

UNIT 3

Similarity

Focus

Explore proportional relationships between similar triangles, the relationships among the angles and sides of right triangles, and transformations in the coordinate plane.

CHAPTER 7

Proportions and Similarity

BIG Idea Prove basic theorems involving similarity and that triangles are similar.

BIG Idea Determine how changes in dimensions affect the perimeter and area of common geometric figures.

CHAPTER 8

Right Triangles and Trigonometry

BIG Idea Prove the Pythagorean Theorem, use it to determine distance, and find missing right triangle lengths.

BIG Idea Know and use the definitions of the basic trigonometric functions defined by the angles of a right triangle.

BIG Idea Know and use angle and side relationships in problems with special right triangles.

CHAPTER 9

Transformations

BIG Idea Know the effect of rigid motions on figures in the coordinate plane, including rotations, translations, and reflections.



Cross-Curricular Project

Geometry and Social Studies

Hidden Treasure Are you intrigued by the idea of hidden treasure? Did you know that a fantastic gold mine might exist in the Superstition Mountains east of Phoenix? According to legend, Jacob Waltz discovered gold there in the 1870s and kept the location a secret. Hundreds of would-be prospectors have searched the Superstition Mountain region in vain to find the mine. In this project, you will use quadrilaterals, circles, and geometric transformations to give clues for a treasure hunt.

Math  **online** Log on to geometryonline.com to begin.

Proportions and Similarity

BIG Ideas

- Identify similar polygons and use ratios and proportions to solve problems.
- Recognize and use proportional parts, corresponding perimeters, altitudes, angle bisectors, and medians of similar triangles to solve problems.

Key Vocabulary

- proportion (p. 381)
- cross products (p. 381)
- similar polygons (p. 388)
- scale factor (p. 389)
- midsegment (p. 406)

Real-World Link

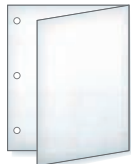
Proportion The seven-story tall Longaberger Home Office in Newark, Ohio, is a replica of a Longaberger Medium Market Basket, reproduced on a 1:160 scale.



FOLDABLES Study Organizer

Proportions and Similarity Make this Foldable to help you organize your notes. Begin with one sheet of 11" × 17" paper.

- 1 Fold** widthwise. Leave space to punch holes so it can be placed in your binder.



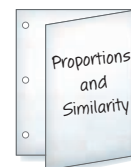
- 2 Open** the flap and draw lines to divide the inside into six equal parts.



- 3 Label** each part using the lesson numbers.

○	7-1	7-2
○	7-3	7-4
○	7-5	Vocabulary

- 4 Write** the name of the chapter on the front.



GET READY for Chapter 7

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2



Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Solve each equation. (Prerequisite Skill)

1. $\frac{2}{3}y - 4 = 6$

2. $\frac{5}{6} = \frac{x-4}{12}$

3. $\frac{4}{3} = \frac{y+2}{y-1}$

4. $\frac{2y}{4} = \frac{32}{y}$

5. **BICYCLING** Randy rode his bicycle 15 miles in 2 hours. At this rate, how far can he ride in 5 hours? (Prerequisite Skill)

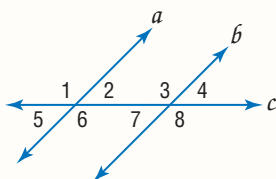
Find the slope of the line given the coordinates of two points on the line. (Lesson 3-3)

6. $(-6, -3)$ and $(2, -3)$

7. $(-3, 4)$ and $(2, -2)$

8. **SPACE** The budget for space research was \$7215 million in 2003 and \$7550 million in 2004. What is the rate of change? (Lesson 3-3)

Given the following information, determine whether $a \parallel b$. State the postulate or theorem that justifies your answer. (Lesson 3-5)



9. $\angle 1 \cong \angle 8$
10. $\angle 3 \cong \angle 6$
11. $\angle 5 \cong \angle 3$
12. $\angle 2 \cong \angle 4$

QUICK Review

EXAMPLE 1

Solve the equation $\frac{5n+2}{n-1} = 2$.

$$\frac{5n+2}{n-1} = 2$$

$$(n-1)\left(\frac{5n+2}{n-1}\right) = 2(n-1) \text{ Multiply.}$$

$$5n+2 = 2n-2 \text{ Simplify.}$$

$$3n = -4 \text{ Combine like terms.}$$

$$n = -\frac{4}{3} \text{ Divide each side by 3.}$$

EXAMPLE 2

Find the slope of the line that contains the points $(-41, 17)$ and $(31, 29)$.

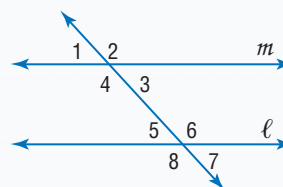
$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} \text{ Slope formula}$$

$$= \frac{29 - 17}{31 - (-41)} \text{ Substitution}$$

$$= \frac{12}{72} \text{ or } \frac{1}{6} \text{ Simplify.}$$

EXAMPLE 3

Determine whether $m \parallel \ell$ if $\angle 4 \cong \angle 6$. Justify your answer.



These angles are alternate interior angles. If alternate interior angles are congruent, then the lines are parallel.

Main Ideas

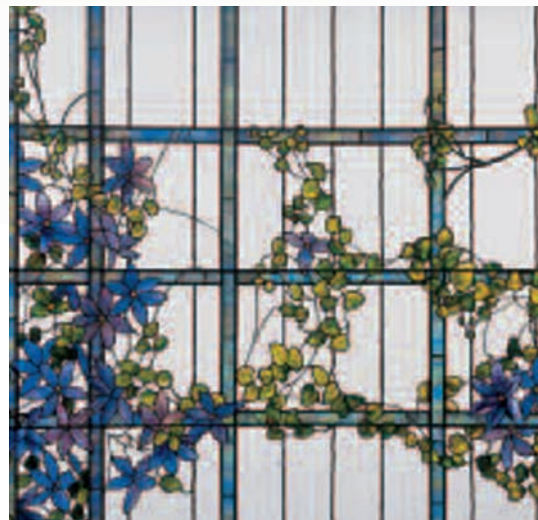
- Write ratios.
- Use properties of proportions.

New Vocabulary

ratio
 proportion
 cross products
 extremes
 means

GET READY for the Lesson

Stained-glass artist Louis Comfort Tiffany used geometric shapes in his designs. In a portion of *Clematis Skylight* shown at the right, rectangular shapes are used as the background for the flowers and vines. Tiffany also used ratio and proportion in the design of this piece.



Write Ratios A **ratio** is a comparison of two quantities using division. The ratio of a to b can be expressed as $\frac{a}{b}$, where b is not zero. This ratio can also be written as $a:b$.

EXAMPLE Write a Ratio

- 1 SOCCER** The U.S. Census Bureau surveyed 9490 schools nationally about their girls' soccer programs. They found that 309,032 girls participated in high school soccer programs in the 2003–2004 school year. Find the ratio of girl soccer players per school rounded to the nearest person.

Divide the number of girl soccer players by the number of schools.

$$\frac{\text{number of girl soccer players}}{\text{number of schools}} = \frac{309,032}{9490} \text{ or about } \frac{33}{1}$$

A ratio in which the denominator is 1 is called a **unit ratio**.

The ratio was 33 girl soccer players per school.

CHECK Your Progress

- 1.** The ratio of football players to high schools in Montgomery County is 546:26. What is the ratio of football players to high schools written as a unit ratio?

Extended ratios can be used to compare three or more numbers. The expression $a:b:c$ means that the ratio of the first two numbers is $a:b$, the ratio of the last two numbers is $b:c$, and the ratio of the first and last numbers is $a:c$.

**Real-World Link**

Mia Hamm is considered to be the greatest female soccer player. She has scored over 149 goals in international soccer.

Source: kidzworld.com

Study Tip

Writing Equations

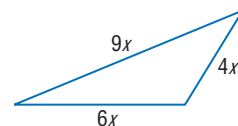
Extended ratio problems require a variable to be the common factor among the terms of the ratio. This will enable you to write an equation to solve the problem.

EXAMPLE Extended Ratios in Triangles

- 2 In a triangle, the ratio of the measures of three sides is 4:6:9, and its perimeter is 190 inches. Find the length of the longest side of the triangle.

Explore You are asked to apply the ratio to the three sides of the triangle and the perimeter to find the longest side.

Plan Recall that equivalent fractions can be found by multiplying the numerator and the denominator by the same number. So, $2:3 = \frac{2}{3} \cdot \frac{x}{x}$ or $\frac{2x}{3x}$. We can rewrite 4:6:9 as $4x:6x:9x$ and use those measures for the triangle's sides.



Solve $4x + 6x + 9x = 190$ Perimeter
 $19x = 190$ Combine like terms.
 $x = 10$ Divide each side by 19.

Now find the measures of the sides: $4x = 4(10)$ or 40, $6x = 6(10)$ or 60, and $9x = 9(10)$ or 90. The longest side is 90 inches.

Check To check the reasonableness of this result, add the lengths of the sides to make sure that the perimeter is 190. $40 + 60 + 90 = 190$ ✓

CHECK Your Progress

2. In a triangle, the ratio of the measures of three sides is 3:3:8, and its perimeter is 392 inches. Find the length of the longest side of the triangle.

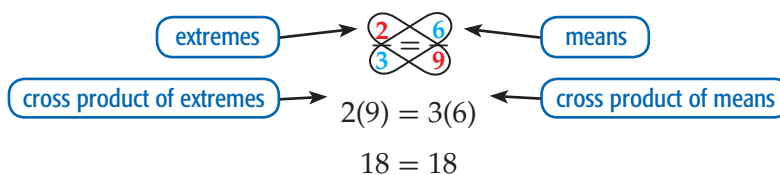
Personal Tutor at geometryonline.com

Reading Math

Proportions When a proportion is written using colons, it is read using the word *to* for the colon. For example, 2:3 is read 2 to 3. The means are the inside numbers, and the extremes are the outside numbers.

extremes
 $2:3 = 6:9$
 means

Use Properties of Proportions An equation stating that two ratios are equal is called a **proportion**. Equivalent fractions set equal to each other form a proportion. Since $\frac{2}{3}$ and $\frac{6}{9}$ are equivalent fractions, $\frac{2}{3} = \frac{6}{9}$ is a proportion. Every proportion has two **cross products**. The cross products in $\frac{2}{3} = \frac{6}{9}$ are 2 times 9 and 3 times 6. The **extremes** of the proportion are 2 and 9. The **means** are 3 and 6.



In a proportion, the product of the means equals the product of the extremes.

$$\frac{a}{b} = \frac{c}{d} \quad b \neq 0, d \neq 0$$

$$(bd)\frac{a}{b} = (bd)\frac{c}{d} \quad \text{Multiply each side by the common denominator, } bd.$$

$$da = bc \quad \text{Simplify.}$$

$$ad = bc \quad \text{Commutative Property}$$

KEY CONCEPT

Property of Proportions

Words For any numbers a and c and any nonzero numbers b and d , $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

Example $\frac{4}{5} = \frac{12}{15}$ if and only if $4 \cdot 15 = 5 \cdot 12$.

To solve a proportion means to find the value of the variable that makes the proportion true.

EXAMPLE Solve Proportions by Using Cross Products

3 Solve each proportion.

a. $\frac{3}{5} = \frac{x}{75}$

$\frac{3}{5} = \frac{x}{75}$ Original proportion

$3(75) = 5x$ Cross products

$225 = 5x$ Multiply.

$45 = x$ Divide each side by 5.

b. $\frac{3x - 5}{4} = \frac{-13}{2}$

$\frac{3x - 5}{4} = \frac{-13}{2}$ Original proportion

$(3x - 5)2 = 4(-13)$ Cross products

$6x - 10 = -52$ Simplify.

$6x = -42$ Add 10 to each side.

$x = -7$ Divide each side by 6.

CHECK Your Progress

3A. $\frac{x}{4} = \frac{11}{-6}$

3B. $\frac{-4}{7} = \frac{6}{2x + 5}$

Proportions can be used to solve problems involving two objects that are said to be *in proportion*. This means that for ratios comparing the measures of all parts of one object with the measures of comparable parts of the other object, a true proportion always exists.

EXAMPLE Solve Problems Using Proportions

4 **AVIATION** A twinjet airplane has a length of 78 meters and a wingspan of 90 meters. A toy model is made in proportion to the real airplane. If the wingspan of the toy is 36 centimeters, find the length of the toy.

$$\frac{\text{plane's length (m)}}{\text{model's length (cm)}} = \frac{\text{plane's wingspan (m)}}{\text{model's wingspan (cm)}}$$

$\frac{78}{x} = \frac{90}{36}$ Substitution

$(78)(36) = x \cdot 90$ Cross products

$2808 = 90x$ Multiply.

$31.2 = x$ Divide each side by 90.

The length of the model is 31.2 centimeters.

CHECK Your Progress

4. The scale on a map shows that 1.5 centimeters represents 100 miles. If the distance on the map from Atlanta to Los Angeles is 29.2 centimeters, approximately how many miles apart are the two cities?

Study Tip

Common Misconception

The proportion shown in Example 4 is not the only correct proportion. There are four equivalent proportions:

$$\frac{a}{b} = \frac{c}{d}, \frac{a}{d} = \frac{c}{b},$$

$$\frac{b}{a} = \frac{d}{c}, \text{ and } \frac{b}{c} = \frac{d}{a}.$$

All of these have identical cross products.

CHECK Your Understanding

Example 1
(p. 380)

1. **CURRENCY** In a recent month, 107 South African rands was equivalent to 18 United States dollars. Find the ratio of rands to dollars.

Example 2
(p. 381)

2. The ratio of the measures of three sides of a triangle is 9:8:7, and its perimeter is 144 units. Find the measure of each side of the triangle.

3. The ratios of the measures of three angles of a triangle are 5:7:8. Find the measure of each angle of the triangle.

Example 3
(p. 382)

Solve each proportion.

4. $\frac{3}{x} = \frac{21}{6}$

5. $\frac{2.3}{4} = \frac{x}{3.7}$

6. $\frac{x-2}{2} = \frac{4}{5}$

Example 4
(p. 382)

7. **MAPS** The scale on a map indicates that 1.5 centimeters represents 200 miles. If the distance on the map between Norfolk, Virginia, and Chapel Hill, North Carolina, measures 1.2 centimeters, approximately how many miles apart are the cities?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–11	1
12–17	2
18–25	3
26–29	4

8. **PETS** Out of a survey of 1000 households, 460 had at least one dog or cat as a pet. What is the ratio of pet owners to households?

9. **BASKETBALL** During tryouts for the basketball team, 30 students tried out for 15 spots on the team. What is the ratio of open spots to the number of students competing?

10. **EDUCATION** In the 2003–2004 school year, Arizona State University had 58,156 students and 2165 full-time faculty members. What was the ratio of the students per faculty member rounded to the nearest person?

11. **SCULPTURE** A replica of *The Thinker* is 10 inches tall. A statue of *The Thinker*, located in front of Grawemeyer Hall on the Belnap Campus of the University of Louisville, is 10 feet tall. What is the ratio of the replica to the statue in Louisville?



Find the measures of the sides of each triangle.

12. The ratio of the measures of three sides of a triangle is 8:7:5. Its perimeter is 240 feet.

13. The ratio of the measures of the sides of a triangle is 3:4:5. Its perimeter is 72 inches.

14. The ratio of the measures of three sides of a triangle are $\frac{1}{4}:\frac{1}{3}:\frac{1}{6}$, and its perimeter is 31.5 inches.

Find the measure of each side of the triangle.

15. The ratio of the measures of three sides of a triangle are $\frac{1}{2}:\frac{1}{3}:\frac{1}{5}$, and its perimeter is 6.2 centimeters. Find the measure of each side of the triangle.

Find the measures of the angles of each triangle.

16. The ratio of the measures of the three angles is 2:5:3.

17. The ratio of the measures of the three angles is 6:9:10.

Solve each proportion.

18. $\frac{3}{8} = \frac{x}{5}$

19. $\frac{w}{6.4} = \frac{1}{2}$

20. $\frac{4x}{24} = \frac{56}{112}$

21. $\frac{11}{20} = \frac{55}{20x}$

22. $\frac{2x - 13}{28} = \frac{-4}{7}$

23. $\frac{4x + 3}{12} = \frac{5}{4}$

24. $\frac{b + 1}{b - 1} = \frac{5}{6}$

25. $\frac{3x - 1}{2} = \frac{-2}{x + 2}$

26. Use the number line at the right to determine the ratio of AC to BH .



27. A cable that is 42 feet long is divided into lengths in the ratio of 3:4. What are the two lengths into which the cable is divided?

LITERATURE For Exercises 28 and 29, use the following information.

Throughout Lewis Carroll's book *Alice's Adventures in Wonderland*, Alice's size changes. Her normal height was about 50 inches tall. She came across a door, about 15 inches high, that led to a garden. Alice's height changes to 10 inches so she can visit the garden.

28. Find the ratio of the height of the door to Alice's height in Wonderland.

29. How tall would the door have been in Alice's normal world?

30. **ENTERTAINMENT** The Great Moments with Mr. Lincoln presentation at Disneyland in California features a life-size animatronic figure of Abraham Lincoln. Before actual construction of the exhibit, Walt Disney and his design company built models that were in proportion to the displays they planned to build. In the model, Lincoln is 8 inches tall. Mr. Lincoln's actual height was 6 feet 4 inches. What is the ratio of the height of the model of Mr. Lincoln compared to his actual height?

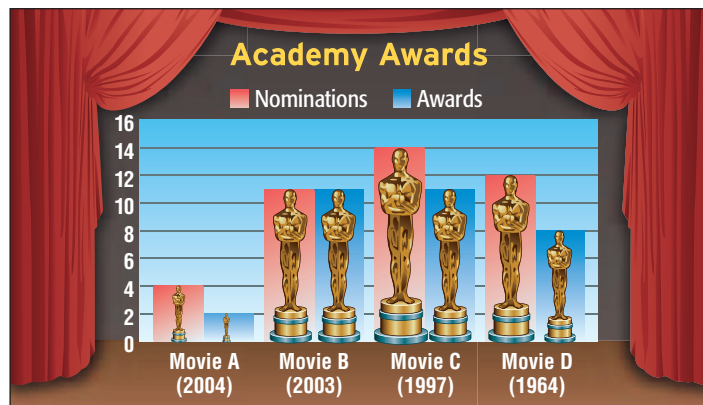


Real-World Link

The attendance at movie theaters in 1970 was 921 million. In 2004 it was 1536 million.

Source: *Statistical Abstract of the United States*

MOVIES For Exercises 31 and 32, refer to the graphic.



Source: oscars.org

31. Of the films listed, which had the greatest ratio of Academy awards to number of nominations?
32. Which film listed had the lowest ratio of awards to nominations?

FOOD For Exercises 33 and 34, use the following information.

There were approximately 295,346,288 people in the United States in a recent year. According to figures from the United States Census, they consumed about 1.4 billion gallons of ice cream that year.

33. Find the approximate consumption of ice cream per person.
34. If there were 8,186,268 people in North Carolina, about how much ice cream might they have been expected to consume?

YEARBOOKS For Exercises 35 and 36, use the following information.

Damon resized a photograph that is 8 inches by 10 inches so that it would fit in a 4-inch by 4-inch area on a yearbook page.

35. Find the maximum dimensions of the reduced photograph.
36. What is the percent of reduction of the length?

GOLDEN RECTANGLES For Exercises 37–39, use the following information.

Many artists have used golden rectangles in their work. In a golden rectangle, the ratio of the length to the width is about 1.618. This ratio is known as the *golden ratio*.

37. A rectangle has dimensions of 19.42 feet and 12.01 feet. Determine if the rectangle is a golden rectangle. Then find the length of the diagonal.
38. **TELEVISION** The *aspect ratio*, or the ratio of the width to the height, of a widescreen television set is 16:9. The aspect ratio of a fullscreen television set is 4:3. Compare these ratios to the golden ratio. Are either television screens golden rectangles? Explain.
39. **RESEARCH** Use the Internet or other sources to find examples of golden rectangles.
40. **OPEN ENDED** Write two proportions with extremes 5 and 8.
41. **REASONING** Explain how you would solve $\frac{28}{48} = \frac{21}{x}$.
42. **Which One Doesn't Belong?** Identify the proportion that doesn't belong with the other three. Explain your reasoning.

$$\frac{3}{8} = \frac{8.4}{22.4}$$

$$\frac{2}{3} = \frac{5}{7.5}$$

$$\frac{5}{6} = \frac{14}{16.8}$$

$$\frac{7}{9} = \frac{19.6}{25.2}$$

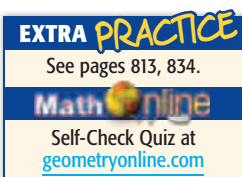
CHALLENGE The ratios of the sides of three polygons are given. Make a conjecture about the type of each polygon described.

43. 2:2:3

44. 3:3:3:3

45. 4:5:4:5

46. *Writing in Math* Refer to page 380. Describe how Louis Comfort Tiffany used ratios. Include four rectangles from the photo that appear to be in proportion, and an estimate in inches of the ratio of the width of the skylight to the length of the skylight given that the dimensions of the rectangle in the bottom left corner are approximately 3.5 inches by 5.5 inches.



H.O.T. Problems



STANDARDIZED TEST PRACTICE

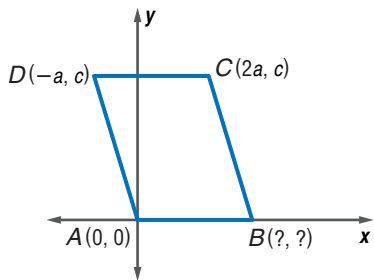
47. A breakfast cereal contains wheat, rice, and oats in the ratio 3:1:2. If the manufacturer makes a mixture using 120 pounds of oats, how many pounds of wheat will be used?
- A 60 lb C 120 lb
B 80 lb D 180 lb

48. **REVIEW** The base of a triangle is 6 centimeters less than twice its height. The area of the triangle is 270 square centimeters. What is the height of the triangle?
- F 12 cm H 18 cm
G 15 cm J 21 cm

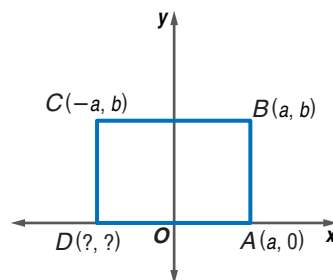
Spiral Review

Name the missing coordinates for each parallelogram or rectangle. (Lesson 6-7)

49.

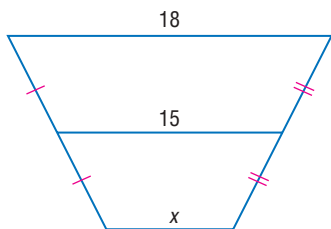


50.

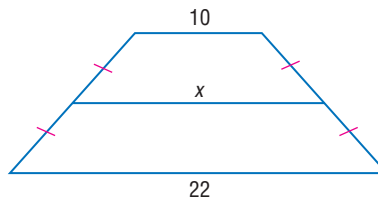


ALGEBRA Find the missing measure for each trapezoid. (Lesson 6-6)

51.



52.

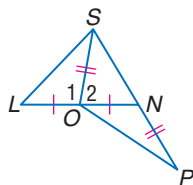


In the figure, \overline{SO} is a median of $\triangle SLN$, $\overline{OS} \cong \overline{NP}$, $m\angle 1 = 3x - 50$, and $m\angle 2 = x + 30$. Determine whether each statement is *always*, *sometimes*, or *never* true. (Lesson 5-5)

53. $LS > SN$

54. $SN < OP$

55. $x = 45$



In the figure, $m\angle 9 = 75$. Find the measure of each angle. (Lesson 3-2)

56. $\angle 3$

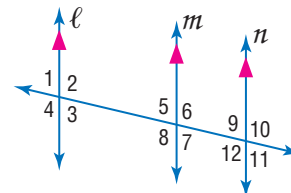
58. $\angle 6$

60. $\angle 11$

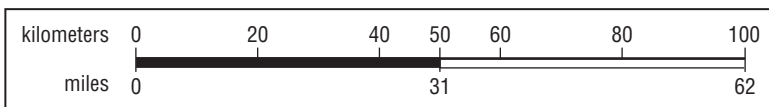
57. $\angle 5$

59. $\angle 8$

61. $\angle 12$



62. **MAPS** On a U.S. map, there is a scale that lists kilometers on the top and miles on the bottom.



Suppose \overline{AB} and \overline{CD} are segments on this map. If $AB = 100$ kilometers and $CD = 62$ miles, is $\overline{AB} \cong \overline{CD}$? Explain. (Lesson 2-7)

GET READY for the Next Lesson

PREREQUISITE SKILL Find the distance between each pair of points to the nearest tenth. (Lesson 1-3)

63. $A(12, 3), B(-8, 3)$

65. $E(\frac{4}{5}, -1), F(2, -\frac{1}{2})$

64. $C(0, 0), D(5, 12)$

66. $G(3, \frac{3}{7}), H(4, -\frac{2}{7})$

Graphing Calculator Lab Fibonacci Sequence and Ratios

Leonardo Pisano (c. 1170–c. 1250), or Fibonacci, was born in Italy but educated in North Africa. As a result, his work is similar to that of other North African authors of that time. His book, *Liber abaci*, published in 1202, introduced what is now called the Fibonacci sequence, in which each term after the first two terms is the sum of the two numbers before it.

Term	1	2	3	4	5	6	7
Fibonacci Number	1	1	2	3	5	8	13
			↑	↑	↑	↑	↑
			1 + 1	1 + 2	2 + 3	3 + 5	5 + 8

ACTIVITY

You can use CellSheet on a TI-83/84 Plus graphing calculator to create terms of the Fibonacci sequence. Then compare each term with its preceding term.

Step 1 Access the CellSheet application by pressing the **APPS** key. Choose the number for CellSheet and press **ENTER**.



Step 2 Enter the column headings in row 1. Use the **ALPHA** key to enter letters and press ["] at the beginning of each label.

Step 3 Enter 1 into cell A2. Then insert the formula =A2+1 in cell A3. Press **STO►** to insert the = in the formula. Then use F3 to copy this formula and use F4 to paste it in each cell in the column.

Step 4 In column B, we will record the Fibonacci numbers. Enter 1 in cells B2 and B3 since you do not have two previous terms to add. Then insert the formula =B2+B3 in cell B4. Copy this formula down the column.



Step 5 In column C, we will find the ratio of each term to its preceding term. Enter 1 in cell C2 since there is no preceding term. Then enter =B3/B2 in cell C3. Copy this formula down the column. *The screens show the results for terms 1 through 10.*

ANALYZE THE RESULTS

1. What happens to the Fibonacci number as the number of the term increases?
2. What pattern of odd and even numbers do you notice in the Fibonacci sequence?
3. As the number of terms gets greater, what pattern do you notice in the ratio column?
4. Extend the spreadsheet to calculate fifty terms of the Fibonacci sequence. Describe any differences in the patterns you described in Exercises 1–3.
5. **MAKE A CONJECTURE** How might the Fibonacci sequence relate to the golden ratio?

Similar Polygons

Main Ideas

- Identify similar figures.
- Solve problems involving scale factors.

New Vocabulary

similar polygons
scale factor

GET READY for the Lesson

M.C. Escher (1898–1972) was a Dutch graphic artist known for drawing impossible structures, spatial illusions, and repeating interlocking geometric patterns. The image at the right is a print of Escher’s *Circle Limit IV*. It includes winged images that have the same shape, but are different in size. Also note that there are similar dark images and similar light images.



Circle Limit IV
M.C. Escher (1960)

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Identify Similar Figures When polygons have the same shape but may be different in size, they are called **similar polygons**.

Study Tip

Similarity and Congruence

If two polygons are congruent, they are also similar. All of the corresponding angles are congruent, and the lengths of the corresponding sides have a ratio of 1:1.

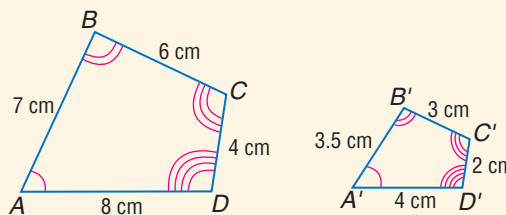
KEY CONCEPT

Similar Polygons

Words Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.

Symbol \sim is read *is similar to*

Example



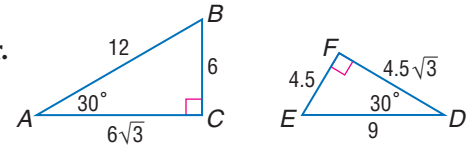
The order of the vertices in a similarity statement is important. It identifies the corresponding angles and the corresponding sides.

similarity statement	congruent angles	corresponding sides
<p>$ABCD \sim EFGH$</p>	$\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $\angle D \cong \angle H$	$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$

Like congruent polygons, similar polygons may be repositioned so that corresponding parts are easy to identify.

EXAMPLE Similar Polygons

- 1 Determine whether the pair of triangles is similar. Justify your answer.



All right angles are congruent, so $\angle C \cong \angle F$. Since $m\angle A = m\angle D$, $\angle A \cong \angle D$. By the Third Angle Theorem, $\angle B \cong \angle E$. Thus, all corresponding angles are congruent.

Now determine whether corresponding sides are proportional.

Sides opposite 90° angle	Sides opposite 30° angle	Sides opposite 60° angle
$\frac{AB}{DE} = \frac{12}{9}$ or $1.\bar{3}$	$\frac{BC}{EF} = \frac{6}{4.5}$ or $1.\bar{3}$	$\frac{AC}{DF} = \frac{6\sqrt{3}}{4.5\sqrt{3}}$ or $1.\bar{3}$

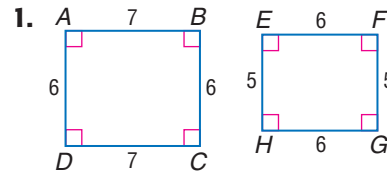
The ratios of the measures of the corresponding sides are equal, and the corresponding angles are congruent, so $\triangle ABC \sim \triangle DEF$.

Study Tip

Common Misconception

When two figures have vertices that are in alphabetical order, this does not mean that the corresponding vertices in the similarity statement will follow alphabetical order.

CHECK Your Progress



Scale Factors When you compare the lengths of corresponding sides of similar figures, you usually get a numerical ratio. This ratio is called the **scale factor** for the two figures.

Real-World EXAMPLE

- 2 **MODEL CARS** This is a miniature replica of a 1923 Checker Cab. The length of the model is 6.5 inches long. If the length of the car is 13 feet, what is the scale factor of the model compared to the car?



Both measurements need to have the same unit of measure.

$$13(12) = 156 \text{ inches} \quad \text{Convert feet to inches.}$$

$$\frac{\text{length of model}}{\text{length of real car}} = \frac{6.5 \text{ inches}}{156 \text{ inches}} \text{ or } \frac{1}{24} \quad \text{Write a proportion and simplify.}$$

The scale factor is $\frac{1}{24}$. So the model is $\frac{1}{24}$ the length of the real car.

CHECK Your Progress

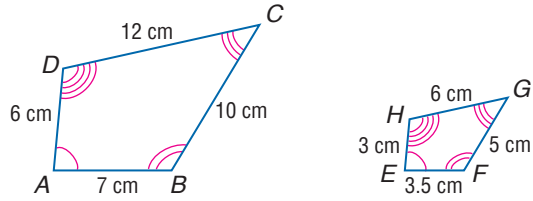
2. **SCALE MODELS** The height of the Soldiers' National Monument in Gettysburg, Pennsylvania, is 60 feet. The height of a model is 10 inches. What is the scale factor of the model compared to the original?

Study Tip

Checking Results

When solving word problems, analyze your answer to make sure it makes sense and that you answered the question asked.

When finding the scale factor for two similar polygons, the scale factor will depend on the order of comparison.



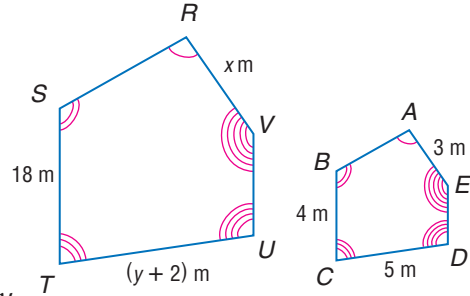
- The scale factor of quadrilateral $ABCD$ to quadrilateral $EFGH$ is 2.
- The scale factor of quadrilateral $EFGH$ to quadrilateral $ABCD$ is $\frac{1}{2}$.

EXAMPLE Proportional Parts and Scale Factor

3 The two polygons are similar.

- a. Write a similarity statement. Then find x , y , and UT .

Use the congruent angles to write the corresponding vertices in order.
 polygon $RSTUV \sim$ polygon $ABCDE$
 Now write proportions to find x and y .



To find x :

$$\frac{ST}{BC} = \frac{VR}{EA} \quad \text{Similarity proportion}$$

$$\frac{18}{4} = \frac{x}{3} \quad \begin{array}{l} ST = 18, BC = 4 \\ VR = x, EA = 3 \end{array}$$

$$18(3) = 4(x) \quad \text{Cross products}$$

$$54 = 4x \quad \text{Multiply.}$$

$$13.5 = x \quad \text{Divide each side by 4.}$$

To find y :

$$\frac{ST}{BC} = \frac{UT}{DC} \quad \text{Similarity proportion}$$

$$\frac{18}{4} = \frac{y + 2}{5} \quad \begin{array}{l} ST = 18, BC = 4 \\ UT = y + 2, DC = 5 \end{array}$$

$$18(5) = 4(y + 2) \quad \text{Cross products}$$

$$90 = 4y + 8 \quad \text{Multiply.}$$

$$82 = 4y \quad \text{Subtract 8 from each side.}$$

$$20.5 = y \quad \text{Divide each side by 4.}$$

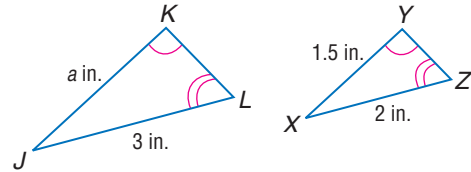
$UT = y + 2$, so $UT = 20.5 + 2$ or 22.5 .

- b. Find the scale factor of polygon $RSTUV$ to polygon $ABCDE$.

The ratio of the lengths of any two corresponding sides is $\frac{ST}{BC} = \frac{18}{4}$ or $\frac{9}{2}$.

CHECK Your Progress

3. Write a similarity statement. Then find a and the scale factor of $\triangle JKL$ to $\triangle XYZ$.



EXAMPLE Enlargement or Reduction of a Figure

- 4 Triangle ABC is similar to $\triangle XYZ$ with a scale factor of $\frac{2}{3}$. If the sides of $\triangle ABC$ are 6, 8, and 10 inches, what are the lengths of the sides of $\triangle XYZ$?

Write proportions for finding side measures.

$$\frac{\triangle ABC}{\triangle XYZ} \rightarrow \frac{6}{x} = \frac{2}{3}$$

$$18 = 2x$$

$$9 = x$$

$$\frac{\triangle ABC}{\triangle XYZ} \rightarrow \frac{8}{y} = \frac{2}{3}$$

$$24 = 2y$$

$$12 = y$$

$$\frac{\triangle ABC}{\triangle XYZ} \rightarrow \frac{10}{z} = \frac{2}{3}$$

$$30 = 2z$$

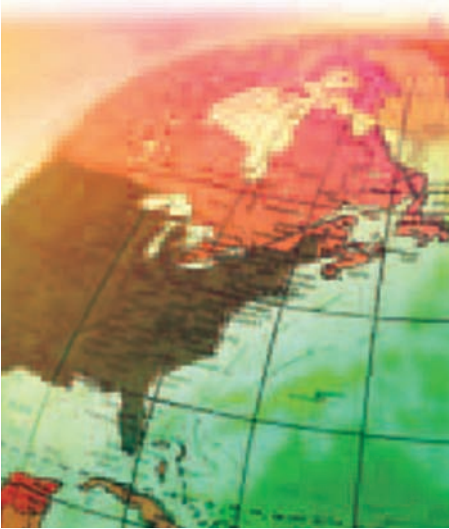
$$15 = z$$

The lengths of the sides of $\triangle XYZ$ are 9, 12, and 15 inches.

Study Tip

Checking Solutions

To verify the scale factor, find the ratio of two other corresponding sides.



Real-World Link

The earliest mapmakers placed grid lines over a map to locate places. This is an early example of coordinate geometry. In modern cartography, latitude and longitude are used to determine the position of places on Earth.

Source: *History of Cartography*

Study Tip

Units of Time

Remember that there are 60 minutes in an hour. When rewriting $\frac{328}{60}$ as a mixed number, you could also write $5\frac{28}{60}$, which means 5 hours 28 minutes.

CHECK Your Progress

4. Rectangle $QRST$ is similar to rectangle $JKLM$ with a scale factor of $\frac{4}{5}$. If the lengths of the sides of rectangle $QRST$ are 5 centimeters and 12 centimeters, what are the lengths of the sides of rectangle $JKLM$?

EXAMPLE Scales on Maps

- 5 **MAPS** The scale on the map of New Mexico is 2 centimeters = 160 miles. The width of New Mexico through Albuquerque on the map is 4.1 centimeters. How long would it take to drive across New Mexico if you drove at an average of 60 miles per hour?



Explore Every 2 centimeters represents 160 miles. The distance across the map is 4.1 centimeters.

Plan Create a proportion relating the measurements to the scale to find the distance in miles. Then use the formula $d = rt$ to find the time.

Solve

$$\begin{array}{l} \text{centimeters} \rightarrow \\ \text{miles} \rightarrow \end{array} \frac{2}{160} = \frac{4.1}{x} \quad \begin{array}{l} \leftarrow \text{centimeters} \\ \leftarrow \text{miles} \end{array}$$

$$2x = 656 \quad \text{Cross products}$$

$$x = 328 \quad \text{Divide each side by 2.}$$

The indicated distance is 328 miles.

$$d = rt$$

$$328 = 60t \quad d = 328 \text{ and } r = 60$$

$$\frac{328}{60} = t \quad \text{Divide each side by 60.}$$

$$5\frac{7}{15} = t \quad \text{Simplify.}$$

It would take $5\frac{7}{15}$ hours or 5 hours and 28 minutes to drive across New Mexico at an average of 60 miles per hour.

Check Reexamine the scale. The map is about 4 centimeters wide, so the distance across New Mexico is about 320 miles. The answer is about 5.5 hours and at 60 miles per hour, the trip would be 330 miles. The two distances are close estimates, so the answer is reasonable.

CHECK Your Progress

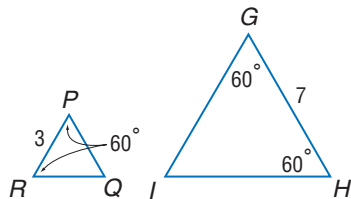
5. The distance on the map from Las Cruces to Roswell is 1.8 centimeters. How long would it take to drive if you drove an average of 55 miles per hour?

Online Personal Tutor at geometryonline.com

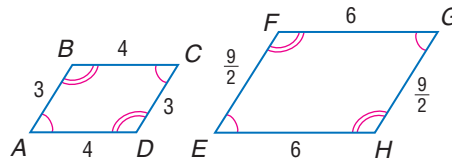
Example 1
(p. 389)

Determine whether each pair of figures is similar. Justify your answer.

1.



2.



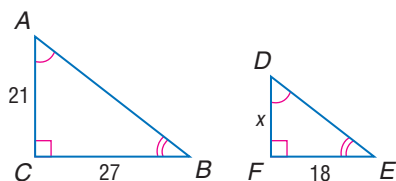
Example 2
(p. 389)

3. MODELS Suki made a scale model of a local bridge. If the span of the bridge was 50 feet and the span of the model was 6 inches, what scale factor did Suki use to build her model?

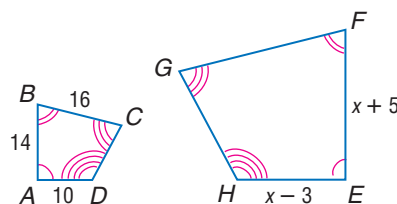
Example 3
(p. 390)

Each pair of polygons is similar. Write a similarity statement, and find x , the measure(s) of the indicated side(s), and the scale factor.

4. \overline{DF}



5. \overline{FE} , \overline{EH} , and \overline{GF}



Example 4
(p. 390)

6. Triangle JKL is similar to $\triangle TUV$ with a scale factor of $\frac{3}{4}$. If the lengths of the sides of $\triangle TUV$ are 4, 6, and 8 centimeters, what are the lengths of the sides of $\triangle JKL$?

Example 5
(p. 391)

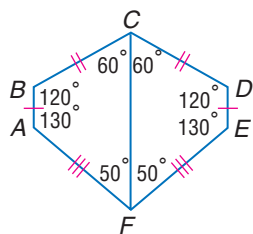
7. MAPS Refer to Example 5 on page 391. The distance on the map from Albuquerque to Roswell is 1.9 centimeters. How long would it take to drive if you drove at an average of 65 miles per hour?

Exercises

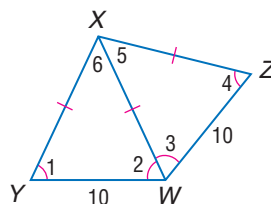
HOMESCHOOL HELP	
For Exercises	See Examples
8–11	1
12, 13	2
14–19	3
22, 23	4
20, 21	5

Determine whether each pair of figures is similar. Justify your answer.

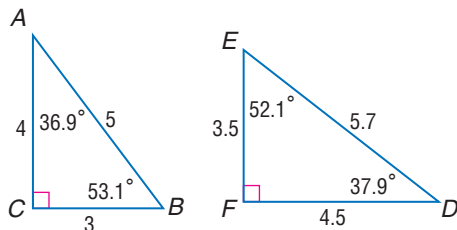
8.



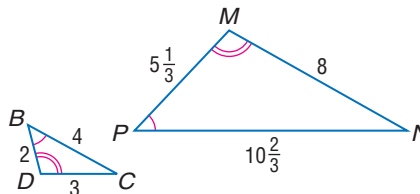
9.



10.



11.





Real-World Link

Scale replicas of the Statue of Liberty are also found in Alabama, Colorado, and Georgia.

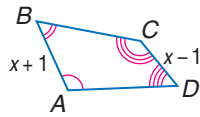
Source: worldweb.com

12. PHOTOCOPYING Mrs. Barojas walked to a copier in her office, made a copy of her proposal, and sent the original to one of her customers. When Mrs. Barojas looked at her copy before filing it, she saw that the copy had been made at an 80% reduction. She needs her filing copy to be the same size as the original. What enlargement scale factor must she use on the first copy to make a second copy the same size as the original?

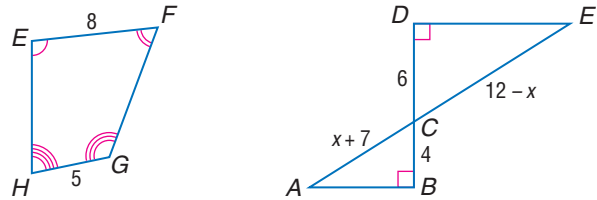
13. ARCHITECTURE A replica of the Statue of Liberty in Austin, Texas, is $16\frac{3}{4}$ feet tall. The statue in New York Harbor is 151 feet. What is the scale factor comparing the actual statue to the replica?

Each pair of polygons is similar. Write a similarity statement, and find x , the measures of the indicated sides, and the scale factor.

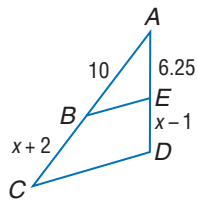
14. \overline{AB} and \overline{CD}



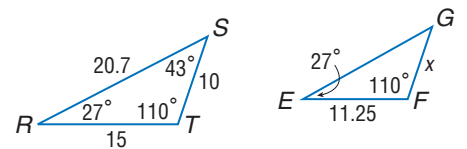
15. \overline{AC} and \overline{CE}



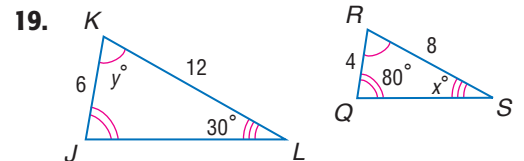
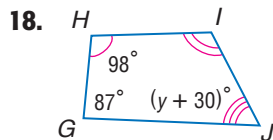
16. \overline{BC} and \overline{ED}



17. \overline{GF} and \overline{EG}



Each pair of polygons is similar. Find x and y . Round to the nearest hundredth if necessary.



MAPS For Exercises 20 and 21 use the following information.

The scale on the map of Georgia is 1 in. = 40 miles.

20. The width of Georgia from Columbus to Dublin is $2\frac{3}{4}$ inches on the map. How long would it take to drive this distance if you drove at an average of 55 miles per hour?

21. The distance from Atlanta to Savannah is $5\frac{3}{4}$ inches on the map. How long would it take to drive from Atlanta to Savannah if you drove at an average of 65 miles per hour?



Cross-Curricular Project

MAPS You can use a map to begin to find the hidden treasure. Visit geometryonline.com.



Real-World Link

Crew Stadium in Columbus, Ohio, was specifically built for Major League Soccer. The dimensions of the field are about 69 meters by 105 meters.

Source: MLSnet.com

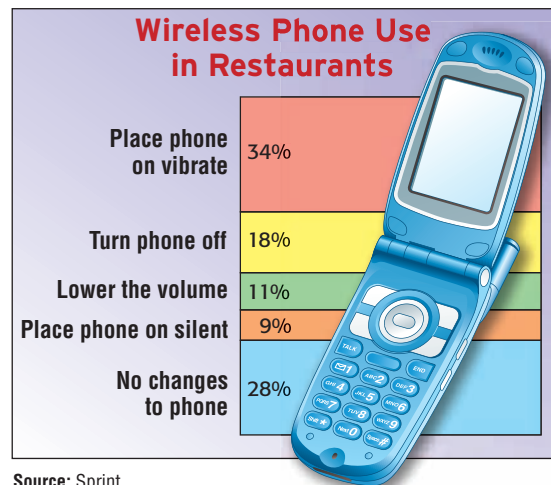
22. A triangle has side lengths of 3 meters, 5 meters, and 4 meters. The triangle is enlarged so that the larger triangle is similar to the original and the scale factor is 5. Find the perimeter of the larger triangle.
23. A rectangle with length 60 centimeters and height 40 centimeters is reduced so that the new rectangle is similar to the original and the scale factor is $\frac{1}{4}$. Find the length and width of the new rectangle.

SPORTS Make a scale drawing of each playing field using the given scale.

24. A baseball diamond is a square 90 feet on each side. Use the scale $\frac{1}{4}$ in. = 9 ft.
25. A high school football field is a rectangle with length 300 feet and width 160 feet. Use the scale $\frac{1}{16}$ in. = 5 ft.

ANALYZE GRAPHS For Exercises 26 and 27, refer to the graphic, which uses rectangles to represent percents.

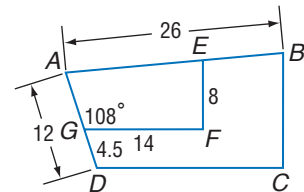
26. Are the rectangles representing 34% and 18% similar? Explain.
27. What is the ratio of the areas of the rectangles representing 18% and 9%? Compare the ratio of the areas to the ratio of the percents.



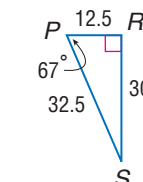
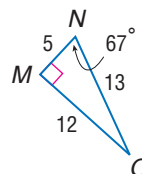
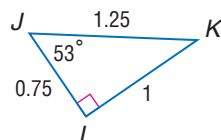
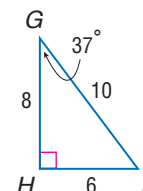
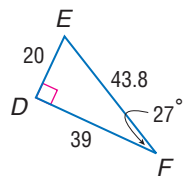
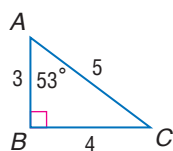
Source: Sprint

For Exercises 28–35, use the following information to find each measure. Polygon $ABCD \sim$ polygon $AEFG$, $m\angle AGF = 108^\circ$, $GF = 14$, $AD = 12$, $DG = 4.5$, $EF = 8$, and $AB = 26$.

28. scale factor of trapezoid $ABCD$ to trapezoid $AEFG$
29. AG
30. DC
31. $m\angle ADC$
32. BC
33. perimeter of trapezoid $ABCD$
34. perimeter of trapezoid $AEFG$
35. ratio of the perimeter of polygon $ABCD$ to the perimeter of polygon $AEFG$



36. Determine which of the following right triangles are similar. Justify your answer.



Determine whether each statement is *always*, *sometimes*, or *never* true.

37. Two congruent triangles are similar.
38. Two squares are similar.
39. Two isosceles triangles are similar.
40. Two obtuse triangles are similar.
41. Two equilateral triangles are similar.

COORDINATE GEOMETRY For Exercises 42–47, use the following information.

Scale factors can be used to produce similar figures. The resulting figure is an enlargement or reduction of the original figure depending on the scale factor. Triangle ABC has vertices $A(0, 0)$, $B(8, 0)$, and $C(2, 7)$. Suppose the coordinates of each vertex are multiplied by 2 to create the similar triangle $A'B'C'$.

42. Find the coordinates of the vertices of $\triangle A'B'C'$.
43. Graph $\triangle ABC$ and $\triangle A'B'C'$.
44. Use the Distance Formula to find the measures of the sides of each triangle.
45. Find the ratios of the sides that appear to correspond.
46. How could you use slope to determine if angles are congruent?
47. Is $\triangle ABC \sim \triangle A'B'C'$? Explain your reasoning.

EXTRA PRACTICE
See pages 814, 834.
Math online
Self-Check Quiz at
geometryonline.com

H.O.T. Problems

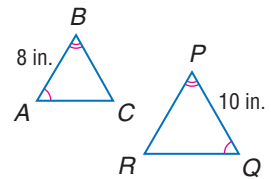
48. **FIND THE ERROR** Roberto and Garrett have calculated the scale factor for two similar triangles. Who is correct? Explain your reasoning.

Roberto

$$\frac{AB}{QP} = \frac{8}{10} = \frac{4}{5}$$

Garrett

$$\frac{QP}{AB} = \frac{10}{8} = \frac{5}{4}$$



49. **OPEN ENDED** Find a counterexample for the statement *All rectangles are similar*.

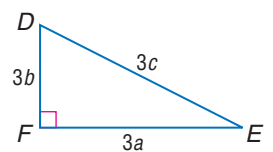
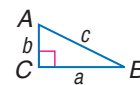
CHALLENGE For Exercises 50–52, use the following information.

Rectangle $ABCD$ is similar to rectangle $WXYZ$ with sides in a ratio of 4:1.

50. What is the ratio of the areas of the two rectangles?
51. Suppose the dimension of each rectangle is tripled. What is the new ratio of the sides of the rectangles?
52. What is the ratio of the areas of these larger rectangles?

CHALLENGE For Exercises 53 and 54, $\triangle ABC \sim \triangle DEF$.

53. Show that the perimeters of $\triangle ABC$ and $\triangle DEF$ have the same ratio as their corresponding sides.



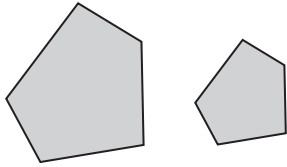
54. If 6 units are added to the lengths of each side, are the new triangles similar? Explain.

Writing in Math Refer to *Circle Limit IV* on page 388.

55. Describe how M.C. Escher used similar figures to create the artwork.
56. **RESEARCH** This art print is a model of a non-Euclidean geometry called *hyperbolic geometry*. Hyperbolic geometry is a two-dimensional space. In this geometry system, lines are arcs with ends that are perpendicular to the edge of the disk. Use the Internet or other source to research hyperbolic geometry. Compare and contrast this geometry system with Euclidean geometry.

STANDARDIZED TEST PRACTICE

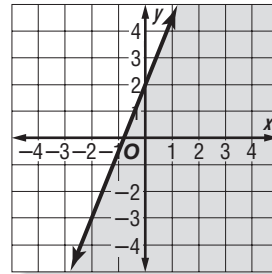
57. A scale factor of $\frac{2}{3}$ was used to produce the smaller pentagon from the larger one.



How does the perimeter of the smaller pentagon compare to the perimeter of the larger pentagon?

- A The perimeter is $\frac{2}{3}$ as large.
 B The perimeter is $\frac{4}{9}$ as large.
 C The perimeter is $\frac{8}{27}$ as large.
 D The perimeter is $\frac{1}{3}$ as large.

58. **REVIEW** Which inequality *best* represents the graph below?



- F $2y - 5x < 4$
 G $2y + 5x \geq 2$
 H $2y + 5x < 4$
 J $2y - 5x > 2$

Spiral Review

Solve each proportion. (Lesson 7-1)

59. $\frac{b}{7.8} = \frac{2}{3}$

60. $\frac{c-2}{c+3} = \frac{5}{4}$

61. $\frac{2}{4y+5} = \frac{-4}{y}$

62. $\frac{2x+3}{x-1} = \frac{-4}{5}$

63. $\frac{2d-8}{6} = \frac{3d+4}{-2}$

64. $\frac{-5}{3k+1} = \frac{-3}{2k-6}$

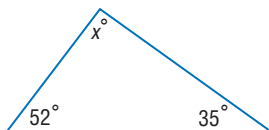
Position and label each quadrilateral on a coordinate plane. (Lesson 6-7)

65. parallelogram with height c and width b

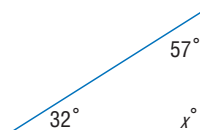
66. rectangle with width $2a$ and height b

Find x . (Lesson 4-2)

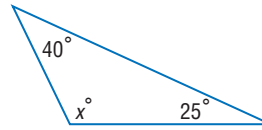
67.



68.



69.

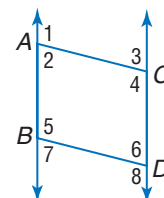


70. Suppose two parallel lines are cut by a transversal and $\angle 1$ and $\angle 2$ are alternate interior angles. Find $m\angle 1$ and $m\angle 2$ if $m\angle 1 = 10x - 9$ and $m\angle 2 = 9x + 3$. (Lesson 3-2)

GET READY for the Next Lesson

PREREQUISITE SKILL In the figure, $\overline{AB} \parallel \overline{CD}$, $\overline{AC} \parallel \overline{BD}$, and $m\angle 4 = 118$. Find the measure of each angle. (Lesson 3-2)

71. $\angle 1$
 72. $\angle 2$
 73. $\angle 3$
 74. $\angle 5$
 75. $\angle 6$
 76. $\angle 8$



7-3

Similar Triangles



▶ GET READY for the Lesson

Main Ideas

- Identify similar triangles.
- Use similar triangles to solve problems.

The Eiffel Tower was built in Paris for the 1889 world exhibition by Gustave Eiffel. Eiffel (1832–1923) was a French engineer who specialized in revolutionary steel constructions. He used thousands of triangles, some the same shape but different in size, to build the Eiffel Tower because triangular shapes result in rigid construction.

Identify Similar Triangles In Chapter 4, you learned several tests to determine whether two triangles are congruent. There are also tests to determine whether two triangles are similar.

GEOMETRY LAB

Similar Triangles

- Draw $\triangle DEF$ with $m\angle D = 35$, $m\angle F = 80$, and $DF = 4$ centimeters.
- Draw $\triangle RST$ with $m\angle T = 35$, $m\angle S = 80$, and $ST = 7$ centimeters.
- Measure \overline{EF} , \overline{ED} , \overline{RS} , and \overline{RT} .
- Calculate the ratios $\frac{FD}{ST}$, $\frac{EF}{RS}$, and $\frac{ED}{RT}$.

ANALYZE THE RESULTS

1. What can you conclude about all of the ratios?
2. Repeat the activity with two more triangles with the same angle measures but different side measures. Then repeat the activity with a third pair of triangles. Are all of the triangles similar? Explain.
3. What are the minimum requirements for two triangles to be similar?

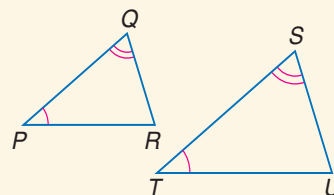
The previous lab leads to the following postulate.

POSTULATE 7.1

Angle-Angle (AA) Similarity

If the two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Example: $\angle P \cong \angle T$ and $\angle Q \cong \angle S$,
so $\triangle PQR \sim \triangle TSU$.

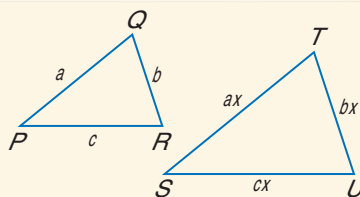


You can use the AA Similarity Postulate to prove two theorems that also verify triangle similarity.

THEOREMS 7.1–7.2

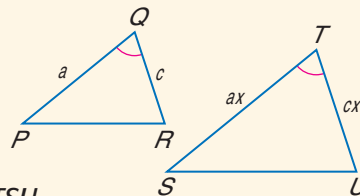
7.1 Side-Side-Side (SSS) Similarity If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

Example: $\frac{PQ}{ST} = \frac{QR}{SU} = \frac{RP}{UT}$, so $\triangle PQR \sim \triangle TSU$.



7.2 Side-Angle-Side (SAS) Similarity If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

Example: $\frac{PQ}{ST} = \frac{QR}{SU}$ and $\angle Q \cong \angle S$, so $\triangle PQR \sim \triangle TSU$.

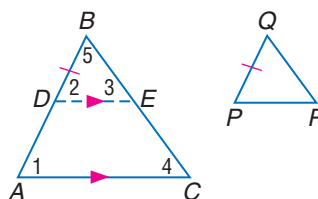


You will prove Theorem 7.2 in Exercise 23.

PROOF Theorem 7.1

Given: $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA}$

Prove: $\triangle BAC \sim \triangle QPR$
Locate D on \overline{AB} so that $\overline{DB} \cong \overline{PQ}$ and draw \overline{DE} so that $\overline{DE} \parallel \overline{AC}$.



Paragraph Proof:

Since $\overline{DE} \parallel \overline{AC}$, $\angle 2$ and $\angle 1$ and $\angle 3$ and $\angle 4$ are corresponding angles. Therefore, $\angle 2 \cong \angle 1$ and $\angle 3 \cong \angle 4$. By AA Similarity, $\triangle BDE \sim \triangle BAC$.

Since $\overline{DB} \cong \overline{PQ}$, $DB = PQ$. By substitution, $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA}$ becomes

$$\frac{DB}{AB} = \frac{QR}{BC} = \frac{RP}{CA}.$$

By the definition of similar polygons, $\frac{DB}{AB} = \frac{BE}{BC} = \frac{ED}{CA}$.

By substitution, $\frac{QR}{BC} = \frac{BE}{BC}$ and $\frac{RP}{CA} = \frac{ED}{CA}$. This means that $QR = BE$ and

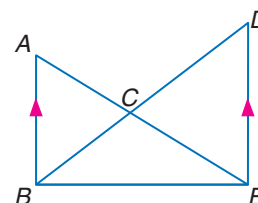
$RP = ED$ or $\overline{QR} \cong \overline{BE}$ and $\overline{RP} \cong \overline{ED}$. With these congruences and $\overline{DB} \cong \overline{PQ}$, $\triangle BDE \cong \triangle QPR$ by SSS. By CPCTC, $\angle B \cong \angle Q$ and $\angle 2 \cong \angle P$. But $\angle 2 \cong \angle A$, so $\angle A \cong \angle P$. By AA Similarity, $\triangle BAC \sim \triangle QPR$.

STANDARDIZED TEST EXAMPLE

Are Triangles Similar?

1 In the figure, $\overline{AB} \parallel \overline{DE}$. Which theorem or postulate can be used to prove $\triangle ACB \sim \triangle ECD$?

- A ASA B SSS C AA D SAS



Read the Test Item You are asked to identify which theorem or postulate can be used to prove that $\triangle ACB$ is similar to $\triangle ECD$.

Solve the Test Item Since $\overline{AB} \parallel \overline{DE}$, $\angle BAE \cong \angle DEA$ by the Alternate Interior Angles Theorem. $\angle ACB \cong \angle ECD$ by the Vertical Angle Theorem. So, by AA Similarity, $\triangle ACB \sim \triangle ECD$. The answer is C.

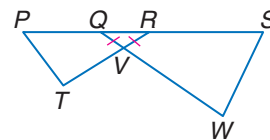
Test-Taking Tip

Overlapping Triangles

When two triangles overlap, you may wish to draw them separately so the corresponding parts are in the same position on the paper. Then write the corresponding angles and sides.

CHECK Your Progress

1. In the figure, $\overline{QV} \cong \overline{RV}$, $PR = 9$, $QS = 15$, $TR = 12$, and $QW = 20$. Which statement *must* be true?



- F $\triangle PTR \sim \triangle QWS$ H $\triangle PTR \sim \triangle QVR$
 G $\triangle QVR \sim \triangle SWQ$ J $\triangle PTR \sim \triangle SWQ$

Like the congruence of triangles, similarity of triangles is reflexive, symmetric, and transitive.

THEOREM 7.3

Similarity of triangles is reflexive, symmetric, and transitive.

Examples:

Reflexive: $\triangle ABC \sim \triangle ABC$ **Symmetric:** If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$.

Transitive: If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle GHI$, then $\triangle ABC \sim \triangle GHI$.

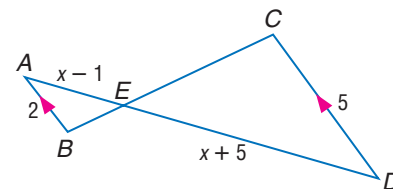
You will prove Theorem 7.3 in Exercise 25.

Use Similar Triangles Similar triangles can be used to solve problems.

EXAMPLE Parts of Similar Triangles

- 2 **ALGEBRA** Find AE and DE .

Since $\overline{AB} \parallel \overline{CD}$, $\angle BAE \cong \angle CDE$ and $\angle ABE \cong \angle DCE$ because they are the alternate interior angles. By AA Similarity, $\triangle ABE \sim \triangle DCE$. Using the definition of similar polygons, $\frac{AB}{DC} = \frac{AE}{DE}$.



$$\frac{AB}{DC} = \frac{AE}{DE}$$

$$\frac{2}{5} = \frac{x-1}{x+5} \quad \text{Substitution}$$

$$2(x+5) = 5(x-1) \quad \text{Cross products}$$

$$2x+10 = 5x-5 \quad \text{Distributive Property}$$

$$-3x = -15 \quad \text{Subtract } 5x \text{ and } 10 \text{ from each side.}$$

$$x = 5 \quad \text{Divide each side by } -3.$$

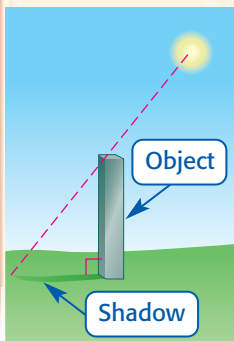
Now find AE and ED .

$$\begin{aligned} AE &= x - 1 & ED &= x + 5 \\ &= 5 - 1 \text{ or } 4 & &= 5 + 5 \text{ or } 10 \end{aligned}$$

Study Tip

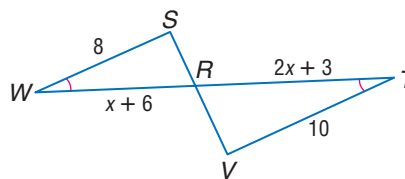
Shadow Problems

In shadow problems, we assume that a right triangle is formed by the Sun's ray from the top of the object to the end of the shadow.



CHECK Your Progress

2. Find WR and RT .



Similar triangles can be used to find measurements indirectly.





Real-World EXAMPLE

Indirect Measurement

3

ROLLER COASTERS For a school project, Hallie needs to determine the height of the Superman roller coaster in Mitchellville, Maryland. She is 5 feet tall and her shadow is 2 feet 9 inches long. If the length of the shadow of the roller coaster is 110 feet, how tall is the roller coaster?

Assuming that the Sun's rays form similar triangles, the following proportion can be written.

$$\frac{\text{height of the roller coaster}}{\text{height of Hallie}} = \frac{\text{roller coaster shadow length}}{\text{Hallie's shadow length}}$$

Now, substitute the known values and let x be the height of the roller coaster.

$$\frac{x}{5} = \frac{110}{2.75} \quad \text{Substitution}$$

$$x \cdot 2.75 = 5(110) \quad \text{Cross products}$$

$$2.75x = 550 \quad \text{Simplify.}$$

$$x = 200 \quad \text{Divide each side by 2.75.}$$

The roller coaster is 200 feet tall.



CHECK Your Progress

3. Alex is standing next to the Palmetto Building in Columbia, South Carolina. He is 6 feet tall and the length of his shadow is 9 feet. If the length of the shadow of the tower is 322.5 feet, how tall is the tower?



Personal Tutor at geometryonline.com

Concepts
in Motion

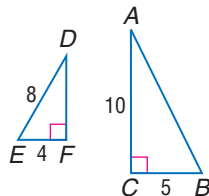
Interactive Lab
geometryonline.com

CHECK Your Understanding

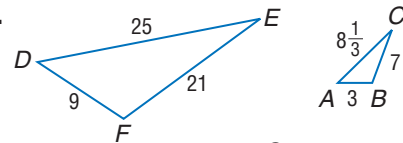
Example 1
(p. 398)

Determine whether each pair of triangles is similar. Justify your answer.

1.



2.



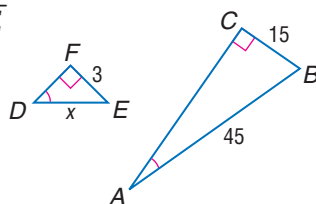
3. **MULTIPLE CHOICE** If $\triangle ABC$ and $\triangle FGH$ are two triangles such that $\angle A \cong \angle F$, which of the following would be sufficient to prove the triangles are similar?

A $\frac{BC}{GH} = \frac{AC}{FH}$ B $\frac{AC}{FH} = \frac{AB}{FG}$ C $\frac{AB}{FG} = \frac{BC}{GH}$ D $\frac{AB}{BC} = \frac{FG}{GH}$

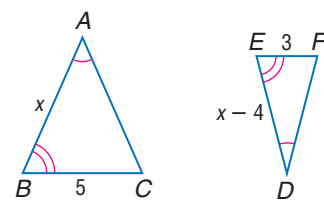
Example 2
(p. 399)

ALGEBRA Identify the similar triangles. Find x and the measures of the indicated sides.

4. \overline{DE}



5. \overline{AB} and \overline{DE}

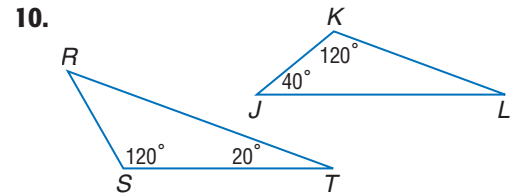
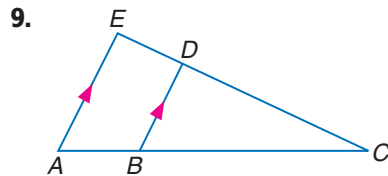
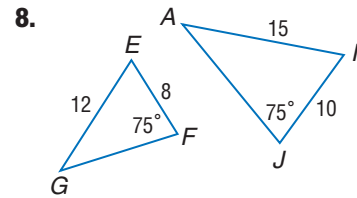
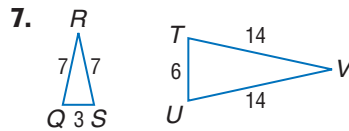


Example 3
(p. 400)

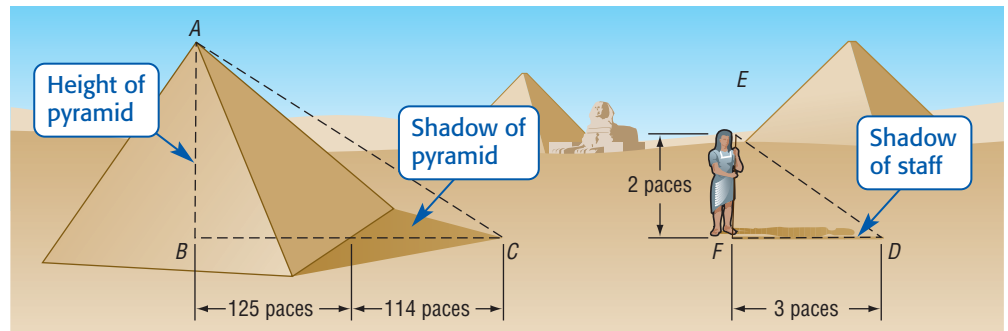
6. **COMMUNICATION** A cell phone tower casts a 100-foot shadow. At the same time, a 4 foot 6 inch post near the tower casts a shadow of 3 feet 4 inches. Find the height of the tower. (*Hint: Make a drawing.*)

HOMEWORK HELP	
For Exercises	See Examples
7–10	1
11–13	3
14–17	2

Determine whether each pair of triangles is similar. Justify your answer.



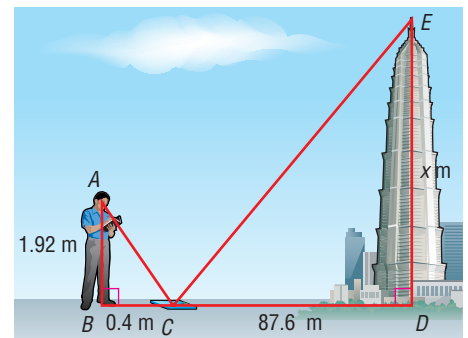
11. **HISTORY** The Greek mathematician Thales was the first to measure the height of a pyramid by using geometry. He showed that the ratio of a pyramid to a staff was equal to the ratio of one shadow to the other. If a pace is about 3 feet, approximately how tall was the pyramid at that time?



TOWERS For Exercises 12 and 13, use the following information.

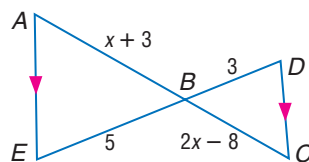
To estimate the height of the Jin Mao Tower in Shanghai, a tourist sights the top of the tower in a mirror that is on the ground and faces upward.

- How tall is the tower?
- Why is the mirror reflection a better way to indirectly measure the tower than by using shadows in this situation?

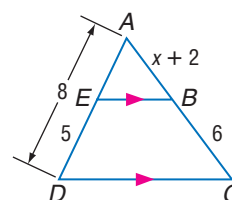


ALGEBRA Identify the similar triangles, and find x and the measures of the indicated sides.

14. \overline{AB} and \overline{BC}

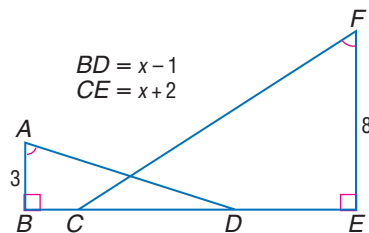


15. \overline{AB} and \overline{AC}

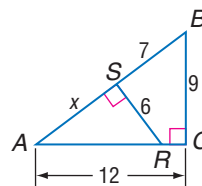


ALGEBRA Identify the similar triangles, and find x and the measures of the indicated sides.

16. \overline{BD} and \overline{EC}



17. \overline{AB} and \overline{AS}



COORDINATE GEOMETRY Triangles ABC and TBS have vertices $A(-2, -8)$, $B(4, 4)$, $C(-2, 7)$, $T(0, -4)$, and $S(0, 6)$.

18. Graph the triangles and prove that $\triangle ABC \sim \triangle TBS$.

19. Find the ratio of the perimeters of the two triangles.

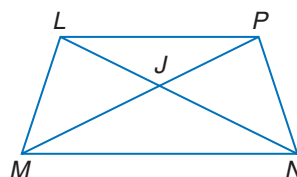
20. The lengths of the sides of triangle ABC are 6 centimeters, 4 centimeters, and 9 centimeters. Triangle DEF is similar to triangle ABC . The length of one side of triangle DEF is 36 centimeters. What is the greatest perimeter possible for triangle DEF ? Explain.

PROOF For Exercises 21–25, write the type of proof specified.

21. Two-column proof

Given: $\overline{LP} \parallel \overline{MN}$

Prove: $\frac{LJ}{JN} = \frac{PJ}{JM}$

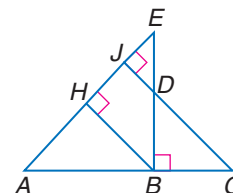


22. Paragraph proof

Given: $\overline{EB} \perp \overline{AC}$, $\overline{BH} \perp \overline{AE}$, $\overline{CJ} \perp \overline{AE}$

Prove: a. $\triangle ABH \sim \triangle DCB$

b. $\frac{BC}{BE} = \frac{BD}{BA}$



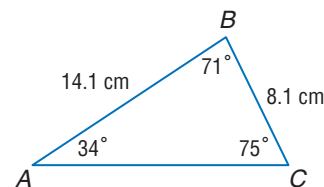
23. Two-column proof: If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar. (Theorem 7.2)

24. Two-column proof: If the measures of the legs of two right triangles are proportional, the triangles are similar.

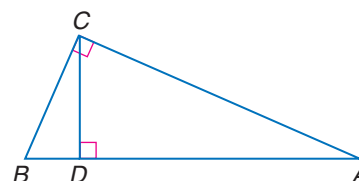
25. Two-column proof: Similarity of triangles is reflexive, symmetric, and transitive. (Theorem 7.3)

26. **OPEN ENDED** Draw a triangle that is similar to $\triangle ABC$. Explain how you know that it is similar.

27. **REASONING** Is it possible that $\triangle ABC$ is not similar to $\triangle RST$ and that $\triangle RST$ is not similar to $\triangle EFG$, but that $\triangle ABC$ is similar to $\triangle EFG$? Explain.



28. **CHALLENGE** Triangle ABC is similar to the two triangles formed by altitude \overline{CD} , and these two triangles are similar to each other. Write three similarity statements about these triangles. Why are the triangles similar to each other?



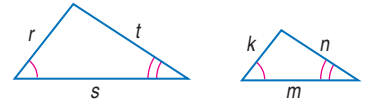
EXTRA PRACTICE
See pages 814, 834.
MathOnline
Self-Check Quiz at
geometryonline.com

H.O.T. Problems...

29. **FIND THE ERROR** Alicia and Jason were writing proportions for the similar triangles shown at the right. Who is correct? Explain your reasoning.

Alicia
 $\frac{r}{k} = \frac{s}{m}$
 $rm = ks$

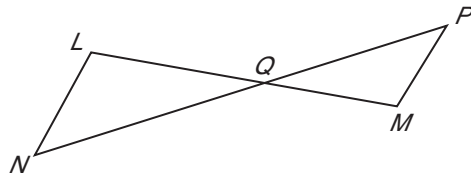
Jason
 $\frac{r}{k} = \frac{m}{s}$
 $rs = km$



30. **Writing in Math** Compare and contrast the tests to prove triangles similar with the tests to prove triangles congruent.

STANDARDIZED TEST PRACTICE

31. In the figure below, \overline{LM} intersects \overline{NP} at point Q.

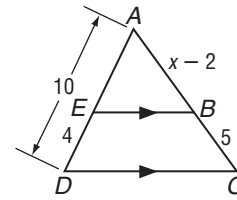


Which additional information would be enough to prove that $\triangle LNQ \sim \triangle MPQ$?

- A \overline{LQ} and \overline{MQ} are congruent.
- B $\angle QMP$ is a right angle.
- C \overline{LN} and \overline{PM} are parallel.
- D $\angle NLQ$ and $\angle PQM$ are congruent.

32. If $\overline{EB} \parallel \overline{DC}$, find the value of x.

- F 9.5
- G 5
- H 4
- J 2



33. **REVIEW** What is the y-coordinate of the solution of the system of linear equations below?

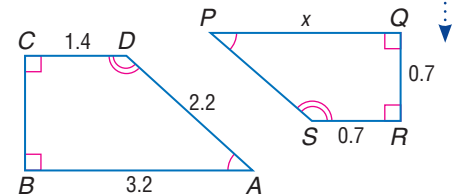
$$5x + 3y = 1$$

$$-3x - 2y = -2$$

- A -5
- B -4
- C 4
- D 7

Spiral Review

34. The pair of polygons is similar. Write a similarity statement, find x, BC, PS, and the scale factor. (Lesson 7-2)



Solve each proportion. (Lesson 7-1)

35. $\frac{1}{y} = \frac{3}{15}$

36. $\frac{6}{8} = \frac{7}{b}$

37. $\frac{20}{28} = \frac{m}{21}$

38. $\frac{16}{7} = \frac{9}{s}$

39. **ROLLER COASTERS** The sign in front of the Electric Storm roller coaster states ALL riders must be at least 54 inches tall to ride. If Adam is 5 feet 8 inches tall, can he ride the Electric Storm? Which law of logic leads you to this conclusion? (Lesson 2-4)

GET READY for the Next Lesson

PREREQUISITE SKILL Simplify. (Pages 790-791)

40. $\sqrt{\frac{24}{64}}$

41. $\sqrt{\frac{75}{81}}$

42. $\sqrt{\frac{72}{144}}$

43. $\sqrt{\frac{32}{108}}$

Solve each proportion. (Lesson 7-1)

1. $\frac{3}{4} = \frac{x}{12}$

2. $\frac{7}{3} = \frac{28}{z}$

3. $\frac{z}{40} = \frac{5}{8}$

4. $\frac{x+2}{5} = \frac{14}{10}$

5. $\frac{3}{7} = \frac{7}{y-3}$

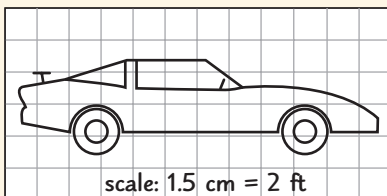
6. $\frac{4-x}{3+x} = \frac{16}{25}$

7. **MAPS** The scale on a map shows that 1.5 centimeters represents 100 miles. If the distance on the map from Seattle, Washington, to Indianapolis, Indiana, is 28.1 centimeters, approximately how many miles apart are the two cities?

(Lesson 7-1)

8. **BASEBALL** A player's slugging percentage is the ratio of the number of total bases from hits to the number of total at-bats. The ratio is converted to a decimal (rounded to three places) by dividing. If a professional baseball player has 281 total bases in 432 at-bats, what is his slugging percentage? (Lesson 7-1)

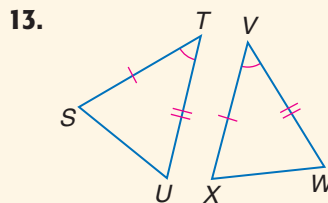
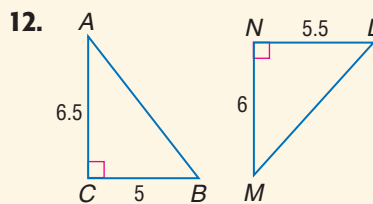
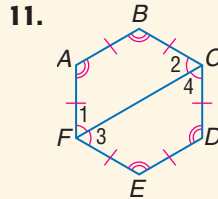
9. **MULTIPLE CHOICE** Miguel is using centimeter grid paper to make a scale drawing of his favorite car. The width of the drawing is 11.25 centimeters. How many feet long is the actual car? (Lesson 7-1)



- A 15.0 ft
- B 18.75 ft
- C 22.5 ft
- D 33.0 ft

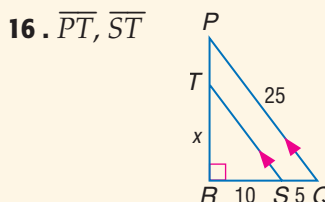
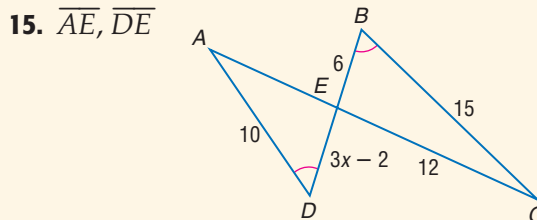
10. A 108-inch-long board is cut into two pieces that have lengths in the ratio 2:7. How long is each new piece? (Lesson 7-1)

Determine whether each pair of figures is similar. Justify your answer. (Lesson 7-2)



14. **ARCHITECTURE** The replica of the Eiffel Tower at an amusement park is $350\frac{2}{3}$ feet tall. The actual Eiffel Tower is 1052 feet tall. What is the scale factor comparing the amusement park tower to the actual tower? (Lesson 7-2)

Identify the similar triangles. Find x and the measures of the indicated sides. (Lesson 7-3)



Parallel Lines and Proportional Parts

Main Ideas

- Use proportional parts of triangles.
- Divide a segment into parts.

New Vocabulary

midsegment

GET READY for the Lesson

Street maps frequently have parallel and perpendicular lines. In Chicago, because of Lake Michigan, Lake Shore Drive runs at an angle between Oak Street and Ontario Street. City planners need to take this angle into account when determining dimensions of available land along Lake Shore Drive.



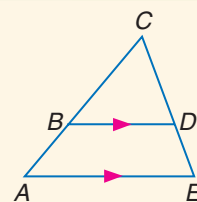
Proportional Parts of Triangles Nonparallel transversals that intersect parallel lines can be extended to form similar triangles. So the sides of the triangles are proportional.

THEOREM 7.4

Triangle Proportionality Theorem

If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths.

Example: If $\overline{BD} \parallel \overline{AE}$, $\frac{BA}{CB} = \frac{DE}{CD}$.

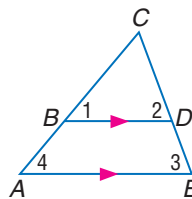


PROOF

Theorem 7.4

Given: $\overline{BD} \parallel \overline{AE}$

Prove: $\frac{BA}{CB} = \frac{DE}{CD}$



Paragraph Proof:

Since $\overline{BD} \parallel \overline{AE}$, $\angle 4 \cong \angle 1$ and $\angle 3 \cong \angle 2$ because they are corresponding angles. Then, by AA Similarity, $\triangle ACE \sim \triangle BCD$. From the definition of similar polygons, $\frac{CA}{CB} = \frac{CE}{CD}$. By the Segment Addition Postulate, $CA = BA + CB$ and $CE = DE + CD$.

Substituting for CA and CE in the ratio, we get the following proportion.

$$\frac{BA + CB}{CB} = \frac{DE + CD}{CD}$$

$$\frac{BA}{CB} + \frac{CB}{CB} = \frac{DE}{CD} + \frac{CD}{CD} \quad \text{Rewrite as a sum.}$$

$$\frac{BA}{CB} + 1 = \frac{DE}{CD} + 1 \quad \frac{CB}{CB} = 1 \text{ and } \frac{CD}{CD} = 1$$

$$\frac{BA}{CB} = \frac{DE}{CD} \quad \text{Subtract 1 from each side.}$$

Study Tip

Overlapping Triangles

Trace two copies of $\triangle ACE$. Cut along \overline{BD} to form $\triangle BCD$. Now $\triangle ACE$ and $\triangle BCD$ are no longer overlapping. Place the triangles side-by-side to compare corresponding angles and sides.

Study Tip

Using Fractions

You can also rewrite $\frac{9}{21}$ as $\frac{3}{7}$. Then use your knowledge of fractions to find the missing denominator.

$$\begin{array}{c} \times 2 \\ \curvearrowright \\ \frac{3}{7} = \frac{6}{?} \\ \curvearrowleft \\ \times 2 \end{array}$$

The correct denominator is 14.

EXAMPLE Find the Length of a Side

- 1 In $\triangle EFG$, $\overline{HL} \parallel \overline{EF}$, $EH = 9$, $HG = 21$, and $FL = 6$. Find LG .

From the Triangle Proportionality Theorem, $\frac{EH}{HG} = \frac{FL}{LG}$.

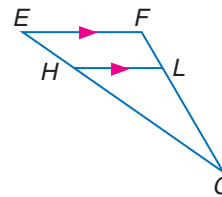
Substitute the known measures.

$$\frac{9}{21} = \frac{6}{LG}$$

$$9(LG) = (21)6 \quad \text{Cross products}$$

$$9(LG) = 126 \quad \text{Multiply.}$$

$$LG = 14 \quad \text{Divide each side by 9.}$$



CHECK Your Progress

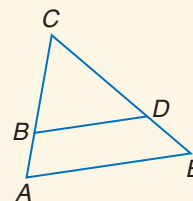
1. In $\triangle EFG$, if $EH = 6$, $FL = 4$, and $LG = 18$, find HG .

Proportional parts of a triangle can also be used to prove the converse of Theorem 7.4.

THEOREM 7.5 Converse of the Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

Example: If $\frac{BA}{CB} = \frac{DE}{CD}$, then $\overline{BD} \parallel \overline{AE}$.



You will prove Theorem 7.5 in Exercise 42.

EXAMPLE Determine Parallel Lines

- 2 In $\triangle HKM$, $HM = 15$, $HN = 10$, and \overline{HJ} is twice the length of \overline{JK} . Determine whether $\overline{NJ} \parallel \overline{MK}$. Explain.

$$HM = HN + NM \quad \text{Segment Addition Postulate}$$

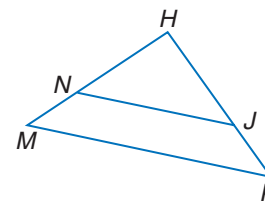
$$15 = 10 + NM \quad \text{HM = 15, HN = 10}$$

$$5 = NM \quad \text{Subtract 10 from each side.}$$

In order to show $\overline{NJ} \parallel \overline{MK}$, we must show that $\frac{HN}{NM} = \frac{HJ}{JK}$.

$$\frac{HN}{NM} = \frac{10}{5} \text{ or } 2. \text{ Let } JK = x. \text{ Then } HJ = 2x. \text{ So, } \frac{HJ}{JK} = \frac{2x}{x} \text{ or } 2.$$

Thus, $\frac{HN}{NM} = \frac{HJ}{JK} = 2$. Since the sides have proportional lengths, $\overline{NJ} \parallel \overline{MK}$.



CHECK Your Progress

2. In $\triangle HKM$, \overline{NM} is half the length of \overline{NH} , $HJ = 10$, and $JK = 6$. Determine whether $\overline{NJ} \parallel \overline{MK}$.

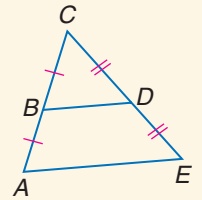
A **midsegment** of a triangle is a segment with endpoints that are the midpoints of two sides of the triangle.

THEOREM 7.6**Triangle Midsegment Theorem**

A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

Example: If B and D are midpoints of \overline{AC} and \overline{EC} , respectively,

$$\overline{BD} \parallel \overline{AE} \text{ and } BD = \frac{1}{2}AE.$$



You will prove Theorem 7.6 in Exercise 43.

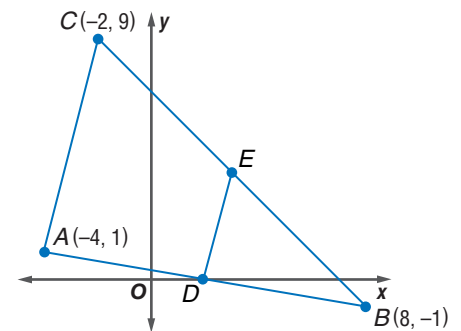
EXAMPLE Midsegment of a Triangle

5 Triangle ABC has vertices $A(-4, 1)$, $B(8, -1)$, and $C(-2, 9)$. \overline{DE} is a midsegment of $\triangle ABC$.

- a.** Find the coordinates of D and E .
Use the Midpoint Formula to find the midpoints of \overline{AB} and \overline{CB} .

$$D\left(\frac{-4 + 8}{2}, \frac{1 + (-1)}{2}\right) = D(2, 0)$$

$$E\left(\frac{-2 + 8}{2}, \frac{9 + (-1)}{2}\right) = E(3, 4)$$



- b.** Verify that \overline{AC} is parallel to \overline{DE} .

If the slopes of \overline{AC} and \overline{DE} are equal, $\overline{AC} \parallel \overline{DE}$.

$$\text{slope of } \overline{AC} = \frac{9 - 1}{-2 - (-4)} \text{ or } 4$$

$$\text{slope of } \overline{DE} = \frac{4 - 0}{3 - 2} \text{ or } 4$$

Because the slopes of \overline{AC} and \overline{DE} are equal, $\overline{AC} \parallel \overline{DE}$.

- c.** Verify that $DE = \frac{1}{2}AC$.

First, use the Distance Formula to find AC and DE .

$$\begin{aligned} AC &= \sqrt{[-2 - (-4)]^2 + (9 - 1)^2} \\ &= \sqrt{4 + 64} \text{ or } \sqrt{68} \end{aligned}$$

$$\begin{aligned} DE &= \sqrt{(3 - 2)^2 + (4 - 0)^2} \\ &= \sqrt{1 + 16} \text{ or } \sqrt{17} \end{aligned}$$

$$\frac{DE}{AC} = \frac{\sqrt{17}}{\sqrt{68}}$$

$$= \sqrt{\frac{17}{68}} \text{ or } \frac{1}{2}$$

$$\text{If } \frac{DE}{AC} = \frac{1}{2}, \text{ then } DE = \frac{1}{2}AC.$$

CHECK Your Progress

Triangle JKL has vertices $J(2, 5)$, $K(-4, -1)$, and $L(6, -3)$. \overline{MN} is the midsegment of JKL and is parallel to \overline{KL} .

3A. Find the coordinates of M and N .

3B. Verify that $\overline{KL} \parallel \overline{MN}$.

3C. Verify that $MN = \frac{1}{2}KL$.

Study Tip**Look Back**

To review the Distance and Midpoint Formulas, see Lesson 1-3.



Divide Segments Proportionally We have seen that parallel lines cut the sides of a triangle into proportional parts. Three or more parallel lines also separate transversals into proportional parts. If the ratio is 1, they separate the transversals into congruent parts.

Study Tip

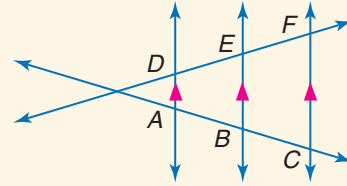
Three Parallel Lines

Corollary 7.1 is a special case of Theorem 7.4. In some drawings, the transversals are not shown to intersect. But, if extended, they will intersect and therefore, form triangles with each parallel line and the transversals.

COROLLARIES

7.1 If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

Example: If $\overleftrightarrow{DA} \parallel \overleftrightarrow{EB} \parallel \overleftrightarrow{FC}$, then $\frac{AB}{BC} = \frac{DE}{EF}$,
 $\frac{AC}{DF} = \frac{BC}{EF}$, and $\frac{AC}{BC} = \frac{DF}{EF}$.



7.2 If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

Example: If $\overline{AB} \cong \overline{BC}$, then $\overline{DE} \cong \overline{EF}$.

You will prove Corollaries 7.1 and 7.2 in Exercises 40 and 41, respectively.

EXAMPLE Proportional Segments

- 4 MAPS** Refer to the map at the beginning of the lesson. The streets from Oak Street to Ontario Street are all parallel to each other. If the distance from Delaware Place to Walton Street along Michigan Avenue is about 411 feet, what is the distance between those streets along Lake Shore Drive?



Notice that the streets form the bottom portion of a triangle that is cut by parallel lines. So you can use the Triangle Proportionality Theorem.

$$\frac{\text{Michigan Avenue}}{\text{Delaware to Walton}} = \frac{\text{Lake Shore Drive}}{\text{Delaware to Walton}}$$

Triangle Proportionality Theorem

$$\frac{411}{3800} = \frac{x}{4430}$$

Substitution

$$3800 \cdot x = 411(4430)$$

Cross products

$$3800x = 1,820,730$$

Multiply.

$$x = 479$$

Divide each side by 3800.

The distance from Delaware Place to Walton Street along Lake Shore Drive is about 479 feet.

CHECK Your Progress

- 4.** The distance from Delaware Place to Ontario Street along Lake Shore Drive is 2555 feet. What is the distance between these streets along Michigan Avenue?

EXAMPLE Congruent Segments

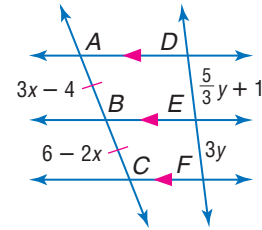
5 Find x and y .

To find x :

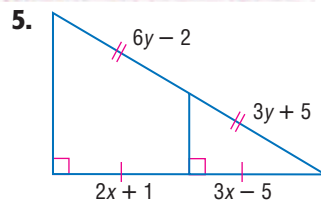
$$\begin{aligned} AB &= BC && \text{Given} \\ 3x - 4 &= 6 - 2x && \text{Substitution} \\ 5x - 4 &= 6 && \text{Add } 2x \text{ to each side.} \\ 5x &= 10 && \text{Add } 4 \text{ to each side.} \\ x &= 2 && \text{Divide each side by } 5. \end{aligned}$$

To find y :

$$\begin{aligned} \overline{DE} &\cong \overline{EF} && \text{Parallel lines that cut off congruent segments on one transversal} \\ &&& \text{cut off congruent segments on every transversal.} \\ DE &= EF && \text{Definition of congruent segments} \\ 3y &= \frac{5}{3}y + 1 && \text{Substitution} \\ 9y &= 5y + 3 && \text{Multiply each side by } 3 \text{ to eliminate the denominator.} \\ 4y &= 3 && \text{Subtract } 5y \text{ from each side.} \\ y &= \frac{3}{4} && \text{Divide each side by } 4. \end{aligned}$$



CHECK Your Progress



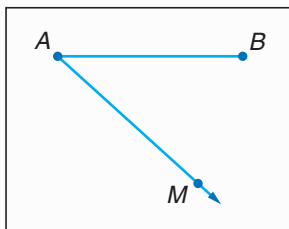
Concepts
in Motion
Animation
geometryonline.com

A segment cannot be separated into three congruent parts by constructing perpendicular bisectors. To do this, you must use parallel lines and the similarity theorems from this lesson. This technique can be used to separate a segment into any number of congruent parts.

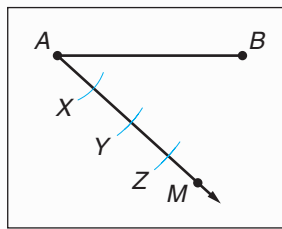
CONSTRUCTION

Trisect a Segment

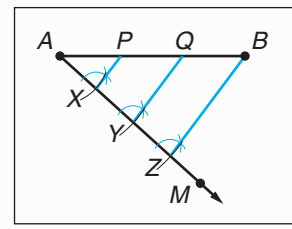
Step 1 Draw \overline{AB} to be trisected. Then draw \overline{AM} .



Step 2 With the compass at A , mark off an arc that intersects \overline{AM} at X . Use the same compass setting to construct \overline{XY} and \overline{YZ} congruent to \overline{AX} .



Step 3 Draw \overline{ZB} . Then construct lines through Y and X that are parallel to \overline{ZB} . Label the intersection points on \overline{AB} as P and Q .

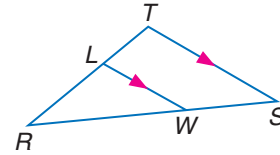


Conclusion: Since parallel lines cut off congruent segments on transversals, $\overline{AP} \cong \overline{PQ} \cong \overline{QB}$.

Example 1
(p. 406)

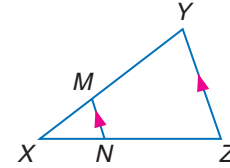
For Exercises 1 and 2, refer to $\triangle RST$.

- If $RL = 5$, $RT = 9$, and $WS = 6$, find RW .
- If $TR = 8$, $LR = 3$, and $RW = 6$, find WS .



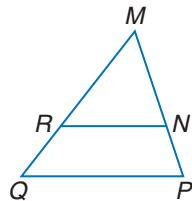
For Exercises 3 and 4, refer to $\triangle XYZ$.

- If $XM = 4$, $XN = 6$, and $NZ = 9$, find XY .
- If $XN = t - 2$, $NZ = t + 1$, $XM = 2$, and $XY = 10$, solve for t .

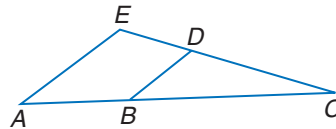


Example 2
(p. 406)

- In $\triangle MQP$, $MP = 25$, $MN = 9$, $MR = 4.5$, and $MQ = 12.5$. Determine whether $\overline{RN} \parallel \overline{QP}$. Justify your answer.



- In $\triangle ACE$, $ED = 8$, $DC = 20$, $BC = 25$, and $AB = 12$. Determine whether $\overline{AE} \parallel \overline{BD}$. Justify your answer.



Example 3
(p. 407)

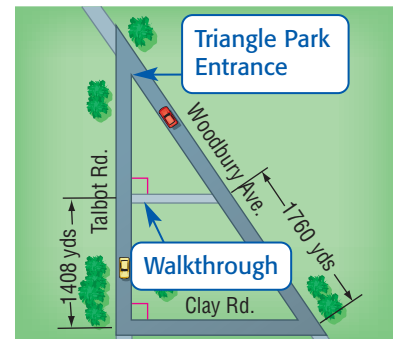
COORDINATE GEOMETRY For Exercises 7–9, use the following information.

Triangle ABC has vertices $A(-2, 6)$, $B(-4, 0)$, and $C(10, 0)$. \overline{DE} is a midsegment parallel to \overline{BC} .

- Find the coordinates of D and E .
- Verify that \overline{DE} is parallel to \overline{BC} .
- Verify that $DE = \frac{1}{2}BC$.

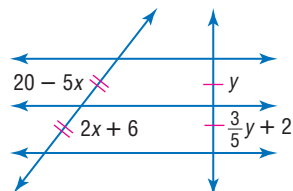
Example 4
(p. 408)

- MAPS** The distance along Talbot Road from the Triangle Park entrance to the Walkthrough is 880 yards. If the Walkthrough is parallel to Clay Road, find the distance from the entrance to the Walkthrough along Woodbury.

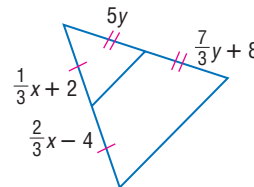


Example 5
(p. 409)

- Find x and y .



- Find x and y .

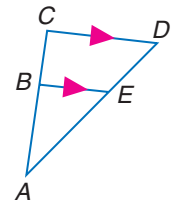


Exercises

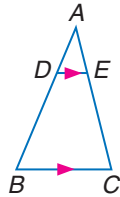
HOMEWORK HELP	
For Exercises	See Examples
13–19	1
20–23	2
24–27	3
28–31	4
32, 33	5

For Exercises 13–15, refer to $\triangle ACD$.

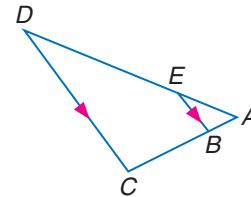
- Find ED if $AB = 6$, $BC = 4$, and $AE = 9$.
- Find AE if $AB = 12$, $AC = 16$, and $ED = 5$.
- Find CD if $AE = 8$, $ED = 4$, and $BE = 6$.



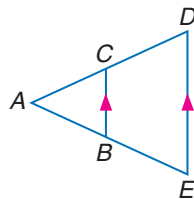
- If $DB = 24$, $AE = 3$, and $EC = 18$, find AD .



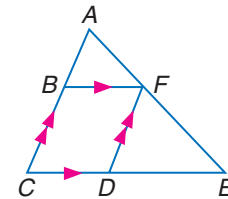
- Find x and ED if $AE = 3$, $AB = 2$, $BC = 6$, and $ED = 2x - 3$.



- Find x , AC , and CD if $AC = x - 3$, $BE = 20$, $AB = 16$, and $CD = x + 5$.

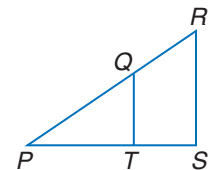


- Find BC , FE , CD , and DE if $AB = 6$, $AF = 8$, $BC = x$, $CD = y$, $DE = 2y - 3$, and $FE = x + \frac{10}{3}$.



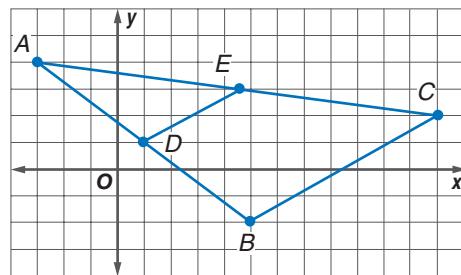
Determine whether $\overline{QT} \parallel \overline{RS}$. Justify your answer.

- $PR = 30$, $PQ = 9$, $PT = 12$, and $PS = 18$
- $QR = 22$, $RP = 65$, and SP is 3 times TS .
- $TS = 8.6$, $PS = 12.9$, and PQ is half RQ .
- $PQ = 34.88$, $RQ = 18.32$, $PS = 33.25$, and $TS = 11.45$

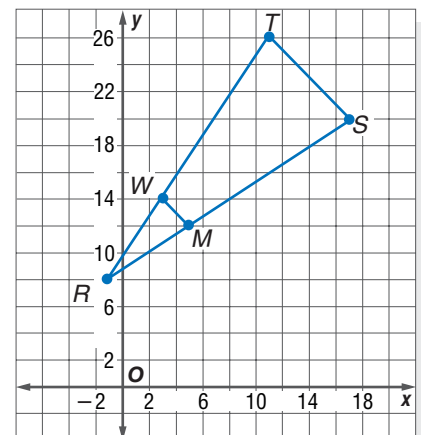


24. COORDINATE GEOMETRY

Find the length of \overline{BC} if $\overline{BC} \parallel \overline{DE}$ and \overline{DE} is a midsegment of $\triangle ABC$.

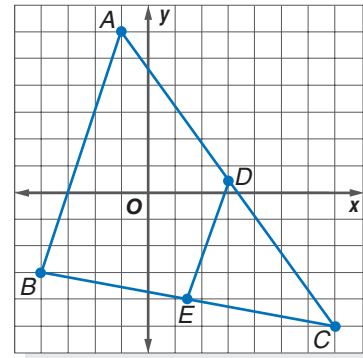


- COORDINATE GEOMETRY** Show that $\overline{WM} \parallel \overline{TS}$ and determine whether \overline{WM} is a midsegment.



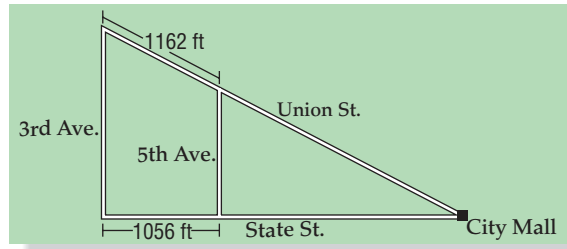
COORDINATE GEOMETRY For Exercises 26 and 27, use the following information.

Triangle ABC has vertices $A(-1, 6)$, $B(-4, -3)$, and $C(7, -5)$. \overline{DE} is a midsegment.



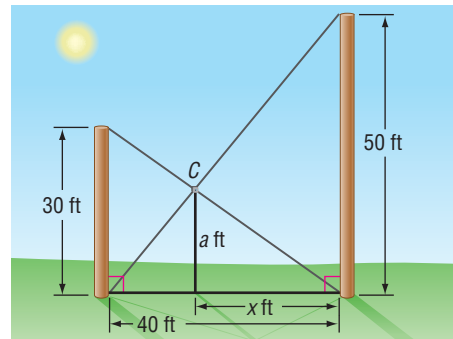
26. Verify that \overline{DE} is parallel to \overline{AB} .
 27. Verify that $DE = \frac{1}{2}AB$.

28. **MAPS** Refer to the map at the right. Third Avenue and 5th Avenue are parallel. If the distance from 3rd Avenue to City Mall along State Street is 3201 feet, find the distance between 5th Avenue and City Mall along Union Street. Round to the nearest tenth.



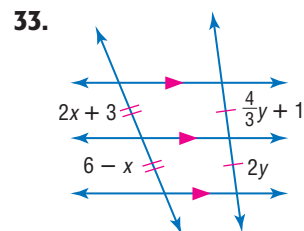
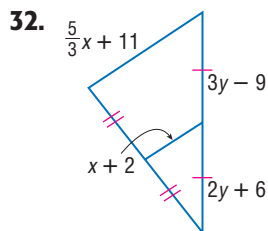
CONSTRUCTION For Exercises 29–31, use the following information and diagram.

Two poles, 30 feet and 50 feet tall, are 40 feet apart and perpendicular to the ground. The poles are supported by wires attached from the top of each pole to the bottom of the other, as in the figure. A coupling is placed at C where the two wires cross.



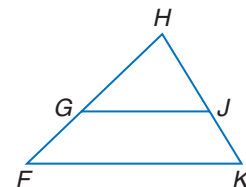
29. Find x , the distance from C to the taller pole.
 30. How high above the ground is the coupling?
 31. How far down the wire from the smaller pole is the coupling?

Find x and y .

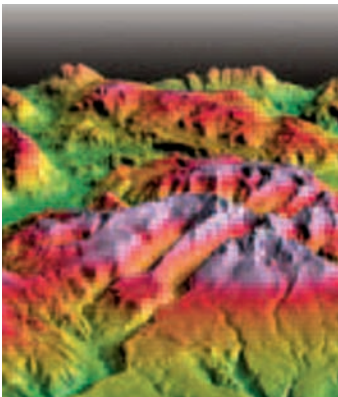


Find x so that $\overline{GJ} \parallel \overline{EK}$.

34. $GE = 12$, $HG = 6$, $HJ = 8$, $JK = x - 4$
 35. $HJ = x - 5$, $JK = 15$, $EG = 18$, $HG = x - 4$
 36. $GH = x + 3.5$, $HJ = x - 8.5$, $EH = 21$, $HK = 7$



37. **COORDINATE GEOMETRY** Given $A(2, 12)$ and $B(5, 0)$, find the coordinates of P such that P separates \overline{AB} into two parts with lengths in a ratio of 2 to 1.



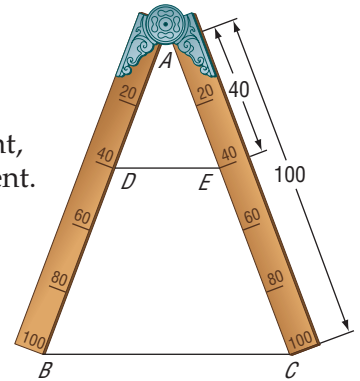
Real-World Link

Modern map-making techniques use images taken from space to produce accurate representations on paper. In February 2000, the crew of the space shuttle *Endeavor* collected a trillion radar images of 46 million square miles of Earth.

Source: www.nima.mil

- 38. COORDINATE GEOMETRY** In $\triangle LMN$, \overline{PR} divides \overline{NL} and \overline{MN} proportionally. If the vertices are $N(8, 20)$, $P(11, 16)$, and $R(3, 8)$ and $\frac{LP}{PN} = \frac{2}{1}$, find the coordinates of L and M .

- 39. MATH HISTORY** The sector compass was a tool perfected by Galileo in the sixteenth century for measurement and calculation. To draw a segment two-fifths the length of a given segment, align the ends of the arms with the given segment. Then draw a segment at the 40 mark. Write a justification that explains why the sector compass works for proportional measurement.



PROOF Write a paragraph proof for each corollary.

40. Corollary 7.1

41. Corollary 7.2

PROOF Write a two-column proof of each theorem.

42. Theorem 7.5

43. Theorem 7.6

CONSTRUCTION Construct each segment as directed.

44. a segment 8 centimeters long, separated into three congruent segments
 45. a segment separated into four congruent segments
 46. a segment separated into two segments in which their lengths have a ratio of 1 to 4

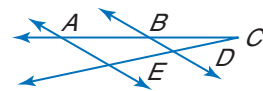
- 47. REASONING** Explain how you would know if a line that intersects two sides of a triangle is parallel to the third side.

- 48. OPEN ENDED** Draw two segments that are intersected by three lines so that the parts are proportional. Then draw a counterexample.

- 49. PROOF** Write a two-column proof.

Given: $AB = 4$ and $BC = 4$, $CD = DE$

Prove: $\overline{BD} \parallel \overline{AE}$



- 50. CHALLENGE** Copy the figure that accompanies Corollary 7.1 on page 408. Draw \overline{DC} . Let G be the intersection point of \overline{DC} and \overline{BE} . Using that segment, explain how you could prove $\frac{AB}{BC} = \frac{DE}{EF}$.

CHALLENGE Draw any quadrilateral $ABCD$ on a coordinate plane. Points E , F , G , and H are midpoints of \overline{AB} , \overline{BC} , \overline{CD} , and \overline{DA} , respectively.

51. Connect the midpoints to form quadrilateral $EFGH$. Describe what you know about the sides of quadrilateral $EFGH$.

52. Will the same reasoning work with five-sided polygons? Explain.

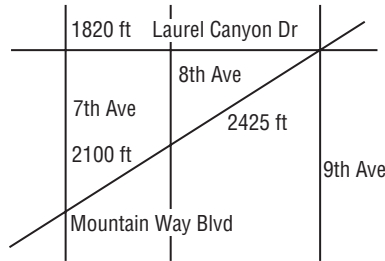
- 53. Writing in Math** Refer to the information on city planning on page 405. Describe the geometry facts a city planner needs to know to explain why the block between Chestnut and Pearson is longer on Lake Shore Drive than on Michigan Avenue.

EXTRA PRACTICE
 See pages 814, 834.
Math online
 Self-Check Quiz at
geometryonline.com



H.O.T. Problems

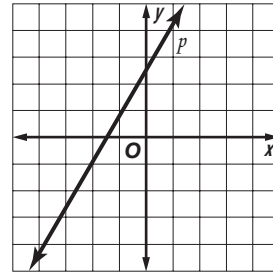
54. The streets 7th Avenue, 8th Avenue, and 9th Avenue are parallel. They all intersect Laurel Canyon Drive and Mountain Way Boulevard.



If all these streets are straight line segments, how long is Laurel Canyon Drive between 7th Avenue and 9th Avenue?

- A 2101.7 ft C 3921.7 ft
 B 2145 ft D 4436 ft

55. **REVIEW** What will happen to the slope of line p if the line is shifted so that the y -intercept stays the same and the x -intercept increases?

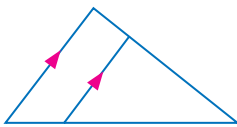


- F The slope will change from negative to positive.
 G The slope will become zero.
 H The slope will decrease.
 J The slope will increase.

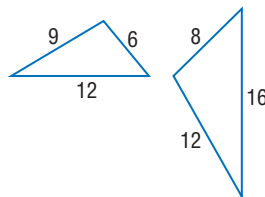
Spiral Review

Determine whether each pair of triangles is similar. Justify your answer. (Lesson 7-3)

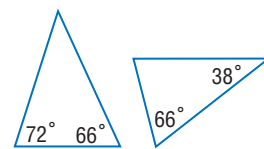
56.



57.

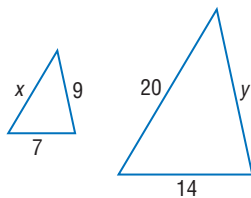


58.

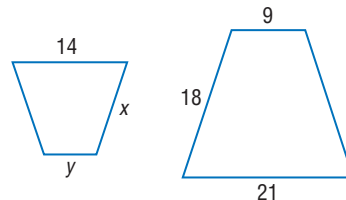


Each pair of polygons is similar. Find x and y . (Lesson 7-2)

59.



60.



61. **ALGEBRA** Quadrilateral $ABCD$ has a perimeter of 95 centimeters. Find the length of each side if $AB = 3a + 2$, $BC = 2(a - 1)$, $CD = 6a + 4$, and $AD = 5a - 5$. (Lesson 1-6)

GET READY for the Next Lesson

PREREQUISITE SKILL Write all the pairs of corresponding parts for each pair of congruent triangles. (Lesson 4-3)

62. $\triangle ABC \cong \triangle DEF$

63. $\triangle RST \cong \triangle XYZ$

64. $\triangle PQR \cong \triangle KLM$

Main Ideas

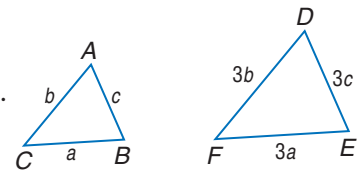
- Recognize and use proportional relationships of corresponding perimeters of similar triangles.
- Recognize and use proportional relationships of corresponding angle bisectors, altitudes, and medians of similar triangles.

GET READY for the Lesson

Professional photographers often use 35-millimeter film cameras for clear images. The camera lens was 6.16 meters from this Dale Chihuly glass sculpture when the photographer took this photograph. The image on the film is 35 millimeters tall. Similar triangles enable us to find the height of the actual sculpture.



Perimeters Triangle ABC is similar to $\triangle DEF$ with a scale factor of 1:3. You can use variables and the scale factor to compare their perimeters. Let the measures of the sides of $\triangle ABC$ be a , b , and c . The measures of the corresponding sides of $\triangle DEF$ would be $3a$, $3b$, and $3c$.



$$\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{a + b + c}{3a + 3b + 3c} = \frac{1(a + b + c)}{3(a + b + c)} \text{ or } \frac{1}{3}$$

The perimeters are in the same proportion as the side measures of the two similar figures. This suggests Theorem 7.7, the Proportional Perimeters Theorem.

THEOREM 7.7 *Proportional Perimeters Theorem*

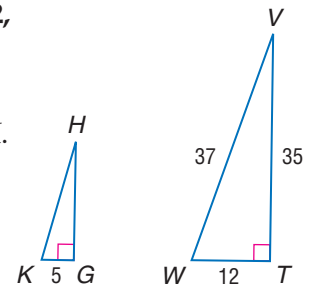
If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides.

You will prove Theorem 7.7 in Exercise 14.

EXAMPLE *Perimeters of Similar Triangles*

I If $\triangle GHK \sim \triangle TVW$, $TV = 35$, $VW = 37$, $WT = 12$, and $KG = 5$, find the perimeter of $\triangle GHK$.

The perimeter of $\triangle TVW = 35 + 37 + 12$ or 84.
Use a proportion to find the perimeter of $\triangle GHK$.
Let x represent the perimeter of $\triangle GHK$.



(continued on the next page)

$$\frac{KG}{WT} = \frac{\text{perimeter of } \triangle GHK}{\text{perimeter of } \triangle TVW} \quad \text{Proportional Perimeter Theorem}$$

$$\frac{5}{12} = \frac{x}{84} \quad \text{Substitution}$$

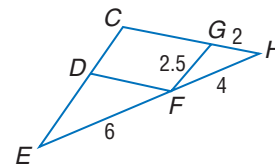
$$12x = 420 \quad \text{Cross products}$$

$$x = 35 \quad \text{Divide each side by 12.}$$

The perimeter of $\triangle GHK$ is 35 units.

CHECK Your Progress

1. If $\triangle DEF \sim \triangle GFH$, find the perimeter of $\triangle DEF$.



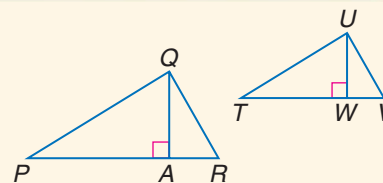
Special Segments of Similar Triangles Think about a triangle drawn on a piece of paper being placed on a copy machine and either enlarged or reduced. The copy is similar to the original triangle. Now suppose you draw in special segments of a triangle, such as the altitudes, medians, or angle bisectors, on the original. When you enlarge or reduce that original triangle, all of those segments are enlarged or reduced at the same rate. This conjecture is formally stated in Theorems 7.8, 7.9, and 7.10.

THEOREMS

Special Segments of Similar Triangles

- 7.8** If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides.

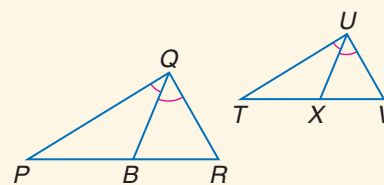
Abbreviation: $\sim \triangle s$ have corr. altitudes proportional to the corr. sides.



$$\frac{QA}{UW} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

- 7.9** If two triangles are similar, then the measures of the corresponding angle bisectors of the triangles are proportional to the measures of the corresponding sides.

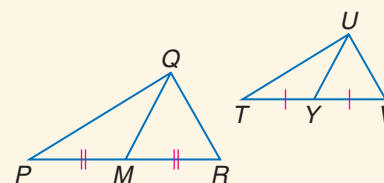
Abbreviation: $\sim \triangle s$ have corr. \angle bisectors proportional to the corr. sides.



$$\frac{QB}{UX} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

- 7.10** If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides.

Abbreviation: $\sim \triangle s$ have corr. medians proportional to the corr. sides.



$$\frac{QM}{UY} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

Study Tip

Points of Concurrency

In a triangle that is enlarged or reduced proportionally, the points of concurrency remain in the same position.

You will prove Theorems 7.8 and 7.10 in Check Your Progress 2 and Exercise 3, respectively.

EXAMPLE Write a Proof

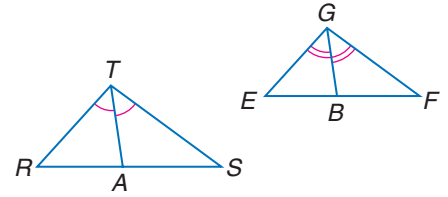
2 Write a paragraph proof of Theorem 7.9.

Since the corresponding angles to be bisected are chosen at random, we need not prove this for every pair of bisectors.

Given: $\triangle RTS \sim \triangle EGF$
 \overline{TA} and \overline{GB} are angle bisectors.

Prove: $\frac{TA}{GB} = \frac{RT}{EG}$

Paragraph Proof: Because corresponding angles of similar triangles are congruent, $\angle R \cong \angle E$ and $\angle RTS \cong \angle EGF$. Since $\angle RTS$ and $\angle EGF$ are bisected, we know that $\frac{1}{2}m\angle RTS = \frac{1}{2}m\angle EGF$ or $m\angle RTA = m\angle EGB$. This makes $\angle RTA \cong \angle EGB$ and $\triangle RTA \sim \triangle EGB$ by AA Similarity. Thus, $\frac{TA}{GB} = \frac{RT}{EG}$.

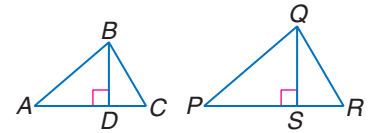


CHECK Your Progress

2. Write a paragraph proof of Theorem 7.8.

Given: $\triangle ABC \sim \triangle PQR$

Prove: $\frac{BD}{QS} = \frac{BA}{QP}$



The medians of similar triangles are also proportional.

EXAMPLE Medians of Similar Triangles

3 In the figure, $\triangle ABC \sim \triangle DEF$. \overline{BG} is a median of $\triangle ABC$, and \overline{EH} is a median of $\triangle DEF$. Find EH if $BC = 30$, $BG = 15$, and $EF = 15$.

Let x represent EH .

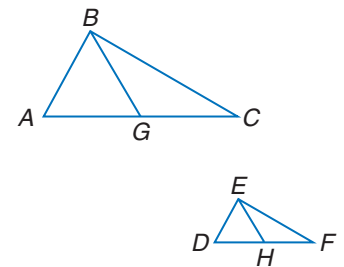
$\frac{BG}{EH} = \frac{BC}{EF}$ Write a proportion.

$\frac{15}{x} = \frac{30}{15}$ $BG = 15$, $EH = x$, $BC = 30$, and $EF = 15$

$30x = 225$ Cross products

$x = 7.5$ Divide each side by 30.

Thus, $EH = 7.5$.



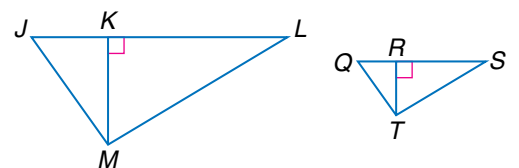
Study Tip

Proportions

This proportion can be written in different ways. The proportion $\frac{BG}{BC} = \frac{EH}{EF}$ gives the same result.

CHECK Your Progress

3. In the figure, $\triangle JLM \sim \triangle QST$.
 \overline{KM} is an altitude of $\triangle JLM$, and
 \overline{RT} is an altitude of $\triangle QST$.
 Find RT if $JL = 12$, $QS = 8$, and
 $KM = 5$.

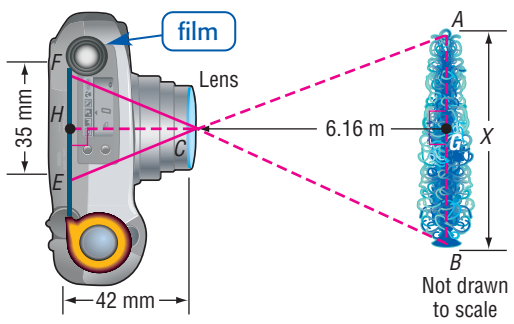


The theorems about the relationships of special segments in similar triangles can be used to solve real-life problems.



EXAMPLE Solve Problems with Similar Triangles

4 PHOTOGRAPHY Refer to the application at the beginning of the lesson. The drawing below illustrates the position of the camera and the distance from the lens of the camera to the film. Find the height of the sculpture.



Real-World Link

The first consumer-oriented digital cameras were produced for sale in 1994 with a 640×480 pixel resolution. In 2005, a 12.8-megapixel camera could take a picture with 4368×2912 pixel resolution, which is a sharper picture than most computer monitors can display.

Source: www.howstuffworks.com

$\triangle ABC$ and $\triangle EFC$ are similar. \overline{CG} and \overline{CH} are altitudes of $\triangle ABC$ and $\triangle EFC$, respectively. If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides. This leads to the proportion $\frac{AB}{EF} = \frac{GC}{HC}$.

$$\frac{AB}{EF} = \frac{GC}{HC} \quad \text{Write the proportion.}$$

$$\frac{x \text{ m}}{35 \text{ mm}} = \frac{6.16 \text{ m}}{42 \text{ mm}} \quad AB = x \text{ m}, EF = 35 \text{ mm}, GC = 6.16 \text{ m}, HC = 42 \text{ mm}$$

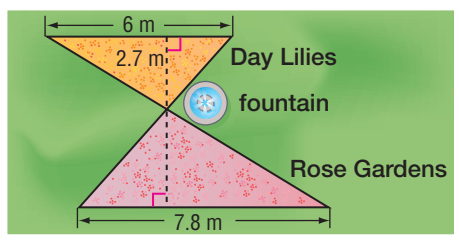
$$x \cdot 42 = 35(6.16) \quad \text{Cross products}$$

$$42x = 215.6 \quad \text{Simplify.}$$

$$x \approx 5.13 \quad \text{The sculpture is about 5.13 meters tall.}$$

CHECK Your Progress

4. LANDSCAPING The landscaping team at a botanical garden is planning to add sidewalks around the fountain. The gardens form two similar triangles. Find the distance from the fountain to the rose gardens.



Personal Tutor at geometryonline.com

An angle bisector also divides the side of the triangle opposite the angle proportionally.

THEOREM 7.11 *Angle Bisector Theorem*

An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides.

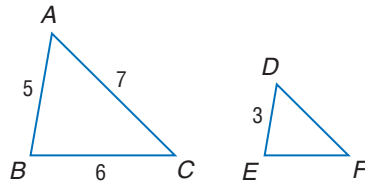
Example: $\frac{AD}{DB} = \frac{AC}{BC}$ ← segments with vertex A
 ← segments with vertex B

You will prove Theorem 7.11 in Exercise 15.

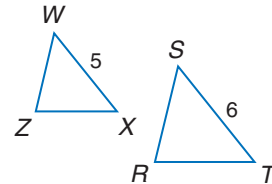
Example 1
(p. 415)

Find the perimeter of the given triangle.

1. $\triangle DEF$, if $\triangle ABC \sim \triangle DEF$,
 $AB = 5$, $BC = 6$, $AC = 7$,
and $DE = 3$



2. $\triangle WZX$, if $\triangle WZX \sim \triangle SRT$, $ST = 6$,
 $WX = 5$, and the perimeter of
 $\triangle SRT = 15$



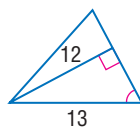
Example 2
(p. 417)

Example 3
(p. 417)

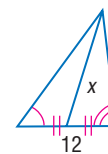
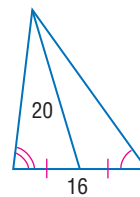
3. Write a two-column proof of Theorem 7.10.

Find x .

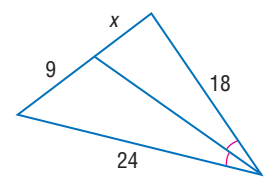
4.



5.



6.



Example 4
(p. 418)

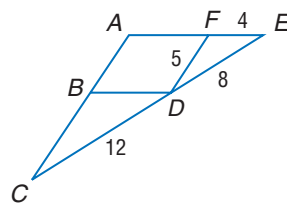
7. **PHOTOGRAPHY** The distance from the film to the lens in a camera is 10 centimeters. The film image is 3 centimeters high. Tamika is 165 centimeters tall. How far should she be from the camera in order for the photographer to take a full-length picture?

Exercises

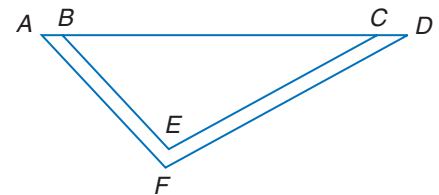
HOMEWORK HELP	
For Exercises	See Examples
8–13	1
14–18	2
19–22	3
23, 24	4

Find the perimeter of the given triangle.

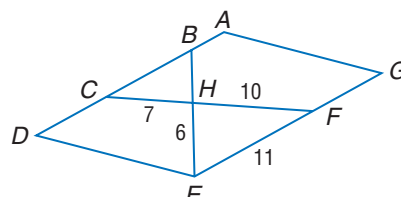
8. $\triangle BCD$, if $\triangle BCD \sim \triangle FDE$,
 $CD = 12$, $FD = 5$, $FE = 4$,
and $DE = 8$



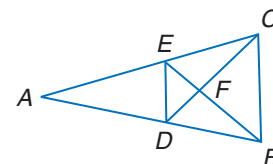
9. $\triangle ADF$, if $\triangle ADF \sim \triangle BCE$,
 $BC = 24$, $EB = 12$, $CE = 18$,
and $DF = 21$



10. $\triangle CBH$, if $\triangle CBH \sim \triangle FEH$,
 $ADEG$ is a parallelogram,
 $CH = 7$, $FH = 10$, $FE = 11$,
and $EH = 6$

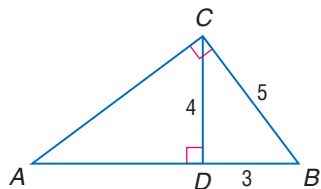


11. $\triangle DEF$, if $\triangle DEF \sim \triangle CBF$,
perimeter of $\triangle CBF = 27$,
 $DF = 6$, $FC = 8$

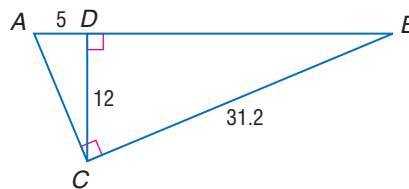


Find the perimeter of the given triangle.

12. $\triangle ABC$, if $\triangle ABC \sim \triangle CBD$,
 $CD = 4$, $DB = 3$, and $CB = 5$



13. $\triangle ABC$, if $\triangle ABC \sim \triangle CBD$,
 $AD = 5$, $CD = 12$, $BC = 31.2$

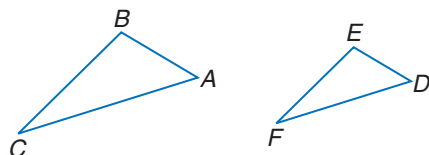


PROOF For Exercises 14–18, write the indicated type of proof.

14. Write a paragraph proof of Theorem 7.7.

Given: $\triangle ABC \sim \triangle DEF$
 and $\frac{AB}{DE} = \frac{m}{n}$

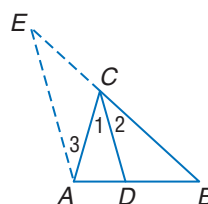
Prove: $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{m}{n}$



15. Two-column proof of the Angle Bisector Theorem (Theorem 7.11)

Given: \overline{CD} bisects $\angle ACB$
 By construction, $\overline{AE} \parallel \overline{CD}$.

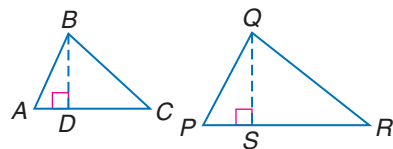
Prove: $\frac{AD}{AC} = \frac{BD}{BC}$



16. Paragraph proof

Given: $\triangle ABC \sim \triangle PQR$
 \overline{BD} is an altitude of $\triangle ABC$.
 \overline{QS} is an altitude of $\triangle PQR$.

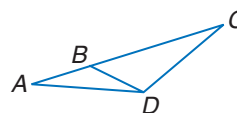
Prove: $\frac{QP}{BA} = \frac{QS}{BD}$



17. Flow proof

Given: $\angle C \cong \angle BDA$

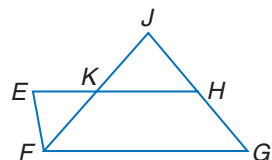
Prove: $\frac{AC}{DA} = \frac{AD}{BA}$



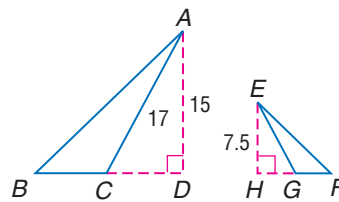
18. Two-column proof

Given: \overline{JF} bisects $\angle EFG$.
 $\overline{EH} \parallel \overline{FG}$, $\overline{EF} \parallel \overline{HG}$

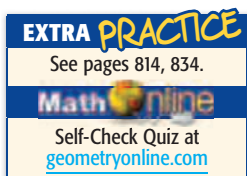
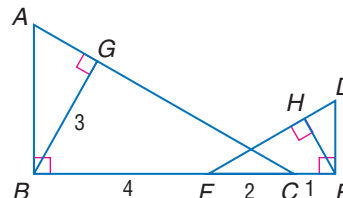
Prove: $\frac{EK}{KF} = \frac{GJ}{JF}$



19. Find EG if $\triangle ACB \sim \triangle EGF$, \overline{AD} is an altitude of $\triangle ACB$, \overline{EH} is an altitude of $\triangle EGF$, $AC = 17$, $AD = 15$, and $EH = 7.5$.



20. Find EH if $\triangle ABC \sim \triangle DEF$, \overline{BG} is an altitude of $\triangle ABC$, \overline{EH} is an altitude of $\triangle DEF$, $BG = 3$, $BF = 4$, $FC = 2$, and $CE = 1$.





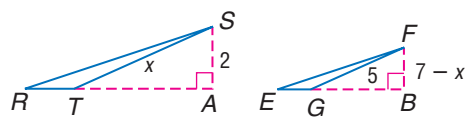
Real-World Career

Photographer

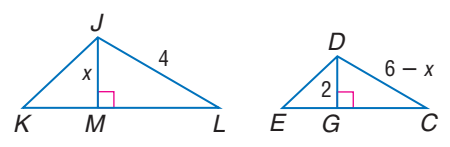
Photographers need a good eye for composition. They are also proficient in developing film. Photographers who own their own business must be good at math and other business skills.

Math Online
For more information, go to geometryonline.com.

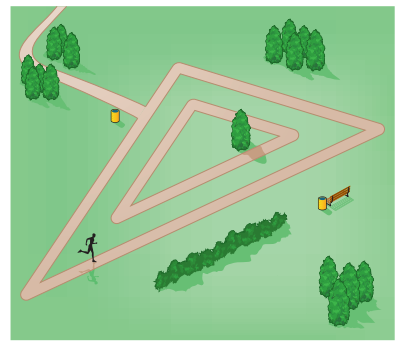
21. Find FB if \overline{SA} and \overline{FB} are altitudes and $\triangle RST \sim \triangle EFG$.



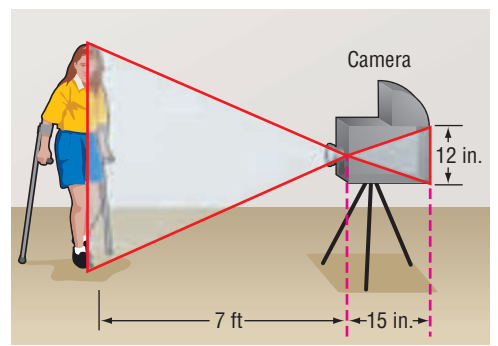
22. Find DC if \overline{DG} and \overline{JM} are altitudes and $\triangle KJL \sim \triangle EDC$.



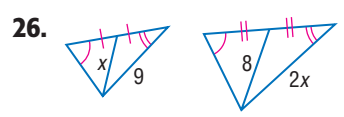
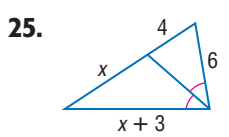
23. **PHYSICAL FITNESS** A park has two similar triangular jogging paths as shown. The dimensions of the inner path are 300 meters, 350 meters, and 550 meters. The shortest side of the outer path is 600 meters. Will a jogger on the inner path run half as far as one on the outer path? Explain.



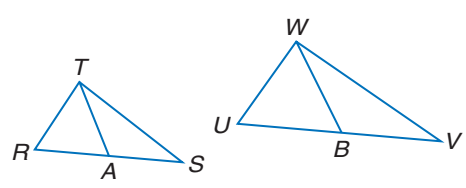
24. **PHOTOGRAPHY** One of the first cameras invented was called a *camera obscura*. Light entered an opening in the front, and an image was reflected in the back of the camera, upside down, forming similar triangles. If the image of the person on the back of the camera is 12 inches, the distance from the opening to the person is 7 feet, and the camera itself is 15 inches long, how tall is the person being photographed?



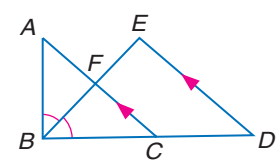
Find x .



27. Find UB if $\triangle RST \sim \triangle UVW$, \overline{TA} and \overline{WB} are medians, $TA = 8$, $RA = 3$, $WB = 3x - 6$, and $UB = x + 2$.

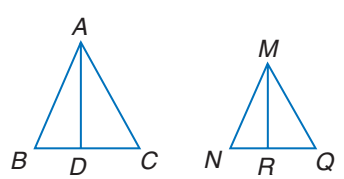


28. Find CF and BD if \overline{BF} bisects $\angle ABC$ and $\overline{AC} \parallel \overline{ED}$, $BA = 6$, $BC = 7.5$, $AC = 9$, and $DE = 9$.

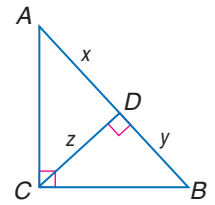


H.O.T. Problems

29. **REASONING** Explain what must be true about $\triangle ABC$ and $\triangle MNQ$ before you can conclude that $\frac{AD}{MR} = \frac{BA}{NM}$.



30. **CHALLENGE** \overline{CD} is an altitude to the hypotenuse \overline{AB} . Make a conjecture about x , y , and z . Justify your reasoning.

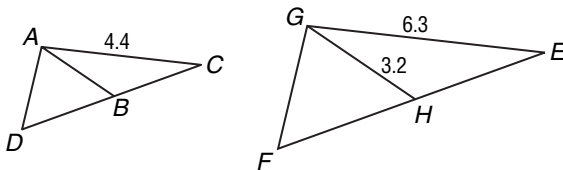


31. **OPEN ENDED** The perimeter of one triangle is 24 centimeters, and the perimeter of a second triangle is 36 centimeters. If the length of one side of the smaller triangle is 6, find possible lengths of the other sides of the triangles so that they are similar.

32. **Writing in Math** Explain how geometry is related to photography. Include a sketch of how a camera works showing the image and the film, and why the two isosceles triangles are similar.

STANDARDIZED TEST PRACTICE

33. In the figures below, $\overline{DB} \cong \overline{BC}$ and $\overline{FH} \cong \overline{HE}$.



If $\triangle ACD \sim \triangle GEF$, find the approximate length of \overline{AB} .

- A 2.2 C 8.7
B 4.6 D 11.1

34. **REVIEW** Which shows 0.00234 written in scientific notation?

- F 2.34×10^5 H 2.34×10^{-2}
G 2.34×10^3 J 2.34×10^{-3}

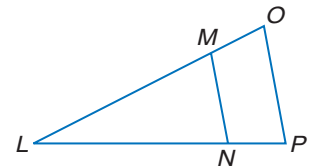
35. **REVIEW** The sum of three numbers is 180. Two of the numbers are the same, and each of them is one third of the greatest number. What is the least number?

- A 30 C 45
B 36 D 72

Spiral Review

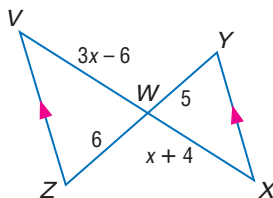
Determine whether $\overline{MN} \parallel \overline{OP}$. Justify your answer. (Lesson 7-4)

36. $LM = 7, LN = 9, LO = 14, LP = 16$
37. $LM = 6, MN = 4, LO = 9, OP = 6$
38. $LN = 12, NP = 4, LM = 15, MO = 5$

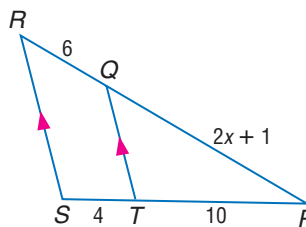


Identify the similar triangles. Find x and the measure(s) of the indicated side(s). (Lesson 7-3)

39. \overline{VW} and \overline{WX}



40. \overline{PQ}



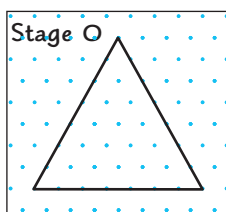
41. **BUSINESS** Elisa charges \$5 to paint mailboxes and \$4 per hour to mow lawns. Write an equation to represent the amount of money Elisa can earn from each homeowner. (Lesson 3-4)

Geometry Lab Sierpinski Triangle

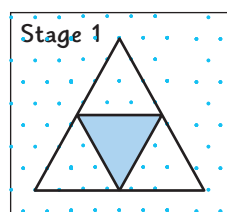
A **fractal** is a geometric figure that is created using iteration. **Iteration** is a process of repeating the same pattern over and over again. Fractals are **self-similar**, which means that the smaller and smaller details of the shape have the same geometric characteristics as the original form.

ACTIVITY

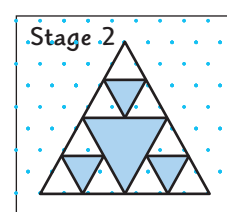
Stage 0 On isometric dot paper, draw an equilateral triangle in which each side is 8 units long.



Stage 1 Connect the midpoints of each side to form another triangle. Shade the center triangle.



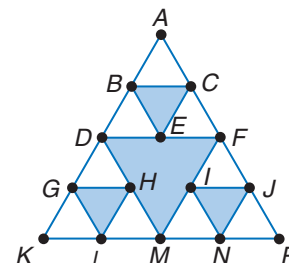
Stage 2 Repeat the process using the three nonshaded triangles. Connect the midpoints of each side to form other triangles.



If you repeat this process indefinitely, the figure that results is called the **Sierpinski Triangle**.

ANALYZE THE RESULTS

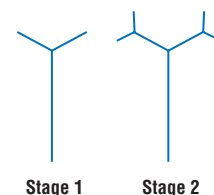
1. Continue the process through Stage 4. How many nonshaded triangles do you have at Stage 4?
2. What is the perimeter of a nonshaded triangle in Stage 0 through Stage 4?
3. If you continue the process indefinitely, describe what will happen to the perimeter of each nonshaded triangle.
4. Study $\triangle DFM$ in Stage 2 of the Sierpinski Triangle shown at the right. Is this an equilateral triangle? Are $\triangle BCE$, $\triangle GHL$, or $\triangle IJN$ equilateral?
5. Is $\triangle BCE \sim \triangle DFM$? Explain your answer.
6. How many Stage 1 Sierpinski triangles are there in Stage 2?



For Exercises 7 and 8, use the following information.

A **fractal tree** can be drawn by making two new branches from the endpoint of each original branch, each one-third as long as the previous branch.

7. Draw Stages 3 and 4 of a fractal tree. How many total branches do you have in Stages 1 through 4? (Do not count the stems.)
8. Find a pattern to predict the number of branches at each stage.



FOLDABLES
Study Organizer

GET READY to Study

Be sure the following
Key Concepts are noted in
your Foldable.



Key Concepts

Proportions (Lesson 7-1)

- For any numbers a and c and any nonzero numbers b and d , $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

Similar Polygons and Triangles (Lessons 7-2 and 7-3)

- Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.
- Two triangles are similar if:
 - AA: the two angles of one triangle are congruent to two angles of another triangle
 - SSS: the measures of the corresponding sides of two triangles are proportional
 - SAS: the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent

Proportional Parts (Lesson 7-4)

- If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional length.
- A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

Parts of Similar Triangles (Lesson 7-5)

- If two triangles are similar, then each of the following are proportional: the perimeters, the measures of the corresponding altitudes, the measures of the corresponding angle bisectors of the triangles, and the measures of the corresponding medians.

Key Vocabulary

- cross products (p. 381)
- extremes (p. 381)
- means (p. 381)
- midsegment (p. 406)
- proportion (p. 381)
- ratio (p. 380)
- scale factor (p. 389)
- similar polygons (p. 388)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

- The symbol \sim means “is congruent to.”
- A midsegment of a triangle is a segment with endpoints that are the midpoints of two sides of the triangle.
- Two polygons are similar if and only if their corresponding angles are congruent and the measures of the corresponding sides are equal.
- AA (Angle-Angle) is a congruence postulate.
- A proportion is a comparison of two quantities by division.
- If two triangles are similar, then the perimeters are proportional to the measures of the corresponding angles.
- A midsegment of a triangle is parallel to one side of the triangle, and its length is twice the length of that side.
- If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional length, then the line is one-half the length of the third side.



Lesson-by-Lesson Review

7-1 Proportions (pp. 380–386)

Solve each proportion.

9. $\frac{x - 12}{6} = \frac{x + 7}{-4}$ 10. $\frac{18}{7w + 5} = \frac{9}{4w - 1}$

11. **BABIES** The average length and weight of a newborn is 20.16 inches and 7.63 pounds, respectively. If length and weight remained proportional over time, what would be the average weight for an adult who is 71 inches tall? Do length and weight remain proportional as children grow? Explain.

Example 1 Solve $\frac{m - 13}{m + 13} = \frac{21}{34}$.

$$\frac{m - 13}{m + 13} = \frac{21}{34} \quad \text{Original proportion}$$

$$34(m - 13) = 21(m + 13) \quad \text{Cross Products}$$

$$34m - 442 = 21m + 273 \quad \text{Distributive Property}$$

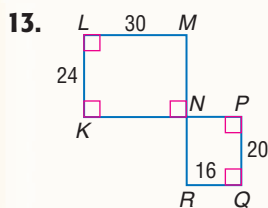
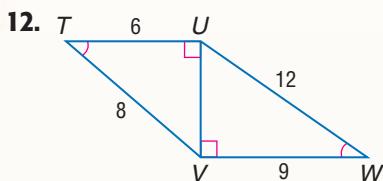
$$13m - 442 = 273 \quad \text{Subtract.}$$

$$13m = 715 \quad \text{Add.}$$

$$m = 55 \quad \text{Divide.}$$

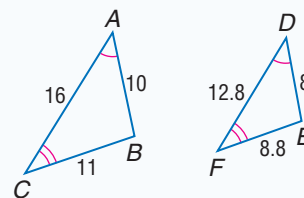
7-2 Similar Polygons (pp. 388–396)

Determine whether each pair of figures is similar. Justify your answer.



14. **SOLAR SYSTEM** In creating an accurate scale model of our solar system, Lana placed Earth 1 foot from the Sun. The actual distance from Earth to the Sun is 93,000,000 miles. If the actual distance from Pluto to the Sun is 3,695,950,000 miles, how far from the Sun would Lana need to place Pluto in her model?

Example 2 Determine whether the pair of triangles is similar. Justify your answer.



$\angle A \cong \angle D$ and $\angle C \cong \angle F$, so by the Third Angle Theorem, $\angle B \cong \angle E$. All of the corresponding angles are congruent.

Next check the corresponding sides.

$$\frac{AB}{DE} = \frac{10}{8} = \frac{5}{4} \text{ or } 1.25 \quad \frac{BC}{EF} = \frac{11}{8.8} = \frac{5}{4} \text{ or } 1.25$$

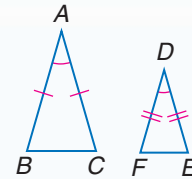
$$\frac{CA}{FD} = \frac{16}{12.8} = \frac{5}{4} \text{ or } 1.25$$

Since the corresponding angles are congruent and the ratios of the measures of the corresponding sides are equal, $\triangle ABC \sim \triangle DEF$.

7-3 Similar Triangles (pp. 397–403)

15. INDIRECT MEASUREMENT To estimate the height of a flagpole, Sonia sights the top of the pole in a mirror on the ground that is facing upward 21 feet from the pole. Sonia is 3 feet from the mirror, and the distance from her eyes to the ground is 5.8 feet. How tall is the flagpole?

Example 3 Determine whether the pair of triangles is similar. Justify your answer.



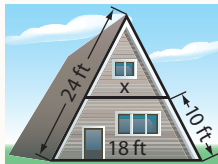
$\triangle ABC \sim \triangle DFE$ by SAS Similarity.

7-4 Parallel Lines and Proportional Parts (pp. 405–414)

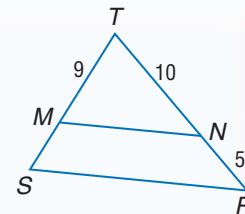
Use the figure in Example 4 to determine whether $\overline{MN} \parallel \overline{SR}$. Justify your answer.

- 16. $TM = 21, MS = 14, RN = 9, NT = 15$
- 17. $SM = 10, MT = 35, TN = 28, TR = 36$

18. HOUSES In an A-frame house, the roof slopes to the ground. Find the width x of the second floor.



Example 4 In $\triangle TRS$, $TS = 12$. Determine whether $\overline{MN} \parallel \overline{SR}$.



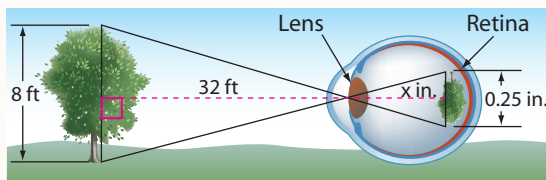
If $TS = 12$, then $MS = 12 - 9$ or 3. Compare the measures of the segments.

$$\frac{TM}{MS} = \frac{9}{3} = 3 \qquad \frac{TN}{NR} = \frac{10}{5} = 2$$

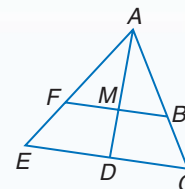
Since $\frac{TM}{MS} \neq \frac{TN}{NR}$, $\overline{MN} \not\parallel \overline{SR}$.

7-5 Parts of Similar Triangles (pp. 415–422)

19. HUMAN EYE The human eye uses similar triangles to invert and reduce an object as it passes through the lens onto the retina. What is the length from your lens to your retina?



Example 5 If $\overline{FB} \parallel \overline{ED}$, AD is an angle bisector of $\angle A$, $BF = 6$, $CE = 10$, and $AD = 5$, find AM .



Since $\angle ACE \cong \angle ABF$ and $\angle EAC \cong \angle FAB$, $\triangle ABF \sim \triangle ACE$ by AA Similarity.

$$\frac{AM}{AD} = \frac{BF}{CE} \quad \sim \triangle s \text{ have corr. } \angle \text{ bisectors proportional to the corr. sides.}$$

$$\frac{x}{5} = \frac{6}{10} \quad AD = 5, BF = 6, CE = 10, AM = x$$

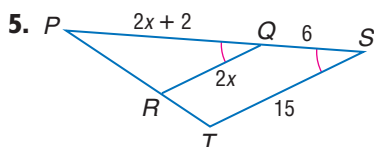
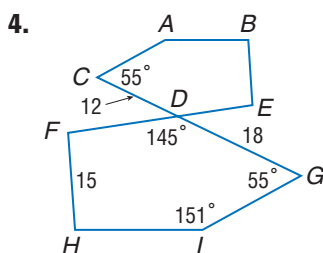
$10x = 30$ Cross products

$x = 3$ Divide. Thus, $AM = 3$.

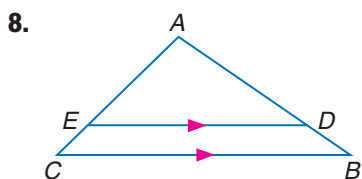
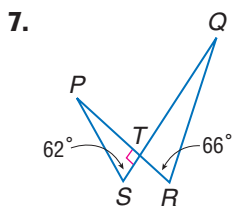
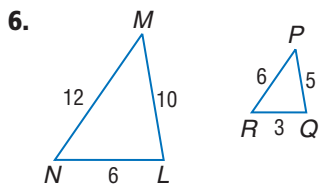
Solve each proportion.

- $\frac{x}{14} = \frac{1}{2}$
- $\frac{4x}{3} = \frac{108}{x}$
- $\frac{k+2}{7} = \frac{k-2}{3}$

Each pair of polygons is similar. Write a similarity statement and find the scale factor.

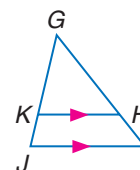


Determine whether each pair of triangles is similar. Justify your answer.



9. **BASKETBALL** Terry wants to measure the height of the top of the backboard of his basketball hoop. At 4:00, the shadow of a 4-foot fence is 20 inches, and the shadow of the backboard is 65 inches. What is the height of the top of the backboard?

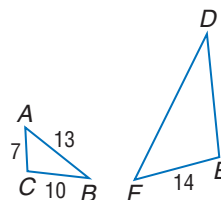
Refer to the figure below.



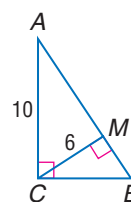
- Find KJ if $GJ = 8$, $GH = 12$, and $HI = 4$.
- Find GK if $GI = 14$, $GH = 7$, and $KJ = 6$.
- Find GI if $GH = 9$, $GK = 6$, and $KJ = 4$.

Find the perimeter of the given triangle.

13. $\triangle DEF$, if $\triangle DEF \sim \triangle ACB$



14. $\triangle ABC$

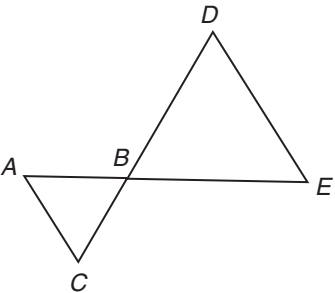


15. **MULTIPLE CHOICE** Joely builds a corkboard that is 45 inches tall and 63 inches wide. She wants to build a smaller corkboard with a similar shape for the kitchen. Which could be the dimensions of that corkboard?
- 4 in. by 3 in.
 - 7 in. by 5 in.
 - 12 in. by 5 in.
 - 21 in. by 14 in.

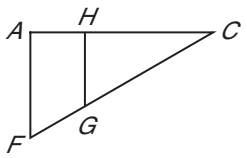
Standardized Test Practice

Cumulative, Chapters 1–7

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

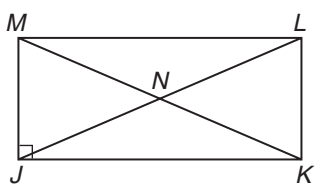
- Which triangles are *not* necessarily similar?
 - two right triangles with one angle measuring 30°
 - two right triangles with one angle measuring 45°
 - two isosceles triangles
 - two equilateral triangles
- Which of the following facts would be sufficient to prove that triangles ABC and EBD are similar?
 
 - $\overline{AB} \cong \overline{EB}$
 - $\overline{AC} \parallel \overline{DE}$
 - $\overline{CB} \cong \overline{DB}$
 - $\angle D \cong \angle E$

- GRIDDABLE** In a quadrilateral, the ratio of the measures of the sides is 2:3:5:9, and its longest side is 13.5 cm. Find the perimeter of the quadrilateral in centimeters.
- Given:** $MNOP$ is an isosceles trapezoid with diagonals \overline{MO} and \overline{NP} . Which of the following is *not* true?
 - $\overline{MO} \cong \overline{NP}$
 - \overline{MO} bisects \overline{NP} .
 - $\angle M \cong \angle N$
 - $\angle O \cong \angle P$

- Which of the following facts would *not* be sufficient to prove that triangles ACF and HCG are similar?
 
 - $\overline{AF} \parallel \overline{HG}$
 - $\frac{AC}{HC} = \frac{FC}{GC}$
 - $\frac{CG}{CF} = \frac{1}{2}$
 - $\angle FAH$ and $\angle CHG$ are right angles.

TEST-TAKING TIP

Question 5 In similar triangles, corresponding angles are congruent and corresponding sides are proportional. When you set up a proportion, be sure that it compares corresponding sides.

- GRIDDABLE** In rectangle $JKLM$ shown below, \overline{JL} and \overline{MK} are diagonals. If $JL = 2x + 5$, and $KM = 4x - 11$, what is x ?
 

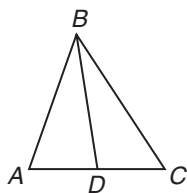
- If the measure of each exterior angle of a regular polygon is less than 50° , which of the following could *not* be the polygon?
 - decagon
 - octagon
 - heptagon
 - pentagon



8. **ALGEBRA** Which equation describes the line that passes through $(-2, 3)$ and is perpendicular to $2x - y = 3$?

F $y = 2x - 2$
 G $y = \frac{1}{2}x - 4$
 H $y = -\frac{1}{2}x + 2$
 J $y = -2x + 4$

9. In $\triangle ABC$, \overline{BD} is a median. If $AD = 3x + 5$, and $CD = 5x - 1$, find AC .



A 3
 B 11
 C 14
 D 28

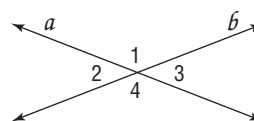
10. At the International Science Fair, a Canadian student recorded temperatures in degrees Celsius. A student from the United States recorded the same temperatures in degrees Fahrenheit. They used their data to plot a graph of Celsius versus Fahrenheit. What is the slope of their graph?

F $\frac{5}{9}$
 G 1
 H $\frac{9}{5}$
 J 2

11. Quadrilateral $HJKL$ is a parallelogram. If the diagonals are perpendicular, which statement must be true?

A Quadrilateral $HJKL$ is a square.
 B Quadrilateral $HJKL$ is a rhombus.
 C Quadrilateral $HJKL$ is a rectangle.
 D Quadrilateral $HJKL$ is an isosceles trapezoid.

12. If $\angle 4$ and $\angle 3$ are supplementary, which reason could you use as the first step in proving that $\angle 1$ and $\angle 2$ are supplementary?

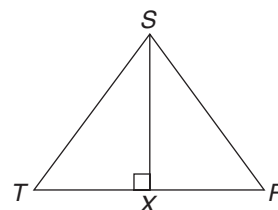


F Definition of a vertical angle
 G Definition of similar angles
 H Definition of perpendicular lines
 J Division Property

Pre-AP

Record your answer on a sheet of paper.
 Show your work.

13. Toby, Rani, and Sasha are practicing for a double Dutch rope-jumping tournament. Toby and Rani are standing at points T and R and are turning the ropes. Sasha is standing at S , equidistant from both Toby and Rani. Sasha will jump into the middle of the turning rope to point X . Prove that when Sasha jumps into the rope, she will be at the midpoint between Toby and Rani.



NEED EXTRA HELP?

If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	7-3	7-4	7-1	6-6	7-3	6-4	6-1	786	5-1	3-3	6-5	2-7	4-6