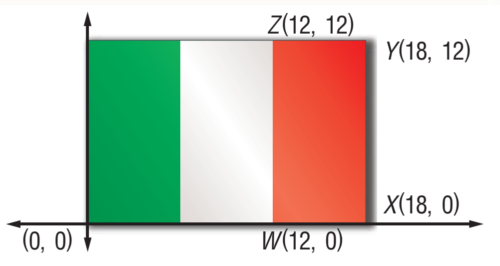
## 6–7 Coordinate Proof with Quadrilaterals (pp. 363–368)

**Position and label each figure on the coordinate plane. Then write a coordinate proof for each of the following.**

**26.** The diagonals of a square are perpendicular.

**27.** A diagonal separates a parallelogram into two congruent triangles.

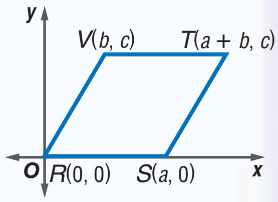
**28. FLAGS** An Italian flag is 12 inches by 18 inches and is made up of three quadrilaterals. Write a coordinate proof to prove that *WXYZ* is a rectangle.




**Example 7** **Write a coordinate proof to prove that each pair of opposite sides of rhombus *RSTV* is parallel.**

**Given:** *RSTV* is a rhombus.

**Prove:** $\overline{RV} \parallel \overline{ST}$, $\overline{RS} \parallel \overline{VT}$




**Coordinate Proof:**

slope of $\overline{RV} = \frac{c - 0}{b - 0}$ or $\frac{c}{b}$

slope of $\overline{RS} = \frac{0 - 0}{a - 0}$ or 0

slope of $\overline{ST} = \frac{c - 0}{(a + b) - a}$ or $\frac{c}{b}$

slope of $\overline{VT} = \frac{c - c}{(a + b) - b}$ or 0

$\overline{RV}$ and $\overline{ST}$ have the same slope, so $\overline{RV} \parallel \overline{ST}$. $\overline{RS}$ and $\overline{VT}$ have the same slope, and $\overline{RS} \parallel \overline{VT}$.

# CHAPTER 6 Practice Test

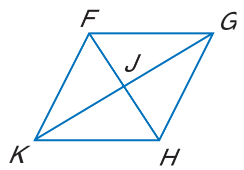
1. What is the measure of one exterior angle of a regular decagon?

2. Find the sum of the measures of the interior angles of a nine-sided polygon.

3. Each interior angle of a regular polygon measures 162°. How many sides does the polygon have?

**Complete each statement about quadrilateral *FGHK*. Justify your answer.**

4. $\overline{HK} \cong$ __?__
5. $\angle FKH \cong$ __?__
6. $\angle FKJ \cong$ __?__
7. $\overline{GH} \parallel$ __?__

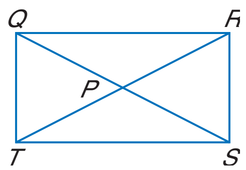



**Determine whether the figure with the given vertices is a parallelogram. Justify your answer.**

8. $A(4, 3)$, $B(6, 0)$, $C(4, -8)$, $D(2, -5)$
9. $S(-2, 6)$, $T(2, 11)$, $V(3, 8)$, $W(-1, 3)$
10. $F(7, -3)$, $G(4, -2)$, $H(6, 4)$, $J(12, 2)$
11. $W(-4, 2)$, $X(-3, 6)$, $Y(2, 7)$, $Z(1, 3)$

**ALGEBRA** ***QRST* is a rectangle.**

12. If $QP = 3x + 11$ and $PS = 4x + 8$, find $QS$.
13. If $m\angle QTR = 2x^2 + 7$ and $m\angle SRT = x^2 + 18$, find $m\angle QTR$.

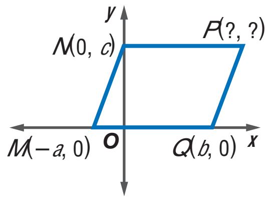



**COORDINATE GEOMETRY** **Determine whether parallelogram *ABCD* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.**

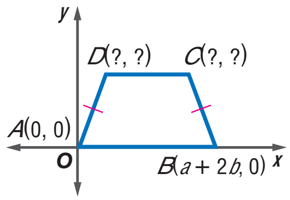
14. $A(12, 0)$, $B(6, -6)$, $C(0, 0)$, $D(6, 6)$
15. $A(-2, 4)$, $B(5, 6)$, $C(12, 4)$, $D(5, 2)$

**Name the missing coordinates for each parallelogram or trapezoid.**

16.

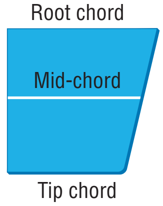



17.




18. Position and label an isosceles trapezoid on the coordinate plane. Write a coordinate proof to prove that the median is parallel to each base.

19. **SAILING** Many large sailboats have a *keel* to keep the boat stable in high winds. A keel is shaped like a trapezoid with its top and bottom parallel. If the root chord is 9.8 feet and the tip chord is 7.4 feet, find the length of the mid-chord.




20. **MULTIPLE CHOICE** If the measure of an interior angle of a regular polygon is 108, what type of polygon is it?

A octagon
B hexagon
C pentagon
D triangle

# Standardized Test Practice

6

**Cumulative, Chapters 1–6**

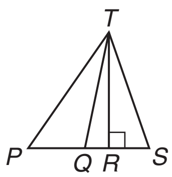
**Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.**

**1.** Which figure can serve as a counterexample to the conjecture below?

> If all the angles of a quadrilateral are right angles, then the quadrilateral is a square.

**A** parallelogram
**B** rectangle
**C** rhombus
**D** trapezoid

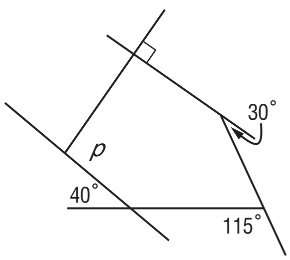
**2.** In the figure below, $\overline{TR}$ is an altitude of $\triangle PST$.




If we assume that $\overline{TQ}$ is the shortest segment from $T$ to $\overline{PS}$, then it follows that $\overline{TQ}$ is an altitude of $\triangle PST$. Since $\triangle PST$ can have only one altitude from vertex $T$, this contradicts the given statement. What conclusion can be drawn from this contradiction?

**F** $TQ > TP$ **H** $TQ < TP$
**G** $TQ > TR$ **J** $TQ < TR$

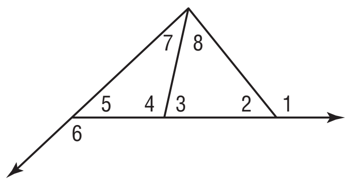
**3. GRIDDABLE** What is $m\angle p$ in degrees?




**4. ALGEBRA** If $x$ is subtracted from $x^2$, the sum is 72. Which of the following could be the value of $x$?

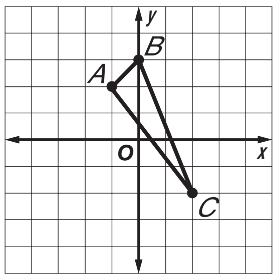
**A** −9
**B** −8
**C** 18
**D** 72

**5.** Which lists contains all of the angles with measures that *must* be less than $m\angle 6$?




**F** ∠1, ∠2, ∠4, ∠7, ∠8
**G** ∠2, ∠3, ∠4, ∠5
**H** ∠2, ∠4, ∠6, ∠7, ∠8
**J** ∠2, ∠4, ∠7, ∠8

**6. GRIDDABLE** Triangle $ABC$ is congruent to $\triangle HIJ$. What is the measure of side $\overline{HJ}$?




**TEST-TAKING TIP**

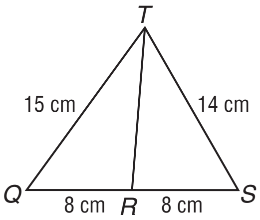
**Question 6** Review any terms and formulas that you have learned before you take the test. Remember that the Distance Formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.




Standardized Test Practice at geometryonline.com

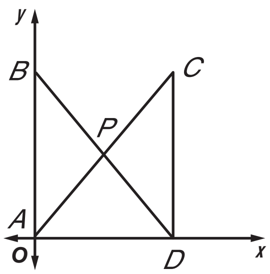
Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

**7.** Which postulate or theorem can be used to prove the measure of $\angle QRT$ is greater than the measure of $\angle SRT$?




**A** AAS Inequality
**B** ASA Inequality
**C** SAS Inequality
**D** SSS Inequality

**8.** Which statement(s) would prove that $\triangle ABP \cong \triangle CDP$?



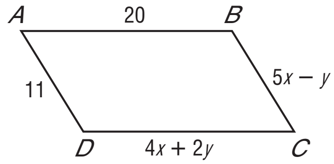

**F** slope $\overline{AB}$ = slope $\overline{CD}$, and the distance from $A$ to $C$ = distance from $B$ to $D$

**G** (slope $\overline{AB}$)(slope $\overline{CD}$) = −1, and the distance from $A$ to $C$ = distance from $B$ to $D$

**H** slope $\overline{AB}$ = slope $\overline{CD}$, and the distance from $B$ to $P$ = distance from $D$ to $P$

**J** (slope $\overline{AB}$)(slope $\overline{CD}$) = 1, and the distance from $A$ to $B$ = distance from $D$ to $C$

**9.** What values of $x$ and $y$ make quadrilateral $ABCD$ a parallelogram?




**A** $x = 4, y = 3$
**B** $x = \frac{31}{9}, y = \frac{11}{9}$
**C** $x = 3, y = 4$
**D** $x = \frac{11}{9}, y = \frac{31}{9}$

**10.** Which is the *converse* of the statement "If I am in La Quinta, then I am in Riverside County"?

**F** If I am not in Riverside County, then I am not in La Quinta.
**G** If I am not in La Quinta, then I am not in Riverside County.
**H** If I am in Riverside County, then I am in La Quinta.
**J** If I am in Riverside County, then I am not in La Quinta.

## Pre-AP

**Record your answer on a sheet of paper. Show your work.**

**11.** Quadrilateral $ABCD$ has vertices with coordinates $A(0, 0)$, $B(a, 0)$, $C(a + b, c)$, and $D(b, c)$.

**a.** Position and label $ABCD$ in the coordinate plane.
**b.** Prove that $ABCD$ is a parallelogram.
**c.** If $a^2 = b^2 + c^2$, determine classify parallelogram $ABCD$. Justify your answer using coordinate geometry.

| NEED EXTRA HELP? | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| If You Missed Question... | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Go to Lesson or Page... | 6-6 | 5-4 | 6-1 | 796 | 5-2 | 4-3 | 5-5 | 4-7 | 6-2 | 2-3 | 6-7 |