CHAPTER 8

Right Triangles and Trigonometry

BIG Ideas

- Solve problems using the geometric mean, the Pythagorean Theorem, and its converse.
- Use trigonometric ratios to solve right triangle problems.
- Solve triangles using the Law of Sines and the Law of Cosines.

Key Vocabulary

trigonometric ratio (p. 456)
Law of Sines (p. 471)
Law of Cosines (p. 479)

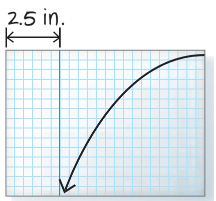
Real-World Link

Bridges The William H. Natcher bridge across the Ohio River has a cable-stayed design. The cables form right triangles with the supports and the length of the bridge.

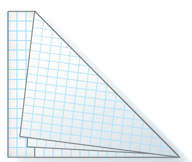
FOLDABLES Study Organizer

Right Triangles and Trigonometry Make this Foldable to help you organize your notes. Begin with seven sheets of grid paper.

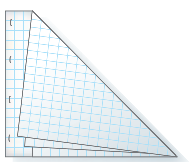
1. **Stack** the sheets. Fold the top right corner to the bottom edge to form a square.



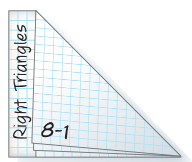
2. **Fold** the rectangular part in half.



3. **Staple** the sheets along the fold in four places.



4. **Label** each sheet with a lesson number and the rectangular part with the chapter title.



GET READY for Chapter 8

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Solve each proportion. Round to the nearest hundredth. (Lesson 7-1)

1. $\frac{3}{4} = \frac{12}{a}$

2. $\frac{c}{5} = \frac{8}{3}$

3. $\frac{d}{20} = \frac{6}{5} = \frac{f}{10}$

4. $\frac{4}{3} = \frac{6}{y} = \frac{1}{z}$

5. MINIATURES The proportion $\frac{1 \text{ in.}}{12 \text{ in.}} = \frac{3.5}{x}$ relates the height of a miniature chair to the height of a real chair. Solve the proportion. (Lesson 7-1)

Find the measure of the hypotenuse of each right triangle having legs with the given measures. Round to the nearest hundredth. (Extend 1-3)

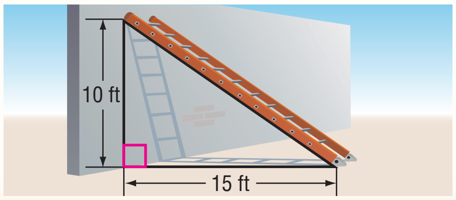
6. 5 and 12

7. 6 and 8

8. 15 and 15

9. 14 and 27

10. PAINTING A ladder is propped against a wall as shown. To the nearest tenth, what is the length of the ladder? (Extend 1-3)



11. The measure of one angle in a right triangle is three times the measure of the second angle. Find the measures of each angle of the triangle. Find x. (Lesson 4-2)

QUICKReview

EXAMPLE 1

Solve the proportion $\frac{a}{30} = \frac{31}{5}$. Round to the nearest hundredth if necessary.

$\frac{a}{30} = \frac{31}{5}$	Write the proportion.
$5a = 30(31)$	Find the cross products.
$5a = 930$	Simplify.
$a = 186$	Divide each side by 5.

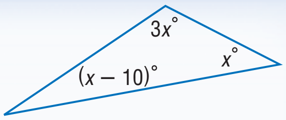
EXAMPLE 2

Find the measure of the hypotenuse of the right triangle having legs with the measures 10 and 24. Round to the nearest hundredth if necessary.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$10^2 + 24^2 = c^2$	Substitution
$100 + 576 = c^2$	Evaluate the exponents.
$676 = c^2$	Simplify.
$\sqrt{676} = \sqrt{c^2}$	Take the square root of each side.
$26 = c$	Simplify.

EXAMPLE 3

Find x.



$$x + 3x + x - 10 = 180$$

$$5x = 190$$

$$x = 38$$

8-1 Geometric Mean

Main Ideas

- Find the geometric mean between two numbers.
- Solve problems involving relationships between parts of a right triangle and the altitude to its hypotenuse.

New Vocabulary

geometric mean

GET READY for the Lesson

When you look at a painting, you should stand at a distance that allows you to see all of the details in the painting. The distance that creates the best view is the geometric mean of the distance from the top of the painting to eye level and the distance from the bottom of the painting to eye level.

Geometric Mean The **geometric mean** between two numbers is the positive square root of their product.

Study Tip

You may wish to review **square roots** and **simplifying radicals** on pp. 790–791.

KEY CONCEPT — Geometric Mean

For two positive numbers a and b, the geometric mean is the positive number x where the proportion $a : x = x : b$ is true. This proportion can be written using fractions as $\frac{a}{x} = \frac{x}{b}$ or with cross products as $x^2 = ab$ or $x = \sqrt{ab}$.

EXAMPLE 1 Geometric Mean

Find the geometric mean between each pair of numbers.

a. 4 and 9

$\frac{4}{x} = \frac{x}{9}$ Definition of geometric mean

$x^2 = 36$ Cross products

$x = \sqrt{36}$ Take the positive square root of each side.

$x = 6$ Simplify.

b. 6 and 15

$\frac{6}{x} = \frac{x}{15}$ Definition of geometric mean

$x^2 = 90$ Cross products

$x = \sqrt{90}$ Take the positive square root of each side.

$x = 3\sqrt{10}$ Simplify.

$x \approx 9.5$ Use a calculator.

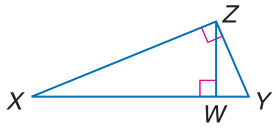
CHECK Your Progress

1A. 5 and 45

1B. 8 and 10

Robert Brenner/PhotoEdit

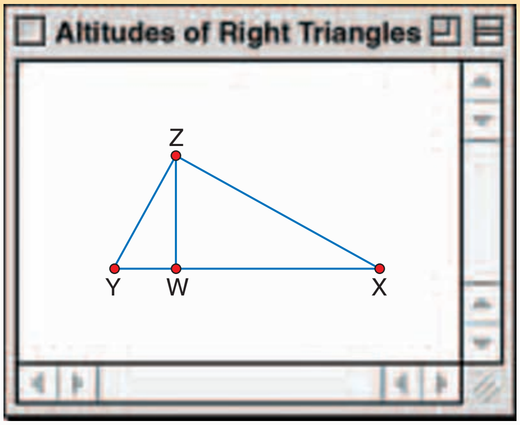
Altitude of a Triangle Consider right triangle XYZ with altitude $\overline{WZ}$ drawn from the right angle Z to the hypotenuse $\overline{XY}$. A special relationship exists for the three right triangles, $\triangle XYZ$, $\triangle XZW$, and $\triangle ZYW$.



GEOMETRY SOFTWARE LAB

Right Triangles Formed by the Altitude

Use The Geometer's Sketchpad to draw a right triangle *XYZ* with right angle *Z*. Draw the altitude $\overline{ZW}$ from the right angle to the hypotenuse.



THINK AND DISCUSS

1. Find the measures of $\angle X$, $\angle XZY$, $\angle Y$, $\angle XWZ$, $\angle XZW$, $\angle YWZ$, and $\angle YZW$.
2. What is the relationship between $m\angle X$ and $m\angle YZW$? between $m\angle Y$ and $m\angle XZW$?
3. Drag point Z to another position. Describe the relationship between the measures of $\angle X$ and $\angle YZW$ and between $m\angle Y$ and $m\angle XZW$.

MAKE A CONJECTURE

4. How are $\triangle XYZ$, $\triangle XZW$, and $\triangle ZYW$ related?

The Geometry Software Lab suggests the following theorem.

Study Tip

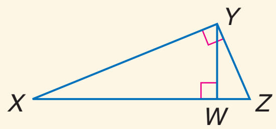
Altitudes of a Right Triangle

The altitude drawn to the hypotenuse originates from the right angle. The other two altitudes of a right triangle are the legs.

THEOREM 8.1

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other.

Example: $\triangle XYZ \sim \triangle XWY \sim \triangle YWZ$



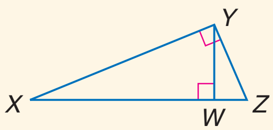
You will prove Theorem 8.1 in Exercise 38.

By Theorem 8.1, since $\triangle XWY \sim \triangle YWZ$, the corresponding sides are proportional. Thus, $\frac{XW}{YW} = \frac{YW}{ZW}$. Notice that $\overline{XW}$ and $\overline{ZW}$ are segments of the hypotenuse of the largest triangle.

THEOREM 8.2

The measure of an altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.

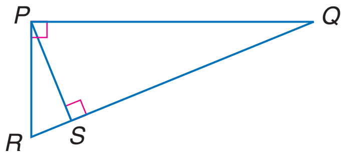
Example: *YW* is the geometric mean of *XW* and *ZW*.



You will prove Theorem 8.2 in Exercise 39.

EXAMPLE Altitude and Segments of the Hypotenuse

2 **In $\triangle PQR$, $RS = 3$ and $QS = 14$. Find PS.**



$\frac{RS}{PS} = \frac{PS}{QS}$ Theorem 8.2

$\frac{3}{x} = \frac{x}{14}$ $RS = 3$, $QS = 14$, and $PS = x$

$x^2 = 42$ Cross products

$x \approx 6.5$ Use a calculator to take the positive square root of each side.

Study Tip

Square Roots

Since these numbers represent measures, you can ignore the negative square root value.

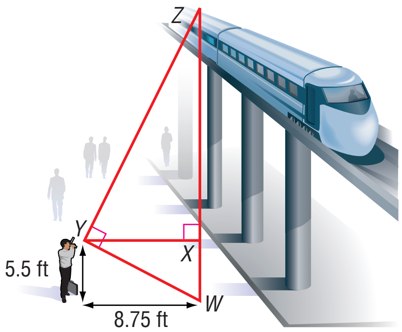
CHECK Your Progress

2. Refer to $\triangle PQR$ above. If $RS = 0.8$ and $QS = 2.2$, find PS.

Real-World EXAMPLE

3 **ARCHITECTURE** **Mr. Martinez is designing a walkway to pass over a train. To find the train height, he holds a carpenter's square at eye level and sights along the edges from the street to the top of the train. If Mr. Martinez's eye level is 5.5 feet above the street and he is 8.75 feet from the train, find the train's height. Round to the nearest tenth.**

Draw a diagram. Let $\overline{YX}$ be the altitude drawn from the right angle of $\triangle WYZ$.



$\frac{WX}{YX} = \frac{YX}{ZX}$ Theorem 8.2

$\frac{5.5}{8.75} = \frac{8.75}{ZX}$ $WX = 5.5$ and $YX = 8.75$

$5.5ZX = 76.5625$ Cross products

$ZX \approx 13.9$ Divide each side by 5.5.

The elevated train is 5.5 + 13.9 or about 19.4 feet high.

CHECK Your Progress

3. Makayla is using a carpenter's square to sight the top of a waterfall. If her eye level is 5 feet from the ground and she is a horizontal distance of 28 feet from the waterfall, find the height of the waterfall to the nearest tenth.

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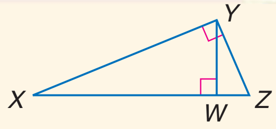
The altitude to the hypotenuse of a right triangle determines another relationship between the segments.

THEOREM 8.3

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

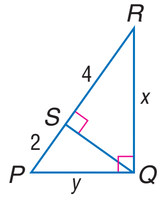


Example: $\frac{XZ}{XY} = \frac{XY}{XW}$ and $\frac{XZ}{YZ} = \frac{YZ}{WZ}$

You will prove Theorem 8.3 in Exercise 40.

EXAMPLE 4 Hypotenuse and Segment of Hypotenuse

Find x and y in $\triangle PQR$.



$\overline{PQ}$ and $\overline{RQ}$ are legs of right triangle PQR. Use Theorem 8.3 to write a proportion for each leg and then solve.

$$\frac{PR}{PQ} = \frac{PQ}{PS}$$

$\frac{6}{y} = \frac{y}{2}$ — $PS = 2$, $PQ = y$, $PR = 6$

$y^2 = 12$ — Cross products

$y = \sqrt{12}$ — Take the square root.

$y = 2\sqrt{3}$ — Simplify.

$y \approx 3.5$ — Use a calculator.

$$\frac{PR}{RQ} = \frac{RQ}{SR}$$

$\frac{6}{x} = \frac{x}{4}$ — $RS = 4$, $RQ = x$, $PR = 6$

$x^2 = 24$ — Cross products

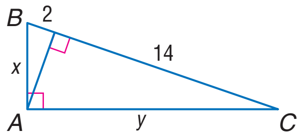
$x = \sqrt{24}$ — Take the square root.

$x = 2\sqrt{6}$ — Simplify.

$x \approx 4.9$ — Use a calculator.

CHECK Your Progress

4. Find x and y in $\triangle ABC$.



CHECK Your Understanding

Example 1 (p. 432)

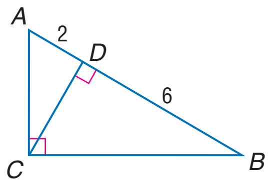
Find the geometric mean between each pair of numbers.

1. 9 and 4 **2.** 36 and 49 **3.** 6 and 8 **4.** $2\sqrt{2}$ and $3\sqrt{2}$

Example 2 (p. 434)

Find the measure of the altitude drawn to the hypotenuse.

5.



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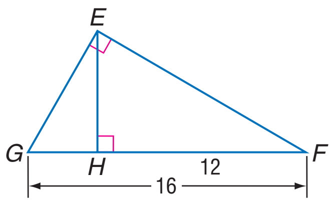
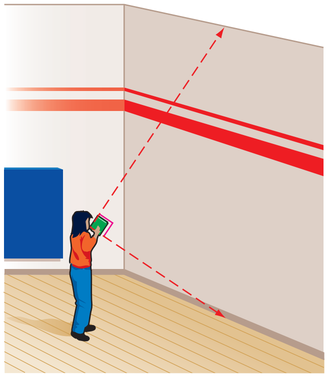


Example 3 (p. 434)

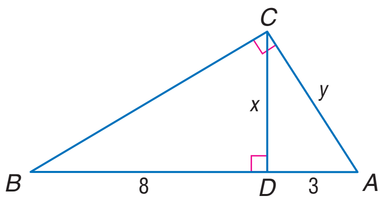
7. DANCES Danielle is making a banner for the dance committee. The banner is to be as high as the wall of the gymnasium. To find the height of the wall, Danielle held a book up to her eyes so that the top and bottom of the wall were in line with the bottom edge and binding of the cover. If Danielle's eye level is 5 feet off the ground and she is standing 12 feet from the wall, how high is the wall?



Example 4 (p. 435)

Find x and y.

8.



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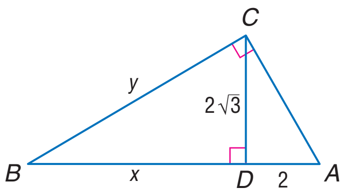


Exercises

HOMEWORK HELP	
For Exercises	See Examples
10–17	1
18–23	2
24–25	3
26–31	4

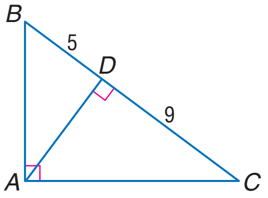
Find the geometric mean between each pair of numbers.

10. 5 and 6 **11.** 24 and 25 **12.** $\sqrt{45}$ and $\sqrt{80}$ **13.** $\sqrt{28}$ and $\sqrt{1372}$

14. $\frac{3}{5}$ and 1 **15.** $\frac{8\sqrt{3}}{5}$ and $\frac{6\sqrt{3}}{5}$ **16.** $\frac{2\sqrt{2}}{6}$ and $\frac{5\sqrt{2}}{6}$ **17.** $\frac{13}{7}$ and $\frac{5}{7}$

Find the measure of the altitude drawn to the hypotenuse.

18.



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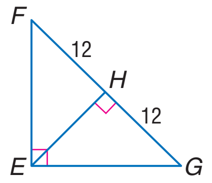

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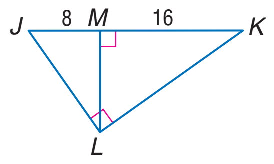

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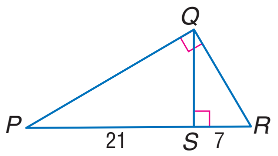

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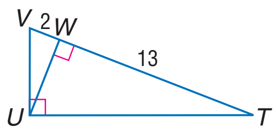

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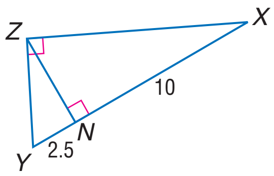
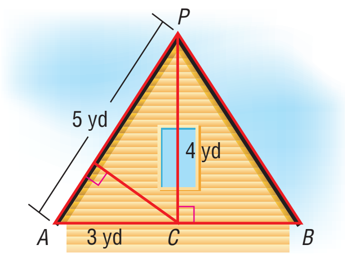
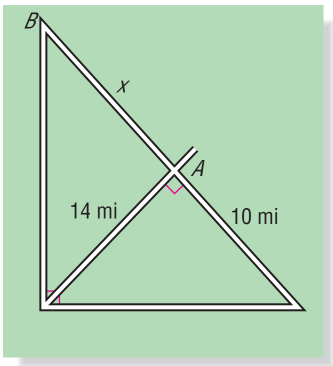


24. CONSTRUCTION The slope of the roof shown below is $\frac{4}{3}$. A builder wants to put a support brace from point C perpendicular to $\overline{AP}$. Find the length of the brace.

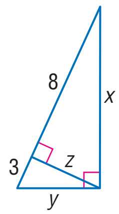


25. ROADS City planners want to build a road to connect points A and B. Find out how long this road will need to be.



Find x, y, and z.

26.



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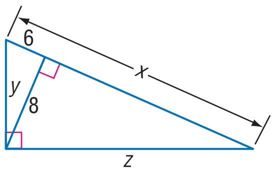

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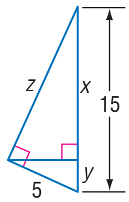

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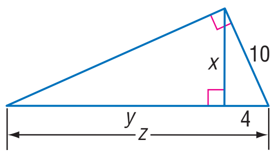

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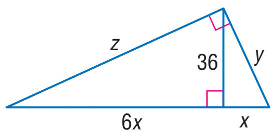

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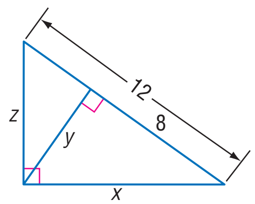


The geometric mean and one extreme are given. Find the other extreme.

32. $\sqrt{17}$ is the geometric mean between a and b. Find b if $a = 7$.

33. $\sqrt{12}$ is the geometric mean between x and y. Find x if $y = \sqrt{3}$.

Determine whether each statement is *always*, *sometimes*, or *never* true.

34. The geometric mean for consecutive positive integers is the average of the two numbers.

35. The geometric mean for two perfect squares is a positive integer.

36. The geometric mean for two positive integers is another integer.

37. The measure of the altitude of a triangle is the geometric mean between the measures of the segments of the side opposite the initial vertex.

PROOF **Write a proof for each theorem.**

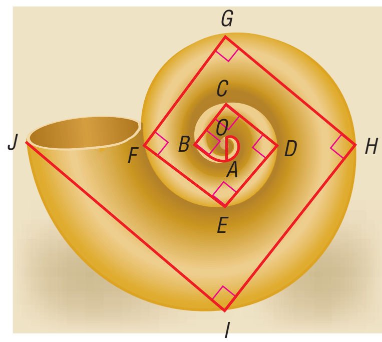
38. Theorem 8.1 **39.** Theorem 8.2 **40.** Theorem 8.3

41. RESEARCH Use the Internet or other resource to write a brief description of the golden ratio, which is also known as the divine proportion, golden mean, or golden section.

EXTRA PRACTICE
See pages 815, 835.

Math Online
Self-Check Quiz at geometryonline.com

42. PATTERNS The spiral of the state shell of Texas, the lightning whelk, can be modeled by a geometric mean. Consider the sequence of segments $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, $\overline{OD}$, $\overline{OE}$, $\overline{OF}$, $\overline{OG}$, $\overline{OH}$, $\overline{OI}$, and $\overline{OJ}$. The length of each of these segments is the geometric mean between the lengths of the preceding segment and the succeeding segment. Explain this relationship. (*Hint*: Consider $\triangle FGH$.)



H.O.T. Problems

43. OPEN ENDED Find two pairs of numbers with a geometric mean of 12.

44. REASONING Draw and label a right triangle with an altitude drawn from the right angle. From your drawing, explain the meaning of *the hypotenuse and the segment of the hypotenuse adjacent to that leg* in Theorem 8.3.

45. FIND THE ERROR $\triangle RST$ is a right isosceles triangle. Holly and Ian are finding the measure of altitude $\overline{SU}$. Who is correct? Explain your reasoning.

Holly

$\frac{RS}{SU} = \frac{SU}{RT}$

$\frac{9.9}{x} = \frac{x}{14}$

$x^2 = 138.5$

$x = \sqrt{138.5}$

$x = 11.8$

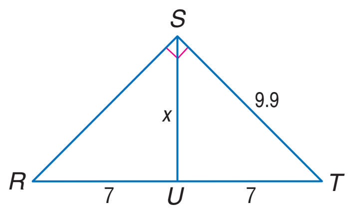
Ian

$\frac{RU}{SU} = \frac{SU}{TU}$

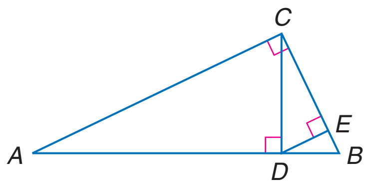
$\frac{7}{x} = \frac{x}{7}$

$x = 49$

$x = 7$



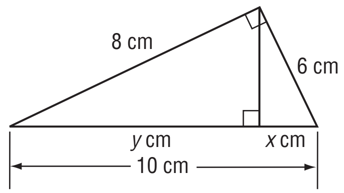
46. CHALLENGE Find the exact value of DE, given $AD = 12$ and $BD = 4$.



47. *Writing in Math* Describe how the geometric mean can be used to view paintings. Include an explanation of what happens when you are too far or too close to a painting.

STANDARDIZED TEST PRACTICE

48. What are the values of x and y?



A 4 and 6

B 2.5 and 7.5

C 3.6 and 6.4

D 3 and 7

49. REVIEW What are the solutions for the quadratic equation $x^2 + 9x = 36$?

F $-3, -12$ H $3, -12$

G $3, 12$ J $-3, 12$

50. REVIEW Tulia borrowed \$300 at 15% simple interest for two years. If she makes no payments either year, how much interest will she owe at the end of the two-year period?

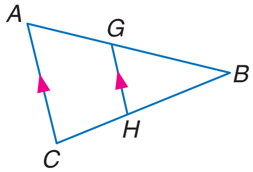
A \$90.00 C \$30.00

B \$45.00 D \$22.50

Spiral Review

51. The measures of the sides of a triangle are 20, 24, and 30. Find the measures of the segments formed where the bisector of the smallest angle meets the opposite side. (Lesson 7-5)

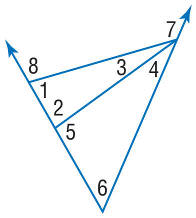
For Exercises 52 and 53, use $\triangle ABC$. (Lesson 7-4)



52. If $AG = 4$, $GB = 6$, and $BH = 8$, find BC.

53. If $AB = 12$, $BC = 14$, and $HC = 4$, find AG.

Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition. (Lesson 5-2)



54. measures less than $m\angle 8$

55. measures greater than $m\angle 1$

56. measures less than $m\angle 7$

57. measures greater than $m\angle 6$

Write an equation in slope-intercept form for the line that satisfies the given conditions. (Lesson 3-4)

58. $m = 2$, y-intercept $= 4$

59. passes through $(2, 6)$ and $(-1, 0)$

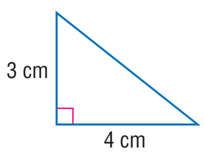
60. $m = -4$, passes through $(-2, -3)$

61. x-intercept is 2, y-intercept $= -8$

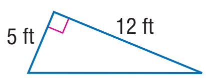
GET READY for the Next Lesson

PREREQUISITE SKILL Use the Pythagorean Theorem to find the length of the hypotenuse of each right triangle. (Lesson 1-4)

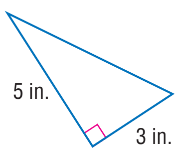
62.



63.



64.



EXPLORE 8-2

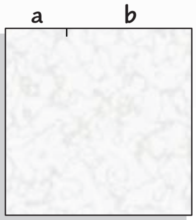
Geometry Lab
The Pythagorean Theorem

In Chapter 1, you learned that the Pythagorean Theorem relates the measures of the legs and the hypotenuse of a right triangle. Ancient cultures used the Pythagorean Theorem before it was officially named in 1909.

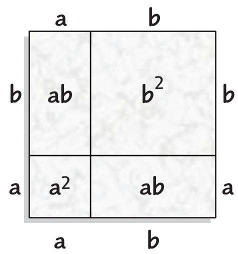
ACTIVITY

Use paper folding to develop the Pythagorean Theorem.

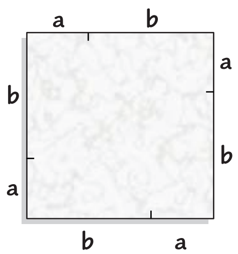
Step 1 On a piece of patty paper, make a mark along one side so that the two resulting segments are not congruent. Label one as *a* and the other as *b*.



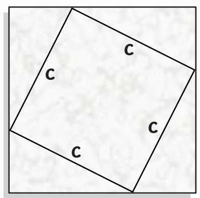
Step 2 Copy these measures on the other sides in the order shown at the right. Fold the paper to divide the square into four sections. Label the area of each section.



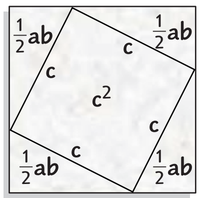
Step 3 On another sheet of patty paper, mark the same lengths *a* and *b* on the sides in the different pattern shown at the right.



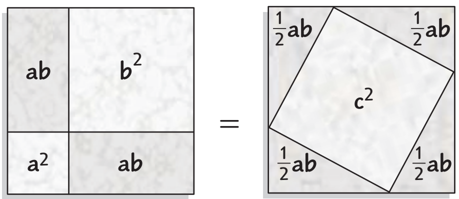
Step 4 Use your straightedge and pencil to connect the marks as shown at the right. Let *c* represent the length of each hypotenuse.



Step 5 Label the area of each section, which is $\frac{1}{2}ab$ for each triangle and c^2 for the square.



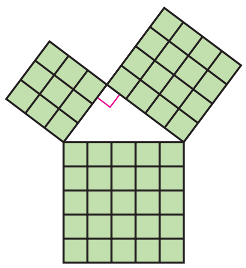
Step 6 Place the squares side by side and color the corresponding regions that have the same area. For example, $ab = \frac{1}{2}ab + \frac{1}{2}ab$.



The parts that are not shaded tell us that $a^2 + b^2 = c^2$.

Analyze the Results

1. Use a ruler to find actual measures for *a*, *b*, and *c*. Do these measures confirm that $a^2 + b^2 = c^2$?
2. Repeat the activity with different *a* and *b* values. What do you notice?
3. Explain why the drawing at the right is an illustration of the Pythagorean Theorem.



4. **CHALLENGE** Use a geometric diagram to show that for any positive numbers *a* and *b*, $a + b > \sqrt{a^2 + b^2}$.

8-2 The Pythagorean Theorem and Its Converse

Main Ideas

- Use the Pythagorean Theorem.
- Use the converse of the Pythagorean Theorem.

New Vocabulary

Pythagorean triple

GET READY for the Lesson

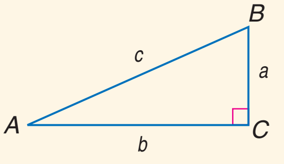
The Talmadge Memorial Bridge over the Savannah River, in Georgia, has two soaring towers of suspension cables. Note the right triangles being formed by the roadway, the perpendicular tower, and the suspension cables. The Pythagorean Theorem can be used to find measures in any right triangle.

The Pythagorean Theorem In Lesson 1-3, you used the Pythagorean Theorem to find the distance between two points by finding the length of the hypotenuse when given the lengths of the two legs of a right triangle. You can also find the measure of any side of a right triangle given the other two measures.

THEOREM 8.4 *Pythagorean Theorem*

In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Symbols: $a^2 + b^2 = c^2$

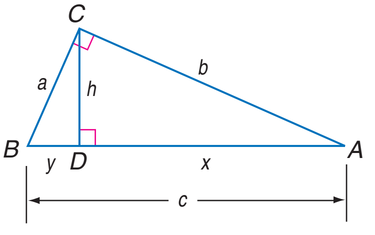


The geometric mean can be used to prove the Pythagorean Theorem.

Proof Pythagorean Theorem

Given: $\triangle ABC$ with right angle at C

Prove: $a^2 + b^2 = c^2$



Proof:

Draw right triangle ABC so C is the right angle. Then draw the altitude from C to $\overline{AB}$. Let $AB = c$, $AC = b$, $BC = a$, $AD = x$, $DB = y$, and $CD = h$.

Two geometric means now exist.

$\frac{c}{a} = \frac{a}{y}$ and $\frac{c}{b} = \frac{b}{x}$

$a^2 = cy$ and $b^2 = cx$ Cross products

Add the equations.

$a^2 + b^2 = cy + cx$

$a^2 + b^2 = c(y + x)$ Factor.

$a^2 + b^2 = c^2$ Since $c = y + x$, substitute c for $(y + x)$.

You can use the Pythagorean Theorem to find the length of the hypotenuse or a leg of a right triangle if the other two sides are known.

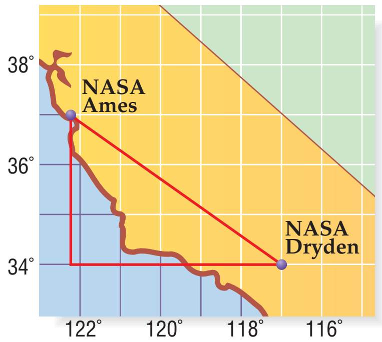
Real-World Link

Due to the curvature of Earth, the distance between two points is often expressed as *degree distance* using latitude and longitude. This measurement closely approximates the distance on a plane.

Source: NASA

Real-World EXAMPLE — Find the Length of the Hypotenuse

1 **GEOGRAPHY** **California's NASA Dryden is located at about 117 degrees longitude and 34 degrees latitude. NASA Ames, also in California, is located at about 122 degrees longitude and 37 degrees latitude. Use the lines of longitude and latitude to find the degree distance to the nearest tenth between NASA Dryden and NASA Ames.**



The change in longitude between the two locations is |117−122| or 5 degrees. Let this distance be a.

The change in latitude is |37 − 34| or 3 degrees latitude. Let this distance be b.

Use the Pythagorean Theorem to find the distance in degrees from NASA Dryden to NASA Ames, represented by c.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$5^2 + 3^2 = c^2$ $a = 5, b = 3$

$25 + 9 = c^2$ Simplify.

$34 = c^2$ Add.

$\sqrt{34} = c$ Take the positive square root of each side.

$5.8 \approx c$ Use a calculator.

The degree distance between NASA Dryden and NASA Ames is about 5.8 degrees.

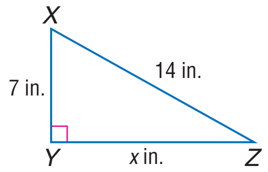
CHECK Your Progress

1. **GEOGRAPHY** Houston, Texas, is located at about 30 degrees latitude and about 95 degrees longitude. Raleigh, North Carolina, is located at about 36 degrees latitude and about 79 degrees longitude. Find the degree distance to the nearest tenth.

Online Personal Tutor at geometryonline.com

EXAMPLE Find the Length of a Leg

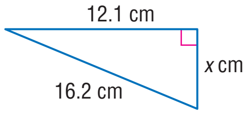
2 **Find x.**



$$
\begin{aligned}
(XY)^2 + (YZ)^2 &= (XZ)^2 && \text{Pythagorean Theorem}\\
7^2 + x^2 &= 14^2 && XY = 7, XZ = 14\\
49 + x^2 &= 196 && \text{Simplify.}\\
x^2 &= 147 && \text{Subtract 49 from each side.}\\
x &= \sqrt{147} && \text{Take the square root of each side.}\\
x &= 7\sqrt{3} && \text{Simplify.}\\
x &\approx 12.1 && \text{Use a calculator.}
\end{aligned}
$$

CHECK Your Progress

2. Find x.

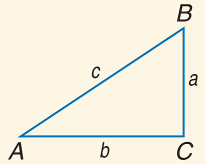


Converse of the Pythagorean Theorem The converse of the Pythagorean Theorem can help you determine whether three measures of the sides of a triangle are those of a right triangle.

THEOREM 8.5 Converse of the Pythagorean Theorem

If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

Symbols: If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.



You will prove Theorem 8.5 in Exercise 30.

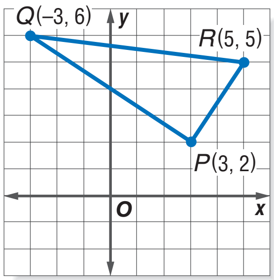
Study Tip

Distance Formula

When using the Distance Formula, be sure to follow the order of operations carefully. Perform the operation inside the parentheses first, square each term, and then add.

EXAMPLE Verify a Triangle is a Right Triangle

3 **COORDINATE GEOMETRY** **Verify that $\triangle PQR$ is a right triangle.**



Use the Distance Formula to determine the lengths of the sides.

$$
\begin{aligned}
PQ &= \sqrt{(-3-3)^2 + (6-2)^2} && x_1 = 3, y_1 = 2, x_2 = -3, y_2 = 6\\
&= \sqrt{(-6)^2 + 4^2} && \text{Subtract.}\\
&= \sqrt{52} && \text{Simplify.}
\end{aligned}
$$

$$
\begin{aligned}
QR &= \sqrt{[5-(-3)]^2 + (5-6)^2} && x_1 = -3, y_1 = 6, x_2 = 5, y_2 = 5\\
&= \sqrt{8^2 + (-1)^2} && \text{Subtract.}\\
&= \sqrt{65} && \text{Simplify.}
\end{aligned}
$$

$$
\begin{aligned}
PR &= \sqrt{(5-3)^2 + (5-2)^2} && x_1 = 3, y_1 = 2, x_2 = 5, y_2 = 5\\
&= \sqrt{2^2 + 3^2} && \text{Subtract.}\\
&= \sqrt{13} && \text{Simplify.}
\end{aligned}
$$

By the converse of the Pythagorean Theorem, if the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

$PQ^2 + PR^2 = QR^2$	Converse of the Pythagorean Theorem
$(\sqrt{52})^2 - (\sqrt{13})^2 \stackrel{?}{=} (\sqrt{65})^2$	$PQ = \sqrt{52}$, $PR = \sqrt{13}$, $QR = \sqrt{65}$
$52 + 13 \stackrel{?}{=} 65$	Simplify.
$65 = 65$	Add.

Since the sum of the squares of two sides equals the square of the longest side, $\triangle PQR$ is a right triangle.

CHECK Your Progress

3. Verify that $\triangle ABC$ with vertices $A(2, -3)$, $B(3, 0)$, and $C(5, -1)$ is a right triangle.

A **Pythagorean triple** is three whole numbers that satisfy the equation $a^2 + b^2 = c^2$, where c is the greatest number. One common Pythagorean triple is 3-4-5. If the measures of the sides of any right triangle are whole numbers, the measures form a Pythagorean triple.

EXAMPLE 4 Pythagorean Triples

Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple.

a. 8, 15, 16

Since the measure of the longest side is 16, 16 must be c, and a or b are 8 and 15, respectively.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$8^2 + 15^2 \stackrel{?}{=} 16^2$	$a = 8$, $b = 15$, $c = 16$
$64 + 225 \stackrel{?}{=} 256$	Simplify.
$289 \neq 256$	Add.

Since $289 \neq 256$, segments with these measures cannot form a right triangle. Therefore, they do not form a Pythagorean triple.

b. $\frac{\sqrt{3}}{5}$, $\frac{\sqrt{6}}{5}$, **and** $\frac{3}{5}$

$a^2 + b^2 = c^2$	Pythagorean Theorem
$\left(\frac{\sqrt{3}}{5}\right)^2 + \left(\frac{\sqrt{6}}{5}\right)^2 \stackrel{?}{=} \left(\frac{3}{5}\right)^2$	$a = \frac{\sqrt{3}}{5}$, $b = \frac{\sqrt{6}}{5}$, $c = \frac{3}{5}$
$\frac{3}{25} + \frac{6}{25} \stackrel{?}{=} \frac{9}{25}$	Simplify.
$\frac{9}{25} = \frac{9}{25}$ ✓	Add.

Since $\frac{9}{25} = \frac{9}{25}$, segments with these measures form a right triangle. However, the three numbers are not whole numbers. Therefore, they do not form a Pythagorean triple.

Study Tip

Comparing Numbers

If you cannot quickly identify the greatest number, use a calculator to find decimal values for each number and compare.

CHECK Your Progress

4A. 20, 48, and 52

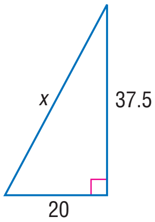
4B. $\frac{\sqrt{2}}{7}$, $\frac{\sqrt{3}}{7}$, and $\frac{\sqrt{5}}{7}$

CHECK Your Understanding

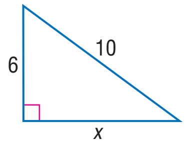
Examples 1 and 2
(pp. 441–442)

Find x.

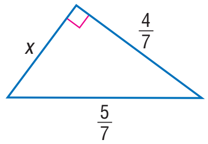
1.



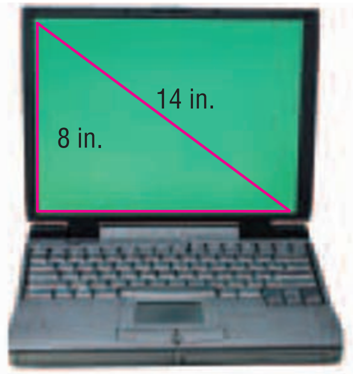
2.



3.



4. COMPUTERS Computer displays are usually measured along the diagonal of the screen. A 14-inch display has a diagonal that measures 14 inches. If the height of the screen is 8 inches, how wide is the screen?



Example 3
(p. 442)

5. COORDINATE GEOMETRY Determine whether $\triangle JKL$ with vertices $J(-2, 2)$, $K(-1, 6)$, and $L(3, 5)$ is a right triangle. Explain.

Example 4
(p. 443)

Determine whether each set of numbers can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

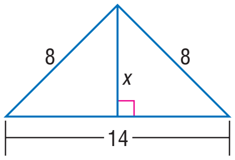
6. 15, 36, 39 **7.** $\sqrt{40}$, 20, 21 **8.** $\sqrt{44}$, 8, $\sqrt{108}$

Exercises

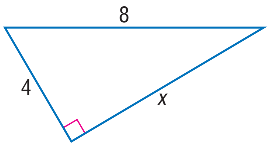
HOMEWORK HELP	
For Exercises	See Examples
9–14	1, 2
15–18	3
19–26	4

Find x.

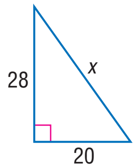
9.



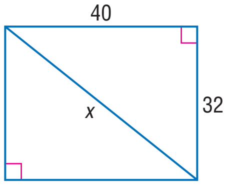
10.



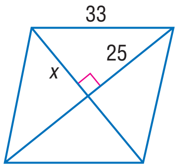
11.



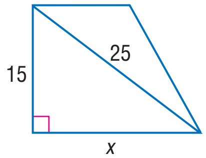
12.



13.



14.



COORDINATE GEOMETRY Determine whether $\triangle QRS$ is a right triangle for the given vertices. Explain.

15. $Q(1, 0)$, $R(1, 6)$, $S(9, 0)$

16. $Q(3, 2)$, $R(0, 6)$, $S(6, 6)$

17. $Q(-4, 6)$, $R(2, 11)$, $S(4, -1)$

18. $Q(-9, -2)$, $R(-4, -4)$, $S(-6, -9)$

Determine whether each set of numbers can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

19. 8, 15, 17 **20.** 7, 24, 25 **21.** 20, 21, 31 **22.** 37, 12, 34

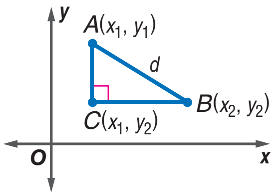
23. $\frac{1}{5}, \frac{1}{7}, \frac{\sqrt{74}}{35}$ **24.** $\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{3}, \frac{35}{36}$ **25.** $\frac{3}{4}, \frac{4}{5}, 1$ **26.** $\frac{6}{7}, \frac{8}{7}, \frac{10}{7}$

27. GARDENING Scott wants to plant flowers in a triangular plot. He has three lengths of plastic garden edging that measure 20 inches, 21 inches, and 29 inches. Discuss whether these pieces form a right triangle. Explain.

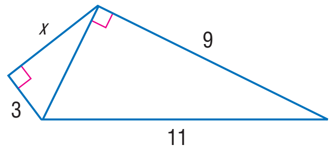
28. NAVIGATION A fishing trawler off the coast of Alaska was ordered by the U.S. Coast Guard to change course. They were to travel 6 miles west and then sail 12 miles south to miss a large iceberg before continuing on the original course. How many miles out of the way did the trawler travel?

29. PROOF Use the Pythagorean Theorem and the figure at the right to prove the Distance Formula.

30. PROOF Write a paragraph proof of Theorem 8.5.



31. Find the value of x in the figure shown.



Real-World Career

Military

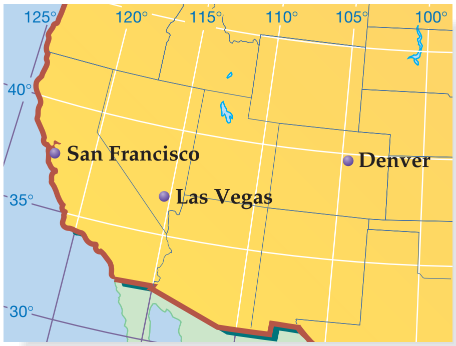
All branches of the military use navigation. Some of the jobs using navigation include radar/sonar operators, boat operators, airplane navigators, and space operations officers.

Math Online

For more information, go to geometryonline.com.

GEOGRAPHY For Exercises 32 and 33, use the following information.

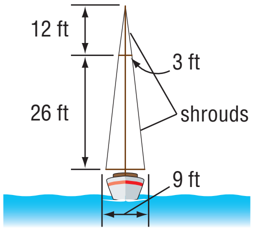
Denver is located at about 105° longitude and 40° latitude. San Francisco is located at about 122° longitude and 38° latitude. Las Vegas is located at about 115° longitude and 36° latitude. Using the lines of longitude and latitude, find each degree distance.



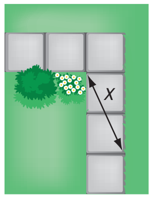
32. San Francisco to Denver

33. Las Vegas to Denver

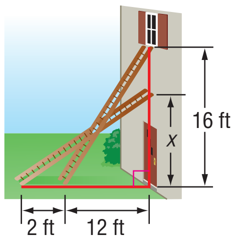
34. SAILING The mast of a sailboat is supported by wires called *shrouds*. What is the total length of wire needed to form these shrouds?



35. LANDSCAPING Six congruent square stones are arranged in an L-shaped walkway through a garden. If $x = 15$ inches, then find the area of the L-shaped walkway.



36. PAINTING A painter sets a ladder up to reach the bottom of a second-story window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint, a neighbor's dog bumps the ladder, which moves the base 2 feet farther away from the house. How far up the side of the house does the ladder reach?



EXTRA PRACTICE

See pages 815, 835.

Math Online

Self-Check Quiz at geometryonline.com

H.O.T. Problems

37. FIND THE ERROR Maria and Colin are determining whether 5-12-13 is a Pythagorean triple. Who is correct? Explain your reasoning.

Colin

$13^2 + 5^2 \stackrel{?}{=} 12^2$

$169 + 25 \stackrel{?}{=} 144$

$193 \neq 144$

no

Maria

$5^2 + 12^2 = 13^2$

$25 + 144 = 169$

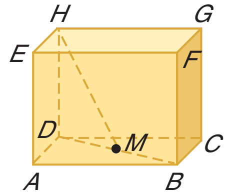
$169 = 169$

yes

38. **OPEN ENDED** Draw a pair of similar right triangles. Are the measures of the sides of each triangle a Pythagorean triple? Explain.

39. **REASONING** *True* or *false*? Any two right triangles with the same hypotenuse have the same area. Explain your reasoning.

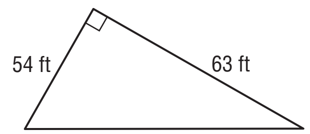
40. **CHALLENGE** The figure at the right is a rectangular prism with $AB = 8$, $BC = 6$, and $BF = 8$. Find HB.



41. ***Writing in Math*** Explain how right triangles are used to build suspension bridges. Which parts of the right triangle are formed by the cables?

STANDARDIZED TEST PRACTICE

42. Miko is going to rope off an area of the park for an upcoming concert. He is going to place a plastic flag for every three feet of rope.



About how many flags is Miko going to place?

A 82 **B** 75 **C** 67 **D** 45

43. A rectangle has an area of 25 square inches. If the dimensions of the rectangle are doubled, what will be the area of the new rectangle?

F 12.5 in^2 **H** 100 in^2
G 50 in^2 **J** 625 in^2

44. **REVIEW** Which equation is equivalent to $5(3 - 2x) = 7 - 2(1 - 4x)$?

A $18x = 10$ **C** $2x = 10$
B $2x = -10$ **D** $10x = -10$

Spiral Review

Find the geometric mean between each pair of numbers. (Lesson 8-1)

45. 3 and 12 46. 9 and 12 47. 11 and 7 48. 6 and 9

49. **GARDENS** A park has a garden plot shaped like a triangle. It is bordered by a path. The triangle formed by the outside edge of the path is similar to the triangular garden. The perimeter of the outside edge of the path is 53 feet, the longest edge is 20 feet. The longest edge of the garden plot is 12 feet. What is the perimeter of the garden? (Lesson 7-5)

50. Could the sides of a triangle have the lengths 12, 13, and 25? Explain. (Lesson 5-4)

GET READY for the Next Lesson

PREREQUISITE SKILL **Simplify each expression by rationalizing the denominator.** (Pages 790–791)

51. $\frac{7}{\sqrt{3}}$ 52. $\frac{18}{\sqrt{2}}$ 53. $\frac{\sqrt{14}}{\sqrt{2}}$ 54. $\frac{3\sqrt{11}}{\sqrt{3}}$ 55. $\frac{24}{\sqrt{2}}$

EXPLORE 8-3

Geometry Lab

Patterns in Special Right Triangles

Triangles with angles 45°-45°-90° measuring or 30°-60°-90° are called *special right triangles*. There are patterns in the measures of the sides of these triangles.

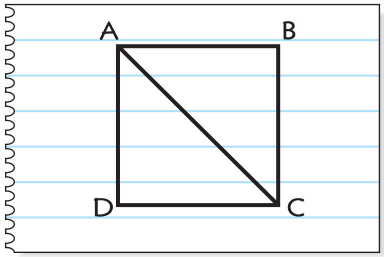
ACTIVITY 1 Identify patterns in 45°-45°-90° triangles.

Step 1 Draw a square with sides 4 centimeters long. Label the vertices *A*, *B*, *C*, and *D*.

Step 2 Draw the diagonal $\overline{AC}$.

Step 3 Use a protractor to measure $\angle CAB$ and $\angle ACB$.

Step 4 Use the Pythagorean Theorem to find *AC*. Write in simplest form.



Analyze the Results

1. Repeat the activity for squares with sides 6 centimeters long and 8 centimeters long.

2. **MAKE A CONJECTURE** What is the length of the hypotenuse of a 45°-45°-90° triangle with legs that are *n* units long?

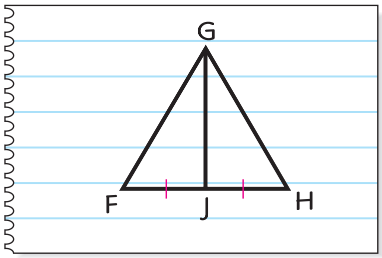
ACTIVITY 2 Identify patterns in 30°-60°-90° triangles.

Step 1 Construct an equilateral triangle with sides 2 inches long. Label the vertices *F*, *G*, and *H*.

Step 2 Find the midpoint of $\overline{FH}$ and label it *J*. Draw median $\overline{GJ}$.

Step 3 Use a protractor to measure $\angle FGJ$, $\angle F$, and $\angle GJF$.

Step 4 Use the Pythagorean Theorem to find *GJ*. Write in simplest form.



Analyze the Results

3. Repeat the activity to complete a table like the one at the right.

4. **MAKE A CONJECTURE** What are the lengths of the long leg and the hypotenuse of a 30°-60°-90° triangle with a short leg *n* units long?

FG	*FJ*	*GJ*
2 in.		
4 in.		
5 in.		

8-3 Special Right Triangles

Main Ideas

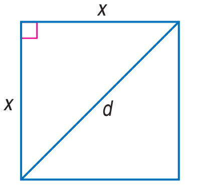
- Use properties of 45°-45°-90° triangles.
- Use properties of 30°-60°-90° triangles.

GET READY for the Lesson

Many quilt patterns use *half square triangles* to create a design. The pinwheel design was created with eight half square triangles rotated around the center. The measures of the angles in the half square triangles are 45°, 45°, and 90°.

Properties of 45°-45°-90° Triangles Facts about 45°-45°-90° triangles are used to solve many geometry problems. The Pythagorean Theorem allows us to discover special relationships that exist among the sides of a 45°-45°-90° triangle.

Draw a diagonal of a square. The two triangles formed are isosceles right triangles. Let x represent the measure of each side and let d represent the measure of the hypotenuse.

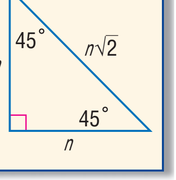


$d^2 = x^2 + x^2$	Pythagorean Theorem
$d^2 = 2x^2$	Add.
$d = \sqrt{2x^2}$	Take the positive square root of each side.
$d = \sqrt{2} \cdot \sqrt{x^2}$	Factor.
$d = x\sqrt{2}$	Simplify.

This algebraic proof verifies that the length of the hypotenuse of any 45°-45°-90° triangle is $\sqrt{2}$ times the length of its leg. The ratio of the sides is $1 : 1 : \sqrt{2}$.

THEOREM 8.6

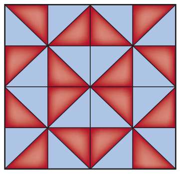
In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.



You can use this relationship to find the measure of the hypotenuse of a 45°-45°-90° triangle given the measure of a leg of the triangle.

EXAMPLE Find the Measure of the Hypotenuse

1 **WALLPAPER TILING** **Assume that the length of one of the legs of the 45°-45°-90° triangles in the wallpaper in the figure is 4 inches. What is the length of the diagonal of the entire wallpaper square?**



The length of each leg of the 45°-45°-90° triangle is 4 inches.

The length of the hypotenuse is $\sqrt{2}$ times as long as a leg. So, the length of the hypotenuse of one of the triangles is $4\sqrt{2}$.

There are four 45°-45°-90° triangles along the diagonal of the square. So, the length of the diagonal of the square is $4\left(4\sqrt{2}\right)$ or $16\sqrt{2}$ inches.

CHECK Your Progress

1. The length of the leg of a 45°-45°-90° triangle is 7 centimeters. What is the length of the hypotenuse?

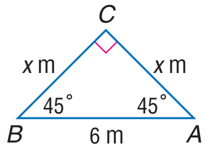
EXAMPLE Find the Measure of the Legs

Study Tip

Rationalizing Denominators

To rationalize a denominator, multiply the fraction by 1 in the form of a radical over itself so that the product in the denominator is a rational number.

2 **Find x.**



The length of the hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of a leg of the triangle.

$AB = (AC)\sqrt{2}$

$6 = x\sqrt{2}$ — $AB = 6, AC = x$

$\frac{6}{\sqrt{2}} = x$ — Divide each side by $\sqrt{2}$.

$\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = x$ — Rationalize the denominator.

$\frac{6\sqrt{2}}{2} = x$ — Multiply.

$3\sqrt{2} = x$ — Divide.

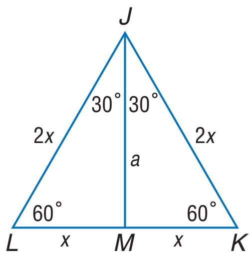
$4.24 \approx x$ — Use a calculator.

CHECK Your Progress

2. Refer to $\triangle ABC$. Suppose $BA = 5m$. Find x.

Properties of 30°-60°-90° Triangles There is also a special relationship among the measures of the sides of a 30°-60°-90° triangle.

When an altitude is drawn from any vertex of an equilateral triangle, two congruent 30°-60°-90° triangles are formed. $\overline{LM}$ and $\overline{KM}$ are congruent segments, so let $LM = x$ and $KM = x$. By the Segment Addition Postulate, $LM + KM = KL$. Thus, $KL = 2x$. Since $\triangle JKL$ is an equilateral triangle, $KL = JL = JK$. Therefore, $JL = 2x$ and $JK = 2x$.



Let a represent the measure of the altitude. Use the Pythagorean Theorem to find a.

$(JM)^2 + (LM)^2 = (JL)^2$	Pythagorean Theorem
$a^2 + x^2 = (2x)^2$	$JM = a$, $LM = x$, $JL = 2x$
$a^2 + x^2 = 4x^2$	Simplify.
$a^2 = 3x^2$	Subtract x^2 from each side.
$a = \sqrt{3x^2}$	Take the positive square root of each side.
$a = \sqrt{3} \cdot \sqrt{x^2}$	Factor.
$a = x\sqrt{3}$	Simplify.

So, in a 30°-60°-90° triangle, the measures of the sides are x, $x\sqrt{3}$, and $2x$. The ratio of the sides is $1:\sqrt{3}:2$.

The relationship of the side measures leads to Theorem 8.7.

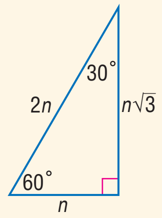
Study Tip

30°-60°-90° Triangle

The shorter leg is opposite the 30° angle, and the longer leg is opposite the 60° angle.

THEOREM 8.7

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

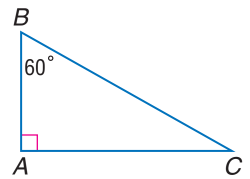


EXAMPLE 30°-60°-90° Triangles

3 Find the missing measures.

a. If $BC = 14$ inches, find AC.

$\overline{AC}$ is the longer leg, $\overline{AB}$ is the shorter leg, and $\overline{BC}$ is the hypotenuse.



$AB = \frac{1}{2}(BC)$	
$= \frac{1}{2}(14)$ or 7	$BC = 14$
$AC = \sqrt{3}(AB)$	
$= \sqrt{3}(7)$ or $7\sqrt{3}$	$AB = 7$
≈ 12.12	AC is $7\sqrt{3}$ or about 12.12 inches.

b. If $AC = 8$ inches, find BC.

$AC = \sqrt{3}(AB)$

$8 = \sqrt{3}(AB)$

$\frac{8}{\sqrt{3}} = AB$

$\frac{8\sqrt{3}}{3} = AB$

$BC = 2AB$

$= 2\left(\frac{8\sqrt{3}}{3}\right)$

$= \frac{16\sqrt{3}}{3}$

≈ 9.24

BC is $\frac{16\sqrt{3}}{3}$ or about 9.24 inches.

CHECK Your Progress

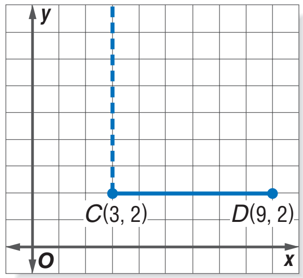
3. Refer to $\triangle ABC$. Suppose $AC = 12$ in. Find BC.

EXAMPLE Special Triangles in a Coordinate Plane

4 COORDINATE GEOMETRY Triangle PCD is a 30°-60°-90° triangle with right angle C. $\overline{CD}$ is the longer leg with endpoints $C(3, 2)$ and $D(9, 2)$. Locate point P in Quadrant I.

> **Study Tip**
>
> **Checking Reasonableness of Results**
>
> To check the coordinates of P in Example 4, use a protractor to draw $\overline{DP}$ such that $m\angle CDP = 30$. Then from the graph, you can estimate the coordinates of P.

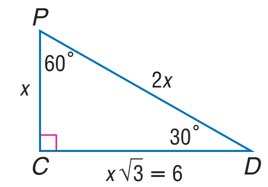
$\overline{CD}$ lies on a horizontal gridline. Since $\overline{PC}$ will be perpendicular to $\overline{CD}$, it lies on a vertical gridline. Find the length of $\overline{CD}$.



$CD = |9 - 3| = 6$

$\overline{CD}$ is the longer leg. $\overline{PC}$ is the shorter leg.

So, $CD = \sqrt{3}(PC)$. Use CD to find PC.



$CD = \sqrt{3}(PC)$

$6 = \sqrt{3}(PC)$ $CD = 6$

$\frac{6}{\sqrt{3}} = PC$ Divide each side by $\sqrt{3}$.

$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = PC$ Rationalize the denominator.

$\frac{6\sqrt{3}}{3} = PC$ Multiply.

$2\sqrt{3} = PC$ Simplify.

Point P has the same x-coordinate as C. P is located $2\sqrt{3}$ units above C. So, the coordinates of P are $(3, 2 + 2\sqrt{3})$ or about (3, 5.46).

CHECK Your Progress

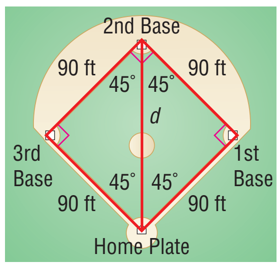
4. Triangle RST is a 30°-60°-90° triangle with right angle RST. $\overline{ST}$ is the shorter leg with endpoints $S(1, 1)$ and $T(4, 1)$. Locate point R in Quadrant I.

Online **Personal Tutor at** geometryonline.com

CHECK Your Understanding

Example 1 (p. 449)

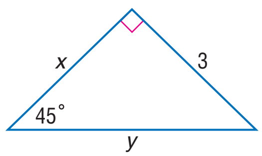
1. SOFTBALL Find the distance from home plate to second base if the bases are 90 feet apart.



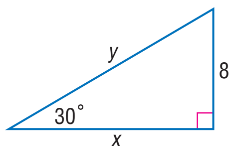
Example 2 (p. 449)

Find x and y.

2.



3.

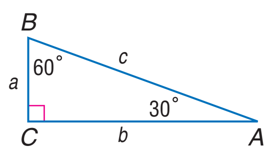


Example 3 (p. 450)

Find the missing measures.

4. If $c = 8$, find a and b.

5. If $b = 18$, find a and c.



Example 4 (p. 451)

Triangle *ABD* is a 30°-60°-90° triangle with right angle *B* and with $\overline{AB}$ as the shorter leg. Graph *A* and *B*, and locate point *D* in Quadrant I.

6. $A(8, 0)$, $B(8, 3)$ **7.** $A(6, 6)$, $B(2, 6)$

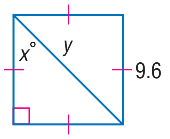
Exercises

HOMEWORK HELP

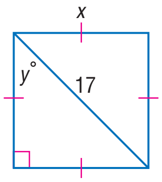
For Exercises	See Examples
8–10, 18, 20	2
11–17, 19, 21–23	3
24–27	4
28–32	1

Find *x* and *y*.

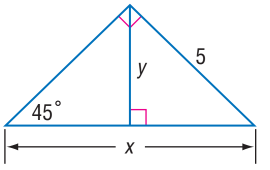
8.



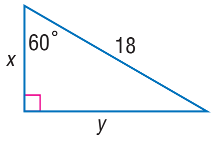
9.



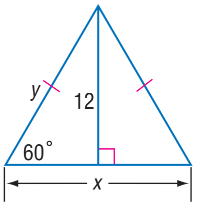
10.



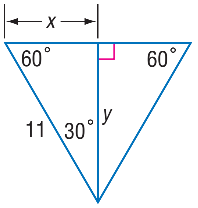
11.



12.



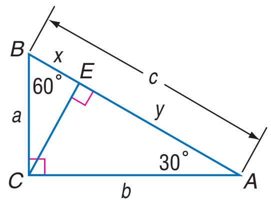
13.



For Exercises 14 and 15, use the figure at the right.

14. If $a = 10\sqrt{3}$, find *CE* and *y*.

15. If $x = 7\sqrt{3}$, find *a*, *CE*, *y*, and *b*.



16. The length of an altitude of an equilateral triangle is 12 feet. Find the length of a side of the triangle.

17. The perimeter of an equilateral triangle is 45 centimeters. Find the length of an altitude of the triangle.

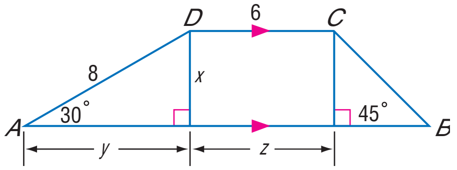
18. The length of a diagonal of a square is $22\sqrt{2}$ millimeters. Find the perimeter of the square.

19. The altitude of an equilateral triangle is 7.4 meters long. Find the perimeter of the triangle.

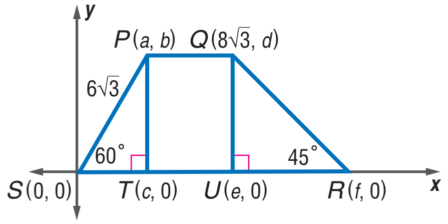
20. The diagonals of a rectangle are 12 inches long and intersect at an angle of 60°. Find the perimeter of the rectangle.

21. The sum of the squares of the measures of the sides of a square is 256. Find the measure of a diagonal of the square.

22. Find *x*, *y*, *z*, and the perimeter of trapezoid *ABCD*.



23. If $\overline{PQ} \parallel \overline{SR}$, find *a*, *b*, *c*, and *d*.



24. $\triangle PAB$ is a 45°-45°-90° triangle with right angle *B*. Find the coordinates of *P* in Quadrant I for $A(-3, 1)$ and $B(4, 1)$.

25. $\triangle PGH$ is a 45°-45°-90° triangle with $m\angle P = 90°$. Find the coordinates of *P* in Quadrant I for $G(4, -1)$ and $H(4, 5)$.

26. $\triangle PCD$ is a 30°-60°-90° triangle with right angle C and $\overline{CD}$ the longer leg. Find the coordinates of P in Quadrant III for $C(-3, -6)$ and $D(-3, 7)$.

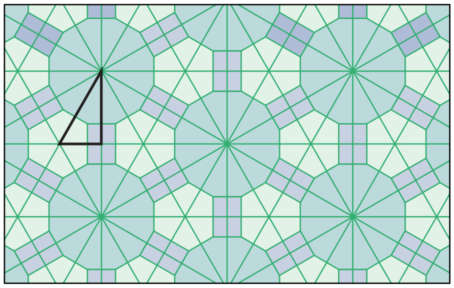
27. $\triangle PCD$ is a 30°-60°-90° triangle with $m\angle C = 30$ and hypotenuse $\overline{CD}$. Find the coordinates of P for $C(2, -5)$ and $D(10, -5)$ if P lies above $\overline{CD}$.

TRIANGLE TILING **For Exercises 28–31, use the following information.**
Triangle tiling refers to the process of taking many copies of a single triangle and laying them next to each other to fill an area. For example, the pattern shown is composed of tiles like the one outlined.

Real-World Link

Triangle Tiling Buildings in Federation Square in Melbourne, Australia, feature a tiling pattern called a pinwheel tiling. The sides of each right triangle are in the ratio $1 : 2 : \sqrt{5}$.

Source: www.federationsquare.com.au



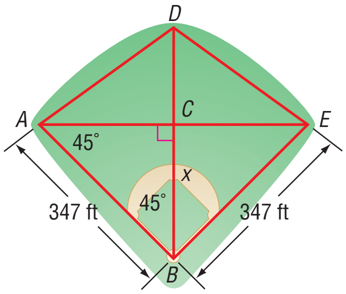
28. How many 30°-60°-90° triangles are used to create the basic pattern, which resembles a circle?

29. Which angle of the 30°-60°-90° triangle is being rotated to make the basic shape?

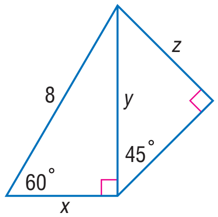
30. Explain why there are no gaps in the basic pattern.

31. Use grid paper to cut out 30°-60°-90° triangles. Color the same pattern on each triangle. Create one basic figure that would be part of a wallpaper tiling.

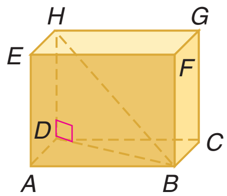
32. **BASEBALL** The diagram at the right shows some dimensions of U.S. Cellular Field in Chicago, Illinois. $\overline{BD}$ is a segment from home plate to dead center field, and $\overline{AE}$ is a segment from the left field foul-ball pole to the right field foul-ball pole. If the center fielder is standing at C, how far is he from home plate?



33. Find x, y, and z.

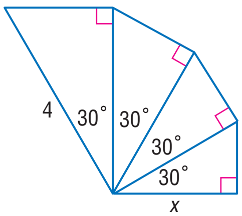


34. If $BD = 8\sqrt{3}$ and $m\angle DHB = 60°$, find BH.

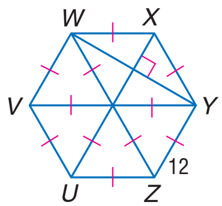


EXTRA PRACTICE
See pages 815, 835.
Math Online
Self-Check Quiz at
geometryonline.com

35. Each triangle in the figure is a 30°-60°-90° triangle. Find x.



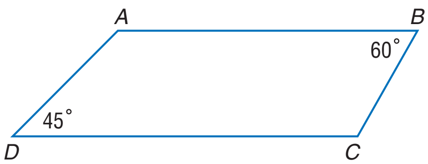
36. In regular hexagon $UVWXYZ$, each side is 12 centimeters long. Find WY.



H.O.T. Problems

37. **OPEN ENDED** Draw a rectangle that has a diagonal twice as long as its width. Then write an equation to find the length of the rectangle.

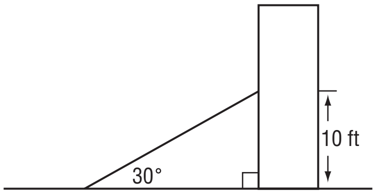
38. **CHALLENGE** Given figure $ABCD$, with $\overline{AB} \parallel \overline{DC}$, $m\angle B = 60°$, $m\angle D = 45°$, $BC = 8$, and $AB = 24$, find the perimeter.



39. ***Writing in Math*** Refer to the information about quilting on page 448. Describe why quilters use the term *half square triangles* to describe 45°-45°-90° triangles. Explain why 45°-45°-90° triangles are used in this pattern instead of 30°-60°-90° triangles.

STANDARDIZED TEST PRACTICE

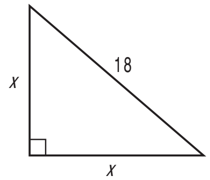
40. A ladder is propped against a building at a 30° angle.



What is the length of the ladder?

A 5 ft
B 10 ft
C $10\sqrt{3}$ ft
D 20 ft

41. Look at the right triangle below. Which of the following could be the triangle's dimensions?



F 9
G $9\sqrt{2}$
H $18\sqrt{2}$
J 36

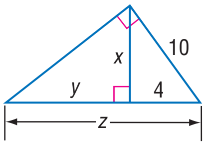
Spiral Review

Determine whether each set of measures contains the sides of a right triangle. Then state whether they form a Pythagorean triple. (Lesson 7-2)

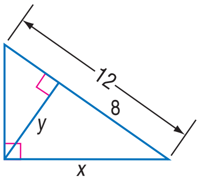
42. 3, 4, 5
43. 9, 40, 41
44. 20, 21, 31
45. 20, 48, 52
46. 7, 24, 25
47. 12, 34, 37

Find x, y, and z. (Lesson 7-1)

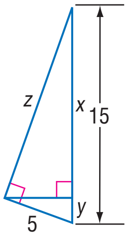
48.



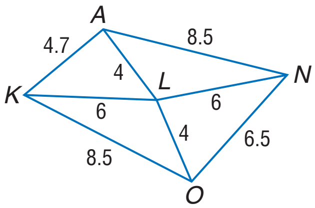
49.



50.



Write an inequality relating each pair of angles. (Lesson 5-5)



51. $m\angle ALK$, $m\angle ALN$
52. $m\angle ALK$, $m\angle NLO$
53. $m\angle OLK$, $m\angle NLO$
54. $m\angle KLO$, $m\angle ALN$

55. SCALE MODELS Taipa wants to build a scale model of the Canadian Horseshoe Falls at Niagara Falls. The height is 52 meters. If she wants the model to be 80 centimeters tall, what scale factor will she use? (Lesson 7-1)

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** (Pages 781–782)

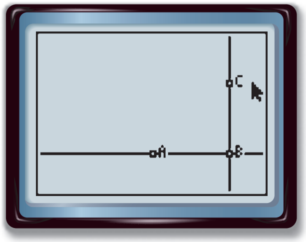
56. $5 = \frac{x}{3}$
57. $\frac{x}{9} = 0.14$
58. $0.5 = \frac{10}{k}$
59. $0.2 = \frac{13}{g}$
60. $\frac{7}{n} = 0.25$
61. $9 = \frac{m}{0.8}$
62. $\frac{24}{x} = 0.4$
63. $\frac{35}{y} = 0.07$

EXPLORE 8-4

Graphing Calculator Lab
Trigonometry

You have investigated the patterns in the measures of special right triangles. The study of the patterns in all right triangles is called *trigonometry*. You can use the Cabri Junior application on a TI-83/84 Plus to investigate these patterns.

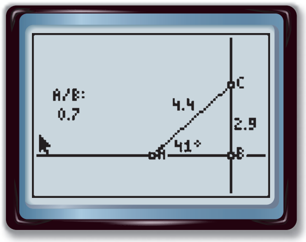
ACTIVITY



Step 1 Use the line tool on the **F2** menu to draw a line. Label the points on the line *A* and *B*.

Step 2 Press **F3** and choose the **Perpendicular** tool to create a perpendicular line through point *B*. Draw and label a point *C* on the perpendicular.

Step 3 Use the segment tool on the **F2** menu to draw $\overline{AC}$.



Step 4 Find and label the measures of $\overline{BC}$ and $\overline{AC}$ using the **Distance** and **Length** tool under **Measure** on the **F5** menu. Use the **Angle** tool for the measure of $\angle A$.

Step 6 Calculate and display the ratio $\frac{BC}{AC}$ using the **Calculate** tool on the **F5** menu. Label the ratio as *A/B*.

Step 7 Press CLEAR. Then use the arrow keys to move the cursor close to point *B*. When the arrow is clear, press and hold the ALPHA key. Drag *B* and observe the ratio.

ANALYZE THE RESULTS

1. Discuss the effect of dragging point *B* on *BC*, *AC*, $m\angle A$, and the ratio $\frac{BC}{AC}$.

2. Use the calculate tool to find the ratios $\frac{AB}{AC}$ and $\frac{BC}{AB}$. Then drag *B* and observe the ratios.

3. **MAKE A CONJECTURE** The *sine*, *cosine*, and *tangent* functions are trigonometric functions based on angle measures. Make a note of $m\angle A$. Exit Cabri Jr. and use SIN, COS and TAN on the calculator to find *sine*, *cosine* and *tangent* for $m\angle A$. Compare the results to the ratios you found in the activity. Make a conjecture about the definitions of sine, cosine, and tangent.

8-4 Trigonometry

Main Ideas

- Find trigonometric ratios using right triangles.
- Solve problems using trigonometric ratios.

New Vocabulary

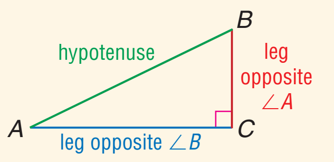
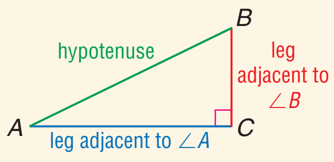
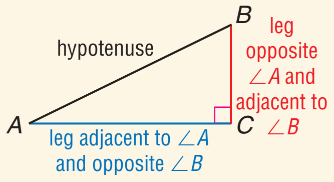
trigonometry
trigonometric ratio
sine
cosine
tangent

GET READY for the Lesson

The branch of mathematics known as *trigonometry* was developed for use by astronomers and surveyors. Surveyors use an instrument called a theodolite (thee AH duh lite) to measure angles. It consists of a telescope mounted on a vertical axis and a horizontal axis. After measuring the angles, surveyors apply trigonometry to calculate distance or height.

Trigonometric Ratios The word **trigonometry** comes from two Greek terms, *trigon*, meaning triangle, and *metron*, meaning measure. The study of trigonometry involves triangle measurement. A ratio of the lengths of sides of a right triangle is called a **trigonometric ratio**. The three most common trigonometric ratios are **sine**, **cosine**, and **tangent**.

KEY CONCEPT — *Trigonometric Ratios*

Words	Symbols	Models
sine of $\angle A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$ sine of $\angle B = \frac{\text{leg opposite } \angle B}{\text{hypotenuse}}$	$\sin A = \frac{BC}{AB}$ $\sin B = \frac{AC}{AB}$	
cosine of $\angle A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$ cosine of $\angle B = \frac{\text{leg adjacent to } \angle B}{\text{hypotenuse}}$	$\cos A = \frac{AC}{AB}$ $\cos B = \frac{BC}{AB}$	
tangent of $\angle A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$ tangent of $\angle B = \frac{\text{leg opposite } \angle B}{\text{leg adjacent to } \angle B}$	$\tan A = \frac{BC}{AC}$ $\tan B = \frac{AC}{BC}$	

Trigonometric ratios are related to the acute angles of a right triangle, not the right angle.

Arthur Thevenart/CORBI

Reading Math

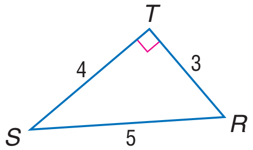
Memory Hint SOH-CAH-TOA is a mnemonic device for learning the ratios for sine, cosine, and tangent using the first letter of each word in the ratios.

$\sin A = \frac{\text{opp}}{\text{hyp}}$

$\cos A = \frac{\text{adj}}{\text{hyp}}$

$\tan A = \frac{\text{opp}}{\text{adj}}$

EXAMPLE Find Sine, Cosine, and Tangent Ratios



1 **Find sin *R*, cos *R*, tan *R*, sin *S*, cos *S*, and tan *S*. Express each ratio as a fraction and as a decimal.**

$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{ST}{RS} = \frac{4}{5} \text{ or } 0.8$

$\cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{RT}{RS} = \frac{3}{5} \text{ or } 0.6$

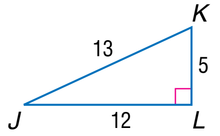
$\tan R = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{ST}{RT} = \frac{4}{3} \text{ or } 1.\overline{3}$

$\sin S = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{RT}{RS} = \frac{3}{5} \text{ or } 0.6$

$\cos S = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{ST}{RS} = \frac{4}{5} \text{ or } 0.8$

$\tan S = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{RT}{ST} = \frac{3}{4} \text{ or } 0.75$

CHECK Your Progress

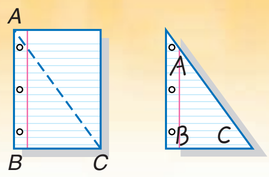


1. Find sin *J*, cos *J*, tan *J*, sin *K*, cos *K*, and tan *K*. Express each ratio as a fraction and as a decimal.

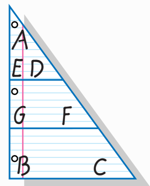
You can use paper folding to investigate trigonometric ratios in similar right triangles.

GEOMETRY LAB

Trigonometric Ratios



- Fold a rectangular piece of paper along a diagonal from *A* to *C*. Then cut along the fold to form right triangle *ABC*. Write the name of each angle on the inside of the triangle.



- Fold the triangle so that there are two segments perpendicular to $\overline{BA}$. Label points *D*, *E*, *F*, and *G* as shown. Use a ruler to measure $\overline{AC}$, $\overline{AB}$, $\overline{BC}$, $\overline{AF}$, $\overline{AG}$, $\overline{FG}$, $\overline{AD}$, $\overline{AE}$, and $\overline{DE}$ to the nearest millimeter.

ANALYZE THE RESULTS

1. What is true of $\triangle AED$, $\triangle AGF$, and $\triangle ABC$?

2. Copy the table. Write the ratio of the side lengths for each ratio. Then calculate a value for each ratio to the nearest ten-thousandth.

	In $\triangle AED$	In $\triangle AGF$	In $\triangle ABC$
sin *A*			
cos *A*			
tan *A*			

3. Study the table. Write a sentence about the patterns you observe.

4. What is true about $m\angle A$ in each triangle?

As the Geometry Lab suggests, the value of a trigonometric ratio depends *only* on the measure of the angle. It does not depend on the size of the triangle.

Study Tip

Graphing Calculator

Be sure your calculator is in degree mode rather than radian mode. Your calculator may require you to input the angle *before* using the trigonometric key.

EXAMPLE Use a Calculator to Evaluate Expressions

2 **Use a calculator to find cos 39° to the nearest ten-thousandth.**

KEYSTROKES: [COS] 39 [ENTER]

$\cos 39° \approx 0.7771$

CHECK Your Progress

2. sin 67°

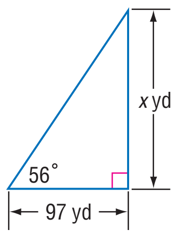
Use Trigonometric Ratios

You can use trigonometric ratios to find the missing measures of a right triangle if you know the measures of two sides of a triangle or the measure of one side and one acute angle.

EXAMPLE Use Trigonometric Ratios to Find a Length

3 **SURVEYING** **Dakota is standing on the ground 97 yards from the base of a cliff. Using a theodolite, he noted that the angle formed by the ground and the line of sight to the top of the cliff is 56°. Find the height of the cliff to the nearest yard.**

Let x be the height of the cliff in yards.



$\tan 56° = \frac{x}{97}$ $\tan = \frac{\text{leg opposite}}{\text{leg adjacent}}$

$97 \tan 56° = x$ Multiply each side by 97.

Use a calculator to find x.

KEYSTROKES: 97 [TAN] 56 [ENTER] 143.8084139

The cliff is about 144 yards high.

CHECK Your Progress

3. **MEASUREMENT** Jonathan is standing 15 yards from a roller coaster. The angle formed by the ground to the top of the roller coaster is 71°. How tall is the roller coaster?

When solving equations like $3x = -27$, you use the inverse of multiplication to find x. In trigonometry, you can find the measure of the angle by using the inverse of sine, cosine, or tangent.

Given equation	To find the angle	Read as
$\sin A = x$	$A = \sin^{-1}(x)$	A equals *the inverse sine of* x.
$\cos A = y$	$A = \cos^{-1}(y)$	A equals *the inverse cosine of* y.
$\tan A = z$	$A = \tan^{-1}(z)$	A equals *the inverse tangent of* z.

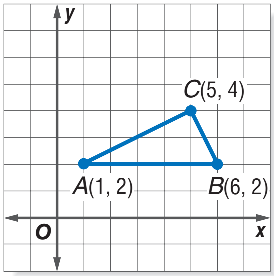
Study Tip

Calculators

The second functions of the [SIN], [COS], and [TAN] keys are usually the inverses.

EXAMPLE 4 Use Trigonometric Ratios to Find an Angle Measure

COORDINATE GEOMETRY **Find $m\angle A$ in right triangle ABC for $A(1, 2)$, $B(6, 2)$, and $C(5, 4)$.**



Explore You know the coordinates of the vertices of a right triangle and that $\angle C$ is the right angle. You need to find the measure of one of the angles.

Plan Use the Distance Formula to find the measure of each side. Then use one of the trigonometric ratios to write an equation. Use the inverse to find $m\angle A$.

Solve

$$AB = \sqrt{(6-1)^2 + (2-2)^2}$$
$$= \sqrt{25 + 0} \text{ or } 5$$

$$BC = \sqrt{(5-6)^2 + (4-2)^2}$$
$$= \sqrt{1+4} \text{ or } \sqrt{5}$$

$$AC = \sqrt{(5-1)^2 + (4-2)^2}$$
$$= \sqrt{16 + 4}$$
$$= \sqrt{20} \text{ or } 2\sqrt{5}$$

Use the cosine ratio.

$$\cos A = \frac{AC}{AB} \qquad \cos = \frac{\text{leg adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{2\sqrt{5}}{5} \qquad AC = 2\sqrt{5} \text{ and } AB = 5$$

$$A = \cos^{-1}\left(\frac{2\sqrt{5}}{5}\right) \qquad \text{Solve for } A.$$

Use a calculator to find $m\angle A$.

KEYSTROKES: [2nd] [COS^{-1}] 2 [2nd] [$\sqrt{\ }$] 5 [)] [÷] 5 [ENTER]

$m\angle A \approx 26.56505118$

The measure of $\angle A$ is about 26.6.

Check Use the sine ratio to check the answer.

$$\sin A = \frac{BC}{AB} \qquad \sin = \frac{\text{leg opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{\sqrt{5}}{5} \qquad BC = \sqrt{5} \text{ and } AB = 5$$

KEYSTROKES: [2nd] [SIN^{-1}] [2nd] [$\sqrt{\ }$] 5 [)] [÷] 5 [ENTER]

$m\angle A \approx 26.56505118$

The answer is correct.

Cross-Curricular Project

Math Online: You can use trigonometry to help you come closer to locating the hidden treasure. Visit geometryonline.com.

CHECK Your Progress

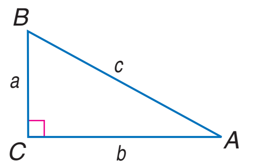
4. Find $m\angle P$ in right $\triangle PQR$ for $P(2, -1)$, $Q(4, 3)$, and $R(8, 1)$.

Online Personal Tutor at geometryonline.com

CHECK Your Understanding

Example 1 (p. 457)

Use $\triangle ABC$ to find sin A, cos A, tan A, sin B, cos B, and tan B. Express each ratio as a fraction and as a decimal to the nearest hundredth.



1. $a = 14$, $b = 48$, and $c = 50$

2. $a = 8$, $b = 15$, and $c = 17$

Example 2 (p. 458)

Use a calculator to find each value. Round to the nearest ten-thousandth.

3. sin 57° **4.** cos 60° **5.** cos 33°

6. tan 30° **7.** tan 45° **8.** sin 85°

Example 3 (p. 458)

9. SURVEYING Maureen is standing on horizontal ground level with the base of the CN Tower in Toronto, Ontario. The angle formed by the ground and the line segment from her position to the top of the tower is 31.2°. She knows that the height of the tower to the top of the antennae is about 1815 feet. Find her distance from the CN Tower to the nearest foot.

Find the measure of each angle to the nearest tenth of a degree.

10. tan $A = 1.4176$ **11.** sin $B = 0.6307$

Example 4 (p. 459)

COORDINATE GEOMETRY Find the measure of the angle to the nearest tenth in each right triangle ABC.

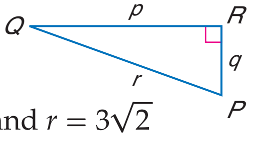
12. $\angle A$ in $\triangle ABC$, for $A(6, 0)$, $B(-4, 2)$, and $C(0, 6)$

13. $\angle B$ in $\triangle ABC$, for $A(3, -3)$, $B(7, 5)$, and $C(7, -3)$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
14–17	1
18–23	2
24, 25	3
26–28	4

Use $\triangle PQR$ with right angle R to find sin P, cos P, tan P, sin Q, cos Q, and tan Q. Express each ratio as a fraction, and as a decimal to the nearest hundredth.



14. $p = 12$, $q = 35$, and $r = 37$ **15.** $p = \sqrt{6}$, $q = 2\sqrt{3}$, and $r = 3\sqrt{2}$

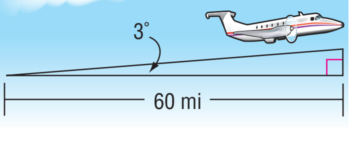
16. $p = \frac{3}{2}$, $q = \frac{3\sqrt{3}}{2}$, and $r = 3$ **17.** $p = 2\sqrt{3}$, $q = \sqrt{15}$, and $r = 3\sqrt{3}$

Use a calculator to find each value. Round to the nearest ten-thousandth.

18. sin 6° **19.** tan 42.8° **20.** cos 77°

21. sin 85.9° **22.** tan 12.7° **23.** cos 22.5°

24. AVIATION A plane is one mile above sea level when it begins to climb at a constant angle of 3° for the next 60 ground miles. About how far above sea level is the plane after its climb?



25. MONUMENTS At 351 feet tall, the Jefferson Davis Monument in Fairview, Kentucky, is the largest concrete obelisk in the world. Pedro is looking at the top of the monument at an angle of 75°. How far away from the monument is he standing?

Real-World Link

The Jefferson Davis Monument in Fairview, Kentucky, is the fourth tallest monument in the United States. The walls are seven feet thick at the base, tapering to two feet thick at the top.

Source: parks.ky.gov

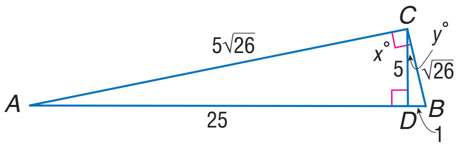
COORDINATE GEOMETRY **Find the measure of each angle to the nearest tenth in each right triangle.**

26. $\angle J$ in $\triangle JCL$ for $J(2, 2)$, $C(2, -2)$, and $L(7, -2)$

27. $\angle C$ in $\triangle BCD$ for $B(-1, -5)$, $C(-6, -5)$, and $D(-1, 2)$

28. $\angle X$ in $\triangle XYZ$ for $X(-5, 0)$, $Y(7, 0)$, and $Z(0, \sqrt{35})$

Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.



29. $\sin A$ **30.** $\tan B$ **31.** $\cos A$

32. $\sin x°$ **33.** $\cos x°$ **34.** $\tan A$

35. $\cos B$ **36.** $\sin y°$ **37.** $\tan x°$

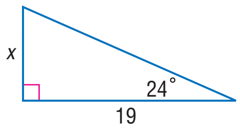
Find the measure of each angle to the nearest tenth of a degree.

38. $\sin B = 0.7245$ **39.** $\cos C = 0.2493$ **40.** $\tan E = 9.4618$

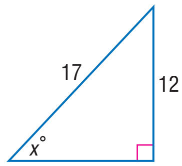
41. $\sin A = 0.4567$ **42.** $\cos D = 0.1212$ **43.** $\tan F = 0.4279$

Find x. Round to the nearest tenth.

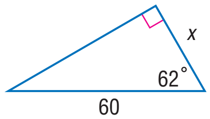
44.



45.



46.



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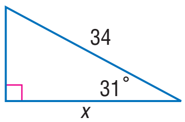

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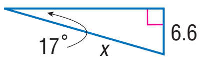

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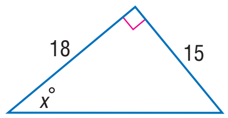
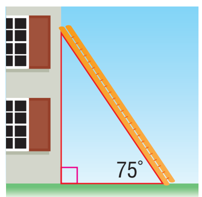


SAFETY **For Exercises 50 and 51, use the following information.**

To guard against a fall, a ladder should make an angle of 75° or less with the ground.

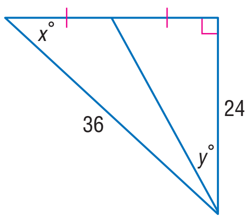


50. What is the maximum height that a 20-foot ladder can reach safely?

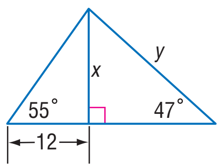
51. How far from the building is the base of the ladder at the maximum height?

Find x and y. Round to the nearest tenth.

52.



53.



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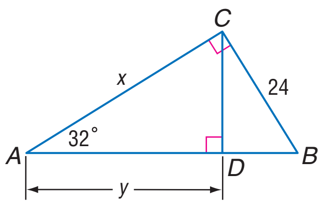


EXTRA PRACTICE

See pages 816, 835.

Math Online

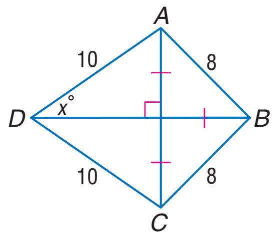
Self-Check Quiz at geometryonline.com

Gibson Stock Photography

H.O.T. Problems

55. OPEN ENDED Draw a right triangle and label the measure of one acute angle and the measure of the side opposite that angle. Then solve for the remaining measures.

56. CHALLENGE Use the figure at the right to find $\sin x°$.

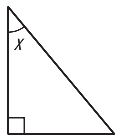


57. REASONING Explain the difference between $\tan A = \frac{x}{y}$ and $\tan^{-1}\left(\frac{x}{y}\right) = A$.

58. *Writing in Math* Refer to the information on theodolites on page 456. Explain how surveyors determine angle measures. Include the kind of information one obtains from a theodolite.

STANDARDIZED TEST PRACTICE

59. In the figure, if $\cos x = \frac{20}{29}$, what are $\sin x$ and $\tan x$?



A $\sin x = \frac{29}{21}$ and $\tan x = \frac{29}{21}$

B $\sin x = \frac{21}{29}$ and $\tan x = \frac{20}{21}$

C $\sin x = \frac{29}{21}$ and $\tan x = \frac{21}{20}$

D $\sin x = \frac{21}{29}$ and $\tan x = \frac{21}{20}$

60. REVIEW What is the solution set of the quadratic equation $x^2 + 4x - 2 = 0$?

F $\{-2, 2\}$

G $\{-2 + \sqrt{6}, -2 - \sqrt{6}\}$

H $\{-2 + \sqrt{2}, -2 - \sqrt{2}\}$

J no real solution

61. REVIEW Which of the following has the same value as $9^{-15} \times 9^3$?

A 9^{-45} C 9^{-12}

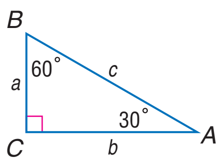
B 9^{-18} D 9^{-5}

Spiral Review

Find each measure. (Lesson 8-3)

62. If $a = 4$, find b and c.

63. If $b = 3$, find a and c.



Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple. (Lesson 8-2)

64. 4, 5, 6 **65.** 5, 12, 13 **66.** 9, 12, 15 **67.** 8, 12, 16

68. TELEVISION During a 30-minute television program, the ratio of minutes of commercials to minutes of the actual show is 4 : 11. How many minutes are spent on commercials? (Lesson 7-1)

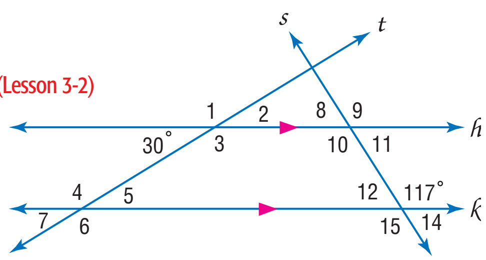
GET READY for the Next Lesson

PREREQUISITE SKILL Find each angle measure if $h \parallel k$. (Lesson 3-2)

69. $m\angle 15$ **70.** $m\angle 7$

71. $m\angle 3$ **72.** $m\angle 12$

73. $m\angle 11$ **74.** $m\angle 4$

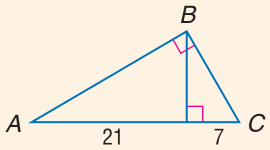


Mid-Chapter Quiz

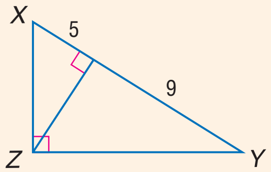
Lessons 8-1 through 8-4

Find the measure of the altitude drawn to the hypotenuse. (Lesson 8-1)

1.

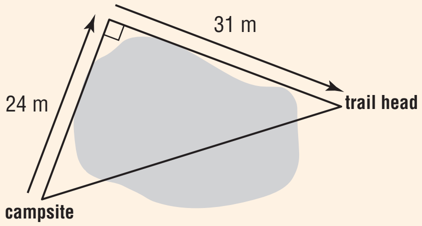


2.



3. Determine whether $\triangle ABC$ with vertices $A(2, 1)$, $B(4, 0)$, and $C(5, 7)$ is a right triangle. Explain. (Lesson 8-2)

4. MULTIPLE CHOICE To get from your campsite to a trail head, you must take the path shown below to avoid walking through a pond.

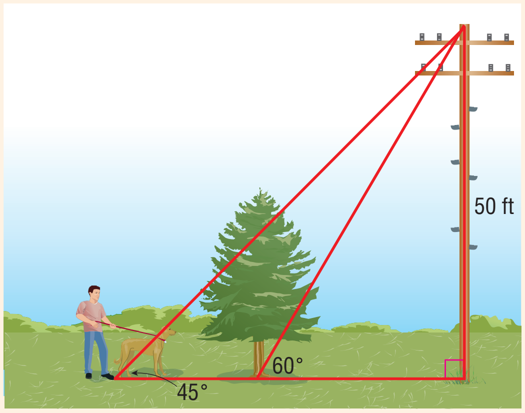


About how many meters would be saved if it were possible to walk through the pond? (Lesson 8-2)

A 55.0 **C** 24.7

B 39.2 **D** 15.8

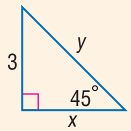
5. DOG WALKING A man is walking his dog on level ground in a straight line with the dog's favorite tree. The angle from the man's present position to the top of a nearby telephone pole is 45°. The angle from the tree to the top of the telephone pole is 60°. If the telephone pole is 50 feet tall, about how far is the man with the dog from the tree? (Lesson 8-3)



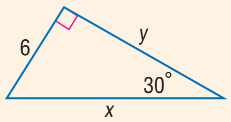
6. WOODWORKING Ginger made a small square table for her workshop with a diagonal that measures 55 inches. What are the measures of the sides? Recall that a square has right angles at the corners and congruent sides. (Lesson 8-3)

Find x and y. (Lesson 8-3)

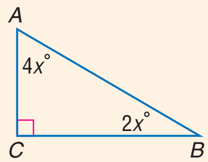
7.



8.



9. MULTIPLE CHOICE In the right triangle, what is AB if $BC = 6$? (Lesson 8-3)

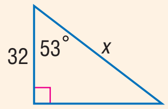


F 12 units **H** $4\sqrt{3}$ units

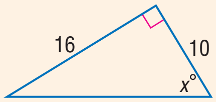
G $6\sqrt{2}$ units **J** $2\sqrt{3}$ units

Find x to the nearest tenth. (Lesson 8-4)

10.



11.



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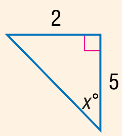

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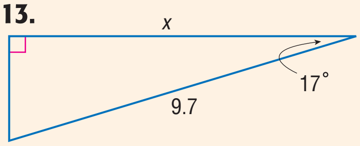


14. GARDENING The lengths of the sides of a triangular garden are 32 feet, 24 feet, and 40 feet. What are the measures of the angles formed on each side of the garden? (Lesson 8-4)

Find the measure of each angle to the nearest tenth of a degree. (Lesson 8-4)

15. $\sin T = 0.5299$

16. $\cos W = 0.0175$

8-5 Angles of Elevation and Depression

Main Ideas

- Solve problems involving angles of elevation.
- Solve problems involving angles of depression.

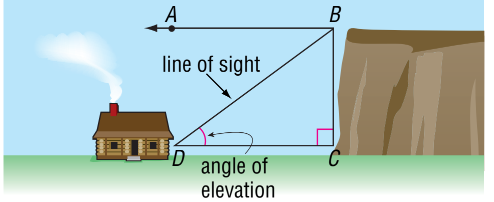
New Vocabulary

angle of elevation
angle of depression

GET READY for the Lesson

A pilot is getting ready to take off from Mountain Valley airport. She looks up at the peak of a mountain immediately in front of her. The pilot must estimate the speed needed and the angle formed by a line along the runway and a line from the plane to the peak of the mountain to clear the mountain.

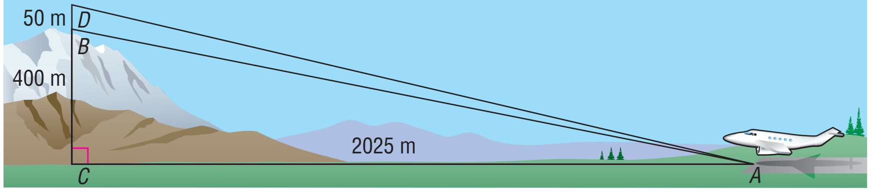
Angles of Elevation An **angle of elevation** is the angle between the line of sight and the horizontal when an observer looks upward.



EXAMPLE 1 Angle of Elevation

AVIATION The peak of Goose Bay Mountain is 400 meters higher than the end of a local airstrip. The peak rises above a point 2025 meters from the end of the airstrip. A plane takes off from the end of the runway in the direction of the mountain at an angle that is kept constant until the peak has been cleared. If the pilot wants to clear the mountain by 50 meters, what should the angle of elevation be for the takeoff to the nearest tenth of a degree?

Make a drawing.



Since *CB* is 400 meters and *BD* is 50 meters, *CD* is 450 meters. Let x represent $m\angle DAC$.

$\tan x^\circ = \frac{CD}{AC}$	$\tan = \frac{\text{opposite}}{\text{adjacent}}$
$\tan x^\circ = \frac{450}{2025}$	$CD = 450$, $AC = 2025$
$x = \tan^{-1}\left(\frac{450}{2025}\right)$	Solve for x.
$x \approx 12.5$	Use a calculator.

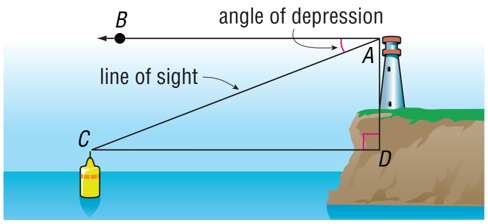
The angle of elevation for the takeoff should be more than 12.5°.

CHECK Your Progress

1. **SHADOWS** Find the angle of elevation of the Sun when a 7.6-meter flagpole casts a 18.2-meter shadow. Round to the nearest tenth of a degree.

Math Online Extra Examples at geometryonline.com

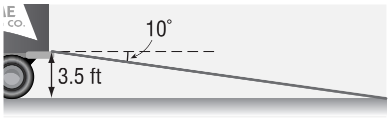
Angles of Depression An **angle of depression** is the angle between the line of sight when an observer looks downward and the horizontal.



STANDARDIZED TEST EXAMPLE

Angle of Depression

2 The tailgate of a moving van is 3.5 feet above the ground. A loading ramp is attached to the rear of the van at an incline of 10°. Which is closest to the length of the ramp?



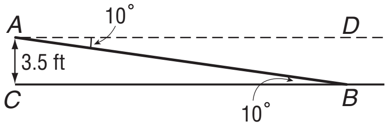
$\sin 10° \approx 0.17$
$\cos 10° \approx 0.98$
$\tan 10° \approx 0.18$

A 3.6 ft
B 12.2 ft
C 19.8 ft
D 20.2 ft

Read the Test Item

The angle of depression between the ramp and the horizontal is 10°. Use trigonometry to find the length of the ramp.

Solve the Test Item



The ground and the horizontal level with the back of the van are parallel. Therefore, $m\angle DAB = m\angle ABC$ since they are alternate interior angles.

$\sin 10° = \frac{3.5}{AB}$ — $\sin = \frac{\text{opposite}}{\text{hypotenuse}}$

$AB \sin 10° = 3.5$ — Multiply each side by AB.

$AB = \frac{3.5}{\sin 10°}$ — Divide each side by sin 10°.

$AB \approx \frac{3.5}{0.17}$ — $\sin 10° \approx 0.17$

$AB \approx 20.2$ — Divide.

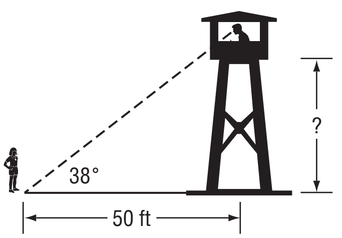
The ramp is about 20.2 feet long. So the correct answer is choice D.

Test-Taking Tip

Check Results Before moving on to the next question, check the reasonableness of your answer. Analyze your result to determine that it makes sense.

CHECK Your Progress

2. HIKING Ayana is hiking in a national park. A forest ranger is standing in a fire tower that overlooks a meadow. She sees Ayana at an angle of depression measuring 38°. If Ayana is 50 feet away from the base of the tower, which is closest to the height of the fire tower?



$\sin 38° \approx 0.62$
$\cos 38° \approx 0.79$
$\tan 38° \approx 0.78$

F 30.8 ft
G 39.1 ft
H 39.4 ft
J 63.5 ft

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Angles of elevation or depression to two different objects can be used to find the distance between those objects.

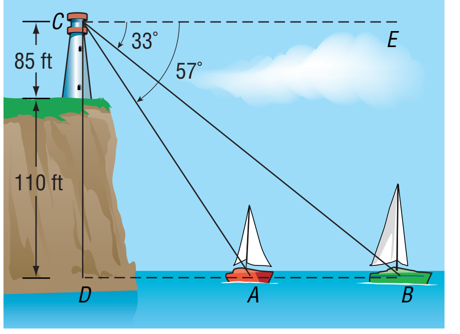
Study Tip

Common Misconception

The angle of depression is often not an angle of the triangle but the complement to an angle of the triangle. In $\triangle DBC$, the angle of depression is $\angle BCE$, not $\angle DCB$.

EXAMPLE Indirect Measurement

3 **Olivia works in a lighthouse on a cliff. She observes two sailboats due east of the lighthouse. The angles of depression to the two boats are 33° and 57°. Find the distance between the two sailboats to the nearest foot.**



$\triangle CDA$ and $\triangle CDB$ are right triangles, and $CD = 110 + 85$ or 195. The distance between the boats is AB or $DB - DA$. Use the right triangles to find these two lengths.

Because $\overline{CE}$ and $\overline{DB}$ are horizontal lines, they are parallel. Thus, $\angle ECB \cong \angle CBD$ and $\angle ECA \cong \angle CAD$ because they are alternate interior angles. This means that $m\angle CBD = 33$ and $m\angle CAD = 57$.

Use the measures of $\triangle CBD$ to find DB.

$\tan 33° = \frac{195}{DB}$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$; $m\angle CBD = 33$

$DB \tan 33° = 195$ Multiply each side by DB.

$DB = \frac{195}{\tan 33°}$ Divide each side by tan 33°.

$DB \approx 300.27$ Use a calculator.

Use the measures of $\triangle CAD$ to find DA.

$\tan 57° = \frac{195}{DA}$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$; $m\angle CAD = 57$

$DA \tan 57° = 195$ Multiply each side by DA.

$DA = \frac{195}{\tan 57°}$ Divide each side by tan 57°.

$DA \approx 126.63$ Use a calculator.

The distance between the boats is $DB - DA$.

$DB - DA \approx 300.27 - 126.63$ or about 174 feet

CHECK Your Progress

3. BOATING Two boats are observed by a parasailer 75 meters above a lake. The angles of depression are 12.5° and 7°. How far apart are the boats?

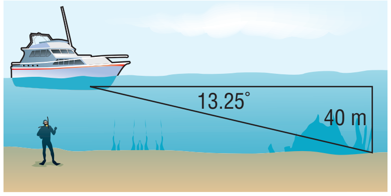
CHECK Your Understanding

Example 1 (p. 464)

1. AVIATION A pilot is flying at 10,000 feet and wants to take the plane up to 20,000 feet over the next 50 miles. What should be his angle of elevation to the nearest tenth? (*Hint:* There are 5280 feet in a mile.)

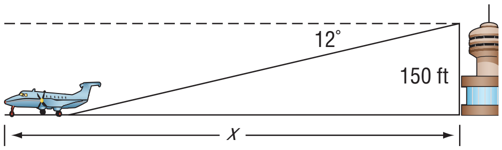
Example 2
(p. 465)

2. **OCEAN ARCHAEOLOGY** A salvage ship uses sonar to determine the angle of depression to a wreck on the ocean floor that is 40 meters below the surface. How far must a diver, lowered from the salvage ship, walk along the ocean floor to reach the wreck?



Example 3
(p. 466)

3. **STANDARDIZED TEST EXAMPLE** From the top of a 150-foot high tower, an air traffic controller observes an airplane on the runway. Which equation would be used to find the distance from the base of the tower to the airplane?

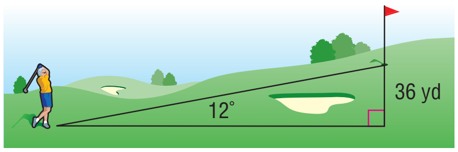


A $x = 150 \tan 12°$ **B** $x = \frac{150}{\cos 12°}$ **C** $x = \frac{150}{\tan 12°}$ **D** $x = \frac{150}{\sin 12°}$

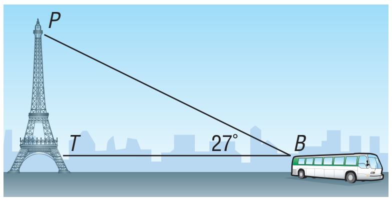
Exercises

HOMEWORK HELP	
For Exercises	See Examples
4–11	1
12, 13	2
14–17	3

4. **GOLF** A golfer is standing at the tee, looking up to the green on a hill. If the tee is 36 yards lower than the green and the angle of elevation from the tee to the hole is 12°, find the distance from the tee to the hole.



5. **TOURISM** Crystal is on a bus in France with her family. She sees the Eiffel Tower at an angle of 27°. If the tower is 986 feet tall, how far away is the bus? Round to the nearest tenth.



CIVIL ENGINEERING For Exercises 6 and 7, use the following information.
The percent grade of a highway is the ratio of the vertical rise or fall over a horizontal distance expressed to the nearest whole percent. Suppose a highway has a vertical rise of 140 feet for every 2000 feet of horizontal distance.

6. Calculate the percent grade of the highway.
7. Find the angle of elevation that the highway makes with the horizontal.

8. **SKIING** A ski run has an angle of elevation of 24.4° and a vertical drop of 1100 feet. To the nearest foot, how long is the ski run?

GEYSERS For Exercises 9 and 10, use the following information.
Kirk visits Yellowstone Park and Old Faithful on a perfect day. His eyes are 6 feet from the ground, and the geyser can reach heights ranging from 90 feet to 184 feet.

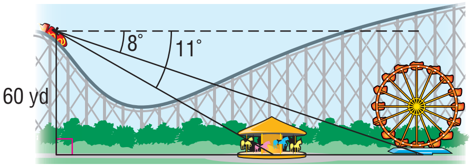
9. If Kirk stands 200 feet from the geyser and the eruption rises 175 feet in the air, what is the angle of elevation to the top of the spray to the nearest tenth?
10. In the afternoon, Kirk returns and observes the geyser's spray reach a height of 123 feet when the angle of elevation is 37°. How far from the geyser is Kirk standing to the nearest tenth of a foot?

11. **RAILROADS** Refer to the information at the left. Determine the incline of the Monongahela Incline.

12. **AVIATION** After flying at an altitude of 500 meters, a helicopter starts to descend when its ground distance from the landing pad is 11 kilometers. What is the angle of depression for this part of the flight?

13. **SLEDDING** A sledding run is 300 yards long with a vertical drop of 27.6 yards. Find the angle of depression of the run.

14. **AMUSEMENT PARKS** From the top of a roller coaster, 60 yards above the ground, a rider looks down and sees the merry-go-round and the Ferris wheel. If the angles of depression are 11° and 8°, respectively, how far apart are the merry-go-round and the Ferris wheel?

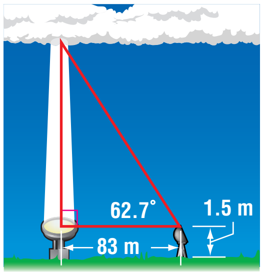


Real-World Link The Monongahela Incline, in Pittsburgh, Pennsylvania, is 635 feet long with a vertical rise of 369.39 feet. Although opened on May 28, 1870, it is still used by commuters to and from Pittsburgh.

Source: www.portauthority.org

15. **BIRD WATCHING** Two observers are 200 feet apart, in line with a tree containing a bird's nest. The angles of elevation to the bird's nest are 30° and 60°. How far is each observer from the base of the tree?

16. **METEOROLOGY** The altitude of the base of a cloud formation is called the ceiling. To find the ceiling one night, a meteorologist directed a spotlight vertically at the clouds. Using a theodolite placed 83 meters from the spotlight and 1.5 meters above the ground, he found the angle of elevation to be 62.7°. How high was the ceiling?



17. **TRAVEL** Kwan-Yong uses a theodolite to measure the angle of elevation from the ground to the top of Ayers Rock to be 15.85°. He walks half a kilometer closer and measures the angle of elevation to be 25.6°. How high is Ayers Rock to the nearest meter?

18. **PHOTOGRAPHY** A digital camera with a panoramic lens is described as having a view with an angle of elevation of 38º. If the camera is on a 3-foot tripod aimed directly at a 124-foot monument, how far from the monument should you place the tripod to see the entire monument in your photograph?

MEDICINE For Exercises 19–21, use the following information.
A doctor is using a treadmill to assess the strength of a patient's heart. At the beginning of the exam, the 48-inch long treadmill is set at an incline of 10°.

19. How far off the horizontal is the raised end of the treadmill at the beginning of the exam?

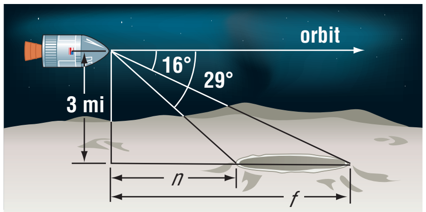
20. During one stage of the exam, the end of the treadmill is 10 inches above the horizontal. What is the incline of the treadmill to the nearest degree?

21. Suppose the exam is divided into five stages and the incline of the treadmill is increased 2° for each stage. Does the end of the treadmill rise the same distance between each stage?

R. Krubner/H. Armstrong Roberts

EXTRA PRACTICE
See pages 816, 835.
Math Online
Self-Check Quiz at
geometryonline.com

22. AEROSPACE On July 20, 1969, Neil Armstrong became the first human to walk on the Moon. During this mission, Apollo 11 orbited the Moon three miles above the surface. At one point in the orbit, the onboard guidance system measured the angles of depression to the far and near edges of a large crater. The angles measured 16° and 29°, respectively. Find the distance across the crater.



H.O.T. Problems

23. OPEN ENDED Find a real-life example of an angle of depression. Draw a diagram and identify the angle of depression.

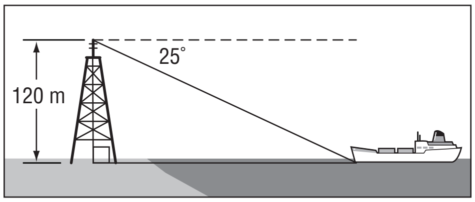
24. REASONING Explain why an angle of elevation is given that name.

25. CHALLENGE Two weather observation stations are 7 miles apart. A weather balloon is located between the stations. From Station 1, the angle of elevation to the weather balloon is 33°. From Station 2, the angle of elevation to the balloon is 52°. Find the altitude of the balloon to the nearest tenth of a mile.

26. ***Writing in Math*** Describe how an airline pilot would use angles of elevation and depression. Make a diagram and label the angles of elevation and depression. Then describe the difference between the two.

STANDARDIZED TEST PRACTICE

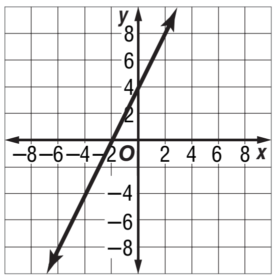
27. The top of a signal tower is 120 meters above sea level. The angle of depression from the top of the tower to a passing ship is 25°. Which is closest to the distance from the foot of the tower to the ship?



sin 25° ≈ 0.42
cos 25° ≈ 0.91
tan 25° ≈ 0.47

A 283.9 m
B 257.3 m
C 132.4 m
D 56.0 m

28. REVIEW What will happen to the slope of line p if the line is shifted so that the y-intercept decreases and the x-intercept remains the same?



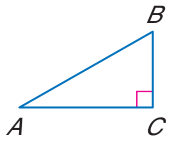
F The slope will change from negative to positive.
G The slope will become undefined.
H The slope will decrease.
J The slope will increase.

Spiral Review

Find the measure of each angle to the nearest tenth of a degree. (Lesson 8-4)

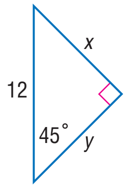
29. $\cos A = 0.6717$ **30.** $\sin B = 0.5127$ **31.** $\tan C = 2.1758$

32. If $\cos B = \frac{1}{4}$, find $\tan B$. (Lesson 8-4)

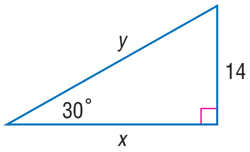


Find x and y. (Lesson 8-3)

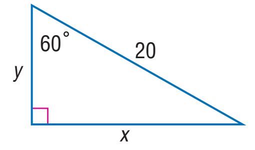
33.



34.



35.



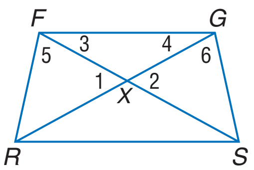
36. LANDSCAPING Paulo is designing two gardens shaped like similar triangles. One garden has a perimeter of 53.5 feet, and the longest side is 25 feet. He wants the second garden to have a perimeter of 32.1 feet. Find the length of the longest side of this garden. (Lesson 7-5)

37. MODEL AIRPLANES A twin-engine airplane used for medium-range flights has a length of 78 meters and a wingspan of 90 meters. If a scale model is made with a wingspan of 36 centimeters, find its length. (Lesson 6-2)

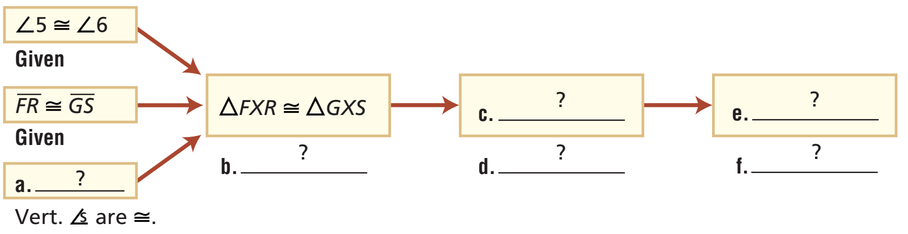
38. Copy and complete the flow proof. (Lesson 4-6)

Given: $\angle 5 \cong \angle 6$
$\overline{FR} \cong \overline{GS}$

Prove: $\angle 4 \cong \angle 3$



Proof:



Determine the truth value of the following statement for each set of conditions.
If you have a fever, then you are sick. (Lesson 2-3)

39. You do not have a fever, and you are sick.

40. You have a fever, and you are not sick.

41. You do not have a fever, and you are not sick.

42. You have a fever, and you are sick.

GET READY for the Next Lesson

PREREQUISITE SKILL **Solve each proportion.** (Lesson 7-1)

43. $\frac{x}{6} = \frac{35}{42}$ **44.** $\frac{3}{x} = \frac{5}{45}$ **45.** $\frac{12}{17} = \frac{24}{x}$ **46.** $\frac{24}{36} = \frac{x}{15}$

8-6 The Law of Sines

Main Ideas

- Use the Law of Sines to solve triangles.
- Solve problems by using the Law of Sines.

New Vocabulary

Law of Sines
solving a triangle

GET READY for the Lesson

The Statue of Liberty was designed by Frederic-Auguste Bartholdi between 1865 and 1875. Copper sheets were hammered and fastened to an interior skeletal framework, which was designed by Alexandre-Gustave Eiffel. The skeleton is 94 feet high and composed of wrought iron bars. These bars are arranged in triangular shapes, many of which are not right triangles.

The Law of Sines In trigonometry, the **Law of Sines** can be used to find missing parts of triangles that are not right triangles.

Study Tip

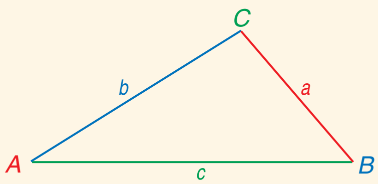
Obtuse Angles

There are also values for sin *A*, cos *A*, and tan *A*, when $A \geq 90°$. Values of the ratios for these angles will be found using the trigonometric functions on your calculator.

THEOREM 8.8 *Law of Sines*

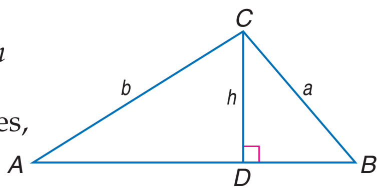
Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



PROOF Law of Sines

$\triangle ABC$ is a triangle with an altitude from C that intersects $\overline{AB}$ at D. Let h represent the measure of $\overline{CD}$. Since $\triangle ADC$ and $\triangle BDC$ are right triangles, we can find $\sin A$ and $\sin B$.



$\sin A = \frac{h}{b}$ $\quad$ $\sin B = \frac{h}{a}$ Definition of sine

$b \sin A = h$ $\quad$ $a \sin B = h$ Cross products

$b \sin A = a \sin B$ Substitution

$\frac{\sin A}{a} = \frac{\sin B}{b}$ Divide each side by *ab*.

The proof can be completed by using a similar technique with the other altitudes to show that $\frac{\sin A}{a} = \frac{\sin C}{c}$ and $\frac{\sin B}{b} = \frac{\sin C}{c}$.

Rohan/Stone/Getty Images

EXAMPLE Use the Law of Sines

1 **Given measures of $\triangle ABC$, find the indicated measure. Round angle measures to the nearest degree and side measures to the nearest tenth.**

a. If $m\angle A = 37$, $m\angle B = 68$, and $a = 3$, find b.

Use the Law of Sines to write a proportion.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{Law of Sines}$$

$$\frac{\sin 37°}{3} = \frac{\sin 68°}{b} \quad m\angle A = 37, a = 3, m\angle B = 68$$

$$b \sin 37° = 3 \sin 68° \quad \text{Cross products}$$

$$b = \frac{3 \sin 68°}{\sin 37°} \quad \text{Divide each side by } \sin 37°.$$

$$b \approx 4.6 \quad \text{Use a calculator.}$$

b. If $b = 17$, $c = 14$, and $m\angle B = 92$, find $m\angle C$.

Write a proportion relating $\angle B$, $\angle C$, b, and c.

$$\frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{Law of Sines}$$

$$\frac{\sin 92°}{17} = \frac{\sin C}{14} \quad m\angle B = 92, b = 17, c = 14$$

$$14 \sin 92° = 17 \sin C \quad \text{Cross products}$$

$$\frac{14 \sin 92°}{17} = \sin C \quad \text{Divide each side by 17.}$$

$$\sin^{-1}\left(\frac{14 \sin 92°}{17}\right) = C \quad \text{Solve for } C.$$

$$55° \approx C \quad \text{Use a calculator.}$$

So, $m\angle C \approx 55$.

Study Tip

Rounding

If you round before the final answer, your results may differ from results in which rounding was not done until the final answer.

CHECK Your Progress

1A. If $m\angle B = 32$, $m\angle C = 51$, $c = 12$, find a.

1B. If $a = 22$, $b = 18$, $m\angle A = 25$, find $m\angle B$.

The Law of Sines can be used to solve a triangle. **Solving a triangle** means finding the measures of all of the angles and sides of a triangle.

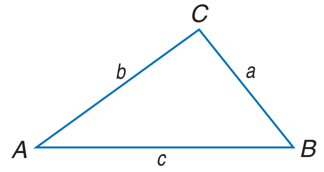
EXAMPLE Solve Triangles

Study Tip

Look Back

To review the **Angle Sum Theorem**, see Lesson 4-2.

2 **a. Solve $\triangle ABC$ if $m\angle A = 33$, $m\angle B = 47$, and $b = 14$. Round angle measures to the nearest degree and side measures to the nearest tenth.**



We know the measures of two angles of the triangle. Use the Angle Sum Theorem to find $m\angle C$.

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Study Tip

An Equivalent Proportion

The Law of Sines may also be written as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. You may wish to use this form when finding the length of a side.

$m\angle A + m\angle B + m\angle C = 180$ Angle Sum Theorem

$33 + 47 + m\angle C = 180$ $m\angle A = 33$, $m\angle B = 47$

$80 + m\angle C = 180$ Add.

$m\angle C = 100$ Subtract 80 from each side.

Since we know $m\angle B$ and b, use proportions involving $\frac{\sin B}{b}$.

To find a:		To find c:
$\frac{\sin B}{b} = \frac{\sin A}{a}$	Law of Sines	$\frac{\sin B}{b} = \frac{\sin C}{c}$
$\frac{\sin 47°}{14} = \frac{\sin 33°}{a}$	Substitute.	$\frac{\sin 47°}{14} = \frac{\sin 100°}{c}$
$a \sin 47° = 14 \sin 33°$	Cross products	$c \sin 47° = 14 \sin 100°$
$a = \frac{14 \sin 33°}{\sin 47°}$	Divide each side by sin 47°.	$c = \frac{14 \sin 100°}{\sin 47°}$
$a \approx 10.4$	Use a calculator.	$c \approx 18.9$

Therefore, $m\angle C = 100$, $a \approx 10.4$, and $c \approx 18.9$.

b. Solve $\triangle ABC$ if $m\angle C = 98$, $b = 14$, and $c = 20$. Round angle measures to the nearest degree and side measures to the nearest tenth.

$\frac{\sin B}{b} = \frac{\sin C}{c}$ Law of Sines

$\frac{\sin B}{14} = \frac{\sin 98°}{20}$ $m\angle C = 98$, $b = 14$, and $c = 20$

$20 \sin B = 14 \sin 98°$ Cross products

$\sin B = \frac{14 \sin 98°}{20}$ Divide each side by 20.

$B = \sin^{-1}\left(\frac{14 \sin 98°}{20}\right)$ Solve for B.

$B \approx 44°$ Use a calculator.

$m\angle A + m\angle B + m\angle C = 180$ Angle Sum Theorem

$m\angle A + 44 + 98 = 180$ $m\angle B = 44$ and $m\angle C = 98$

$m\angle A + 142 = 180$ Add.

$m\angle A = 38$ Subtract 142 from each side.

$\frac{\sin A}{a} = \frac{\sin C}{c}$ Law of Sines

$\frac{\sin 38°}{a} = \frac{\sin 98°}{20}$ $m\angle A = 38$, $m\angle C = 98$, and $c = 20$

$20 \sin 38° = a \sin 98°$ Cross products

$\frac{20 \sin 38°}{\sin 98°} = a$ Divide each side by sin 98°.

$12.4 \approx a$ Use a calculator.

Therefore, $A \approx 38°$, $B \approx 44°$, and $a \approx 12.4$.

CHECK Your Progress

Find the missing angles and sides of $\triangle PQR$. Round angle measures to the nearest degree and side measures to the nearest tenth.

2A. $m\angle R = 66$, $m\angle Q = 59$, $p = 72$ **2B.** $p = 32$, $r = 11$, $m\angle P = 105$

Concepts in Motion

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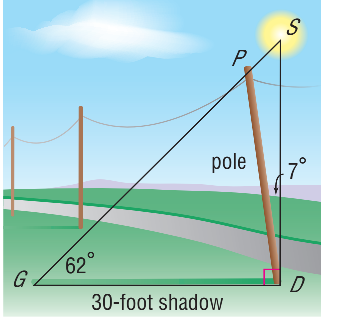
Use the Law of Sines to Solve Problems

The Law of Sines is very useful in solving direct and indirect measurement applications.

Real-World EXAMPLE 3 Indirect Measurement

ENGINEERING When the angle of elevation to the Sun is 62°, a telephone pole tilted at an angle of 7° from the vertical casts a shadow 30 feet long on the ground. Find the length of the telephone pole to the nearest tenth of a foot.



Draw a diagram.

Draw $\overline{SD} \perp \overline{GD}$. Then find $m\angle GDP$ and $m\angle GPD$.

$m\angle GDP = 90 - 7$ or 83

$m\angle GPD + 62 + 83 = 180$ or $m\angle GPD = 35$

Since you know the measures of two angles of the triangle, $m\angle GDP$ and $m\angle GPD$, and the length of a side opposite one of the angles ($\overline{GD}$ is opposite $\angle GPD$) you can use the Law of Sines to find the length of the pole.

$$\frac{PD}{\sin \angle DGP} = \frac{GD}{\sin \angle GPD}$$ Law of Sines

$$\frac{PD}{\sin 62°} = \frac{30}{\sin 35°}$$ $m\angle DGP = 62$, $m\angle GPD = 35$, and $GD = 30$

$$PD \sin 35° = 30 \sin 62°$$ Cross products

$$PD = \frac{30 \sin 62°}{\sin 35°}$$ Divide each side by sin 35°.

$$PD \approx 46.2$$ Use a calculator.

The telephone pole is about 46.2 feet long.

CHECK Your Progress

3. AVIATION Two radar stations that are 35 miles apart located a plane at the same time. The first station indicated that the position of the plane made an angle of 37° with the line between the stations. The second station indicated that it made an angle of 54° with the same line. How far is each station from the plane?

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Study Tip

Law of Sines
Case 2 of the Law of Sines can lead to two different triangles. This is called the *ambiguous case* of the Law of Sines.

KEY CONCEPT *Law of Sines*

The Law of Sines can be used to solve a triangle in the following cases.

Case 1 You know the measures of two angles and any side of a triangle. (AAS or ASA)

Case 2 You know the measures of two sides and an angle opposite one of these sides of the triangle. (SSA)

CHECK Your Understanding

Example 1 (p. 472)

Find each measure using the given measures of $\triangle XYZ$. Round angle measures to the nearest degree and side measures to the nearest tenth.

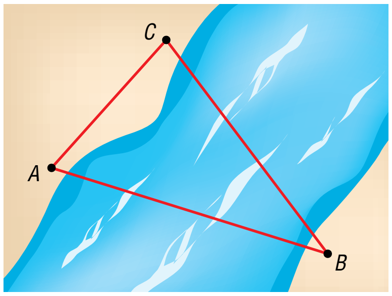
1. If $x = 3$, $m\angle X = 37$, and $m\angle Y = 68$, find y.

2. If $y = 12.1$, $m\angle X = 57$, and $m\angle Z = 72$, find x.

3. If $y = 7$, $z = 11$, and $m\angle Z = 37$, find $m\angle Y$.

4. If $y = 17$, $z = 14$, and $m\angle Y = 92$, find $m\angle Z$.

5. **SURVEYING** To find the distance between two points A and B that are on opposite sides of a river, a surveyor measures the distance to point C on the same side of the river as point A. The distance from A to C is 240 feet. He then measures the angle across from A to B as 62° and measures the angle across from C to B as 55°. Find the distance from A to B.



Example 2 (p. 472)

Solve each $\triangle PQR$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

6. $m\angle P = 33$, $m\angle R = 58$, $q = 22$

7. $p = 28$, $q = 22$, $m\angle P = 120$

8. $m\angle P = 50$, $m\angle Q = 65$, $p = 12$

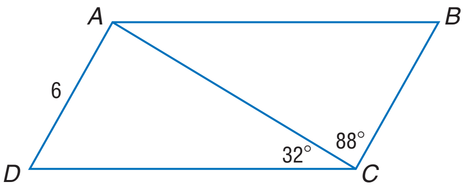
9. $q = 17.2$, $r = 9.8$, $m\angle Q = 110.7$

10. $m\angle P = 49$, $m\angle R = 57$, $p = 8$

11. $m\angle P = 40$, $m\angle Q = 60$, $r = 20$

Example 3 (p. 474)

12. Find the perimeter of parallelogram $ABCD$ to the nearest tenth.



Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–18	1
19–26	2
27, 28	3

Find each measure using the given measures of $\triangle KLM$. Round angle measures to the nearest degree and side measures to the nearest tenth.

13. If $k = 3.2$, $m\angle L = 52$, and $m\angle K = 70$, find ℓ.

14. If $m = 10.5$, $k = 18.2$, and $m\angle K = 73$, find $m\angle M$.

15. If $k = 10$, $m = 4.8$, and $m\angle K = 96$, find $m\angle M$.

16. If $m\angle M = 59$, $\ell = 8.3$, and $m = 14.8$, find $m\angle L$.

17. If $m\angle L = 45$, $m\angle M = 72$, and $\ell = 22$, find k.

18. If $m\angle M = 61$, $m\angle K = 31$, and $m = 5.4$, find ℓ.

Solve each $\triangle WXY$ described below. Round measures to the nearest tenth.

19. $m\angle Y = 71$, $y = 7.4$, $m\angle X = 41$

20. $x = 10.3$, $y = 23.7$, $m\angle Y = 96$

21. $m\angle X = 25$, $m\angle W = 52$, $y = 15.6$

22. $m\angle Y = 112$, $x = 20$, $y = 56$

23. $m\angle W = 38$, $m\angle Y = 115$, $w = 8.5$

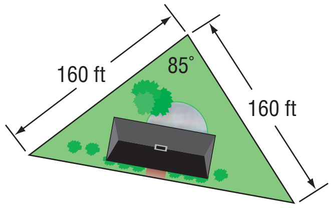
24. $m\angle W = 36$, $m\angle Y = 62$, $w = 3.1$

25. $w = 30$, $y = 9.5$, $m\angle W = 107$

26. $x = 16$, $w = 21$, $m\angle W = 88$

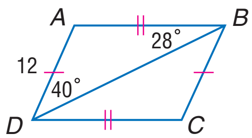
27. **TELEVISIONS** To gain better reception on his antique TV, Mr. Ramirez positioned the two antennae 13 inches apart with an angle between them of approximately 82°. If one antenna is 5 inches long, about how long is the other antenna?

28. **REAL ESTATE** A house is built on a triangular plot of land. Two sides of the plot are 160 feet long, and they meet at an angle of 85°. If a fence is to be placed along the perimeter of the property, how much fencing material is needed?



29. An isosceles triangle has a base of 46 centimeters and a vertex angle of 44°. Find the perimeter.

30. Find the perimeter of quadrilateral *ABCD* to the nearest tenth.



31. **SURVEYING** Maria Lopez is a surveyor who must determine the distance across a section of the Rio Grande Gorge in New Mexico. On one side of the ridge, she measures the angle formed by the edge of the ridge and the line of sight to a tree on the other side of the ridge. She then walks along the ridge 315 feet, passing the tree and measures the angle formed by the edge of the ridge and the new line of sight to the same tree. If the first angle is 80° and the second angle is 85°, find the distance across the gorge.

HIKING For Exercises 32 and 33, use the following information.
Kayla, Jenna, and Paige are hiking at a state park and they get separated. Kayla and Jenna are 120 feet apart. Paige sends up a signal. Jenna turns 95° in the direction of the signal and Kayla rotates 60°.

32. To the nearest foot, how far apart are Kayla and Paige?

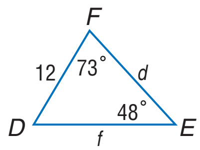
33. To the nearest foot, how far apart are Jenna and Paige?

EXTRA PRACTICE
See pages 816, 835.
Math Online
Self-Check Quiz at geometryonline.com

H.O.T. Problems

34. **FIND THE ERROR** Makayla and Felipe are trying to find *d* in $\triangle DEF$. Who is correct? Explain your reasoning.

Makayla	Felipe
$\sin 59° = \frac{d}{12}$	$\frac{\sin 59°}{d} = \frac{\sin 48°}{12}$



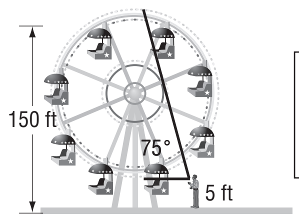
35. **OPEN ENDED** Draw an acute triangle and label the measures of two angles and the length of one side. Explain how to solve the triangle.

36. **CHALLENGE** Does the Law of Sines apply to the acute angles of a right triangle? Explain your answer.

37. ***Writing in Math*** Refer to the information on the Statue of Liberty on page 471. Describe how triangles are used in structural support.

STANDARDIZED TEST PRACTICE

38. Soledad is looking at the top of a 150-foot tall Ferris wheel at an angle of 75°.



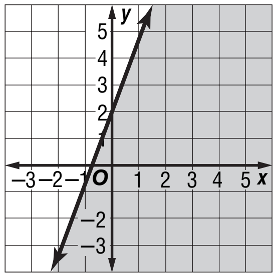
sin 75° ≈ 0.97
cos 75° ≈ 0.26
tan 75° ≈ 3.73

If she is 5 feet tall, how far is Soledad from the Ferris wheel?

A 15.0 ft **C** 75.8 ft

B 38.9 ft **D** 541.1 ft

39. **REVIEW** Which inequality *best* describes the graph below?



F $y \geq -x + 2$

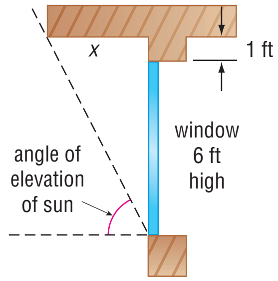
G $y \leq x + 2$

H $y \geq -3x + 2$

J $y \leq 3x + 2$

Spiral Review

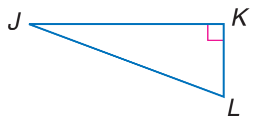
ARCHITECTURE **For Exercises 40 and 41, use the following information.**
Mr. Martinez is an architect who designs houses so that the windows receive minimum Sun in the summer and maximum Sun in the winter. For Columbus, Ohio, the angle of elevation of the Sun at noon on the longest day is 73.5° and on the shortest day is 26.5°. Suppose a house is designed with a south-facing window that is 6 feet tall. The top of the window is to be installed 1 foot below the overhang. (Lesson 8-5)



40. How long should the architect make the overhang so that the window gets no direct sunlight at noon on the longest day?

41. Using the overhang from Exercise 40, how much of the window will get direct sunlight at noon on the shortest day?

Use $\triangle JKL$ to find sin J, cos J, tan J, sin L, cos L, and tan L. Express each ratio as a fraction and as a decimal to the nearest hundredth. (Lesson 8-4)



42. $j = 8, k = 17, l = 15$

43. $j = 20, k = 29, l = 21$

44. $j = 12, k = 24, l = 12\sqrt{3}$

45. $j = 7\sqrt{2}, k = 14, l = 7\sqrt{2}$

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate** $\frac{c^2 - a^2 - b^2}{-2ab}$ **for the given values of a, b, and c.** (Page 780)

46. $a = 7, b = 8, c = 10$

47. $a = 4, b = 9, c = 6$

48. $a = 5, b = 8, c = 10$

49. $a = 16, b = 4, c = 13$

50. $a = 3, b = 10, c = 9$

51. $a = 5, b = 7, c = 11$

EXTEND 8-6

Geometry Software Lab
The Ambiguous Case of the Law of Sines

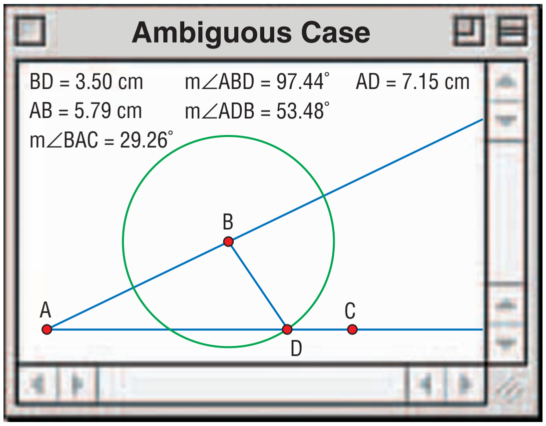
In Lesson 8-6, you learned that you could solve a triangle using the Law of Sines if you know the measures of two angles and any side of the triangle (AAS or ASA). You can also solve a triangle by the Law of Sines if you know the measures of two sides and an angle opposite one of the sides (SSA). When you use SSA to solve a triangle, and the given angle is acute, sometimes it is possible to find two different triangles. You can use The Geometer's Sketchpad to explore this case, called the **ambiguous case**, of the Law of Sines.

ACTIVITY

Step 1 Construct $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Construct a circle whose center is B so that it intersects $\overrightarrow{AC}$ at two points. Then, construct any radius $\overline{BD}$.

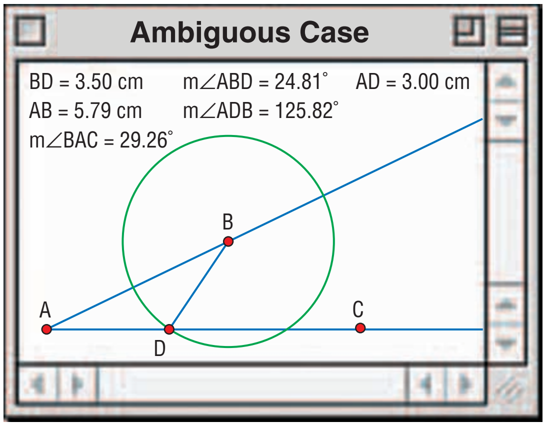
Step 2 Find the measures of $\overline{BD}$, $\overline{AB}$, and $\angle A$.

Step 3 Use the rotate tool to move D so that it lies on one of the intersection points of circle B and $\overrightarrow{AC}$. In $\triangle ABD$, find the measures of $\angle ABD$, $\angle BDA$, and $\overline{AD}$.



Step 4 Using the rotate tool, move D to the other intersection point of circle B and $\overrightarrow{AC}$.

Step 5 Note the measures of $\angle ABD$, $\angle BDA$, and $\overline{AD}$ in $\triangle ABD$.



Analyze the Results

1. Which measures are the same in both triangles?
2. Repeat the activity using different measures for $\angle A$, $\overline{BD}$, and $\overline{AB}$. How do the results compare to the earlier results?
3. Compare your results with those of your classmates. How do the results compare?
4. What would have to be true about circle B in order for there to be one unique solution? Test your conjecture by repeating the activity.
5. Is it possible, given the measures of $\overline{BD}$, $\overline{AB}$, and $\angle A$, to have no solution? Test your conjecture and explain.

8-7 The Law of Cosines

GET READY for the Lesson

German architect Ludwig Mies van der Rohe entered the design at the right in the Friedrichstrasses Skyscraper Competition in Berlin in 1921. The skyscraper was to be built on a triangular plot of land. In order to maximize space, the design called for three towers in a triangular shape. However, the skyscraper was never built.

Main Ideas

- Use the Law of Cosines to solve triangles.
- Solve problems by using the Law of Cosines.

New Vocabulary

Law of Cosines

The Law of Cosines Suppose you know the lengths of the sides of the triangular building and want to solve the triangle. The **Law of Cosines** allows us to solve a triangle when the Law of Sines cannot be used.

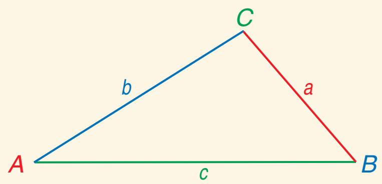
THEOREM 8.9 *Law of Cosines*

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles A, B, and C, respectively. Then the following equations are true.

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$



Study Tip

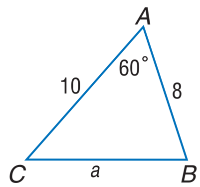
Side and Angle

Note that the letter of the side length on the left-hand side of each equation corresponds to the angle measure used with the cosine.

The Law of Cosines can be used to find missing measures in a triangle if you know the measures of two sides and their included angle.

EXAMPLE 1 Two Sides and the Included Angle

Find a if $c = 8$, $b = 10$, and $m\angle A = 60$.



Use the Law of Cosines since the measures of two sides and the included are known.

$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$a^2 = 10^2 + 8^2 - 2(10)(8) \cos 60°$	$b = 10$, $c = 8$, and $m\angle A = 60$
$a^2 = 164 - 160 \cos 60°$	Simplify.
$a = \sqrt{164 - 160 \cos 60°}$	Take the square root of each side.
$a \approx 9.2$	Use a calculator.

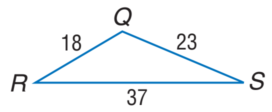
CHECK Your Progress

1. In $\triangle DEF$, $e = 19$, $f = 28$, and $m\angle D = 49$. Find d.

Extra Examples at geometryonline.com

You can also use the Law of Cosines to find the measures of angles of a triangle when you know the measures of the three sides.

EXAMPLE Three Sides



2 **Find $m\angle R$.**

$r^2 = q^2 + s^2 - 2qs \cos R$	Law of Cosines
$23^2 = 37^2 + 18^2 - 2(37)(18) \cos R$	$r = 23$, $q = 37$, $s = 18$
$529 = 1693 - 1332 \cos R$	Simplify.
$-1164 = -1332 \cos R$	Subtract 1693 from each side.
$\frac{-1164}{-1332} = \cos R$	Divide each side by -1332.
$R = \cos^{-1}\left(\frac{1164}{1332}\right)$	Solve for R.
$R \approx 29.1°$	Use a calculator.

CHECK Your Progress

2. In $\triangle TVW$, $v = 18$, $t = 24$, and $w = 30$. Find $m\angle W$.

Use the Law of Cosines to Solve Problems Most problems can be solved using more than one method. Choosing the most efficient way to solve a problem is sometimes not obvious.

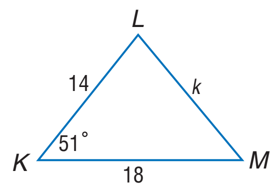
When solving right triangles, you can use sine, cosine, or tangent ratios. When solving other triangles, you can use the Law of Sines or the Law of Cosines. You must decide how to solve each problem depending on the given information.

CONCEPT SUMMARY — *Solving a Triangle*

To solve	Given	Begin by using
Right triangle	two legs	tangent
	leg and hypotenuse	sine or cosine
	angle and hypotenuse	sine or cosine
	angle and a leg	sine, cosine, or tangent
Any triangle	two angles and any side	Law of Sines
	two sides and the angle opposite one of them	Law of Sines
	two sides and the included angle	Law of Cosines
	three sides	Law of Cosines

EXAMPLE Select a Strategy

3 **Solve $\triangle KLM$. Round angle measures to the nearest degree and side measures to the nearest tenth.**



We do not know whether $\triangle KLM$ is a right triangle, so we must use the Law of Cosines or the Law of Sines. We know the measures of two sides and the included angle. This is SAS, so use the Law of Cosines.

$k^2 = \ell^2 + m^2 - 2\ell m \cos K$ — Law of Cosines

$k^2 = 18^2 + 14^2 - 2(18)(14) \cos 51°$ — $\ell = 18$, $m = 14$, and $m\angle K = 51$

$k = \sqrt{18^2 + 14^2 - 2(18)(14) \cos 51°}$ — Take the square root of each side.

$k \approx 14.2$ — Use a calculator.

Study Tip

Law of Cosines

If you use the Law of Cosines to find another measure, your answer may differ slightly from one found using the Law of Sines. This is due to rounding.

Next, we can find $m\angle L$ or $m\angle M$. If we decide to find $m\angle L$, we can use either the Law of Sines or the Law of Cosines to find this value. In this case, we will use the Law of Sines.

$\frac{\sin L}{\ell} = \frac{\sin K}{k}$ — Law of Sines

$\frac{\sin L}{18} \approx \frac{\sin 51°}{14.2}$ — $\ell = 18$, $k \approx 14.2$, and $m\angle K = 51$

$14.2 \sin L \approx 18 \sin 51°$ — Cross products

$\sin L \approx \frac{18 \sin 51°}{14.2}$ — Divide each side by 14.2.

$L \approx \sin^{-1}\left(\frac{18 \sin 51°}{14.2}\right)$ — Take the inverse sine of each side.

$L \approx 80°$ — Use a calculator.

Use the Angle Sum Theorem to find $m\angle M$.

$m\angle K + m\angle L + m\angle M = 180$ — Angle Sum Theorem

$51 + 80 + m\angle M \approx 180$ — $m\angle K = 51$ and $m\angle L \approx 80$

$m\angle M \approx 49$ — Subtract 131 from each side.

Therefore, $k \approx 14.2$, $m\angle K \approx 80$, and $m\angle M \approx 49$.

Check Your Progress

3. Solve $\triangle XYZ$ for $x = 10$, $y = 11$, and $z = 12$.

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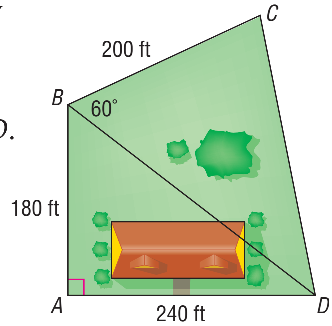
Real-World EXAMPLE — Use the Law of Cosines

4 **REAL ESTATE Ms. Jenkins is buying some property that is shaped like quadrilateral *ABCD*. Find the perimeter of the property.**



Use the Pythagorean Theorem to find BD in $\triangle ABD$.

$(AB)^2 + (AD)^2 = (BD)^2$ — Pythagorean Theorem

$180^2 + 240^2 = (BD)^2$ — $AB = 180$, $AD = 240$

$90{,}000 = (BD)^2$ — Simplify.

$300 = BD$ — Take the square root of each side.

Next, use the Law of Cosines to find CD in $\triangle BCD$.

$(CD)^2 = (BC)^2 + (BD)^2 - 2(BC)(BD) \cos \angle CBD$ — Law of Cosines

$(CD)^2 = 200^2 + 300^2 - 2(200)(300) \cos 60°$ — $BC = 200$, $BD = 300$, $m\angle CBD = 60$

$(CD)^2 = 130{,}000 - 120{,}000 \cos 60°$ — Simplify.

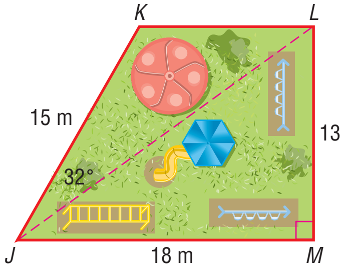
$CD = \sqrt{130{,}000 - 120{,}000 \cos 60°}$ — Take the square root of each side.

$CD \approx 264.6$ — Use a calculator.

The perimeter is 180 + 200 + 264.6 + 240 or about 884.6 feet.

CHECK Your Progress

4. ARCHITECTURE An architect is designing a playground in the shape of a quadrilateral. Find the perimeter of the playground to the nearest tenth.



CHECK Your Understanding

Example 1 (p. 479)

In $\triangle BCD$, given the following measures, find the measure of the missing side.

1. $c = \sqrt{2}, d = 5, m\angle B = 45$

2. $b = 107, c = 94, m\angle D = 105$

Example 2 (p. 480)

In $\triangle RST$, given the lengths of the sides, find the measure of the stated angle to the nearest degree.

3. $r = 33, s = 65, t = 56; m\angle S$

4. $r = 2.2, s = 1.3, t = 1.6; m\angle R$

Example 3 (p. 480)

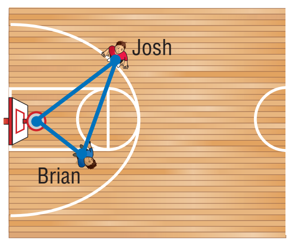
Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

5. $\triangle XYZ$: $x = 5, y = 10, z = 13$

6. $\triangle JKL$: $j = 20, \ell = 24, m\angle K = 47$

Example 4 (p. 481)

7. BASKETBALL Josh and Brian are playing basketball. Josh passes the ball to Brian, who takes a shot. Josh is 12 feet from the hoop and 10 feet from Brian. The angle formed by the hoop, Josh, and Brian is 34°. Find the distance Brian is from the hoop.



Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–11	1
12–15	2
16–22, 25–32	3
23, 24	4

In $\triangle TUV$, given the following measures, find the measure of the missing side.

8. $t = 9.1, v = 8.3, m\angle U = 32$

9. $t = 11, u = 17, m\angle V = 78$

10. $u = 11, v = 17, m\angle T = 105$

11. $v = 11, u = 17, m\angle T = 59$

In $\triangle EFG$, given the lengths of the sides, find the measure of the stated angle to the nearest degree.

12. $e = 9.1, f = 8.3, g = 16.7; m\angle F$

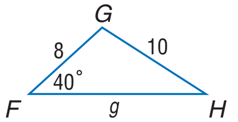
13. $e = 14, f = 19, g = 32; m\angle E$

14. $e = 325, f = 198, g = 208; m\angle F$

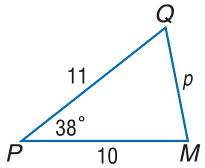
15. $e = 21.9, f = 18.9, g = 10; m\angle G$

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

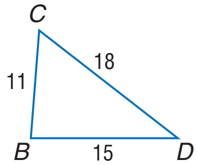
16.



17.



18.



Real-World Link

The Swissôtel in Chicago, Illinois, is built in the shape of a triangular prism. The lengths of the sides of the triangle are 180 feet, 186 feet, and 174 feet.

Source: Swissôtel

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

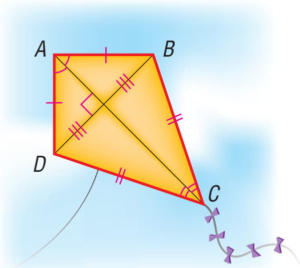
19. $\triangle ABC$: $m\angle A = 42$, $m\angle C = 77$, $c = 6$

20. $\triangle ABC$: $a = 10.3$, $b = 9.5$, $m\angle C = 37$

21. $\triangle ABC$: $a = 15$, $b = 19$, $c = 28$

22. $\triangle ABC$: $m\angle A = 53$, $m\angle C = 28$, $c = 14.9$

23. KITES Beth is building a kite like the one at the right. If $\overline{AB}$ is 5 feet long, $\overline{BC}$ is 8 feet long, and $\overline{BD}$ is $7\frac{2}{3}$ feet long, find the measures of the angle between the short sides and the angle between the long sides to the nearest degree.



24. BUILDINGS Refer to the information at the left. Find the measures of the angles of the triangular building to the nearest tenth.

Solve each $\triangle LMN$ described below. Round measures to the nearest tenth.

25. $m = 44$, $\ell = 54$, $m\angle L = 23$

26. $m\angle M = 46$, $m\angle L = 55$, $n = 16$

27. $m = 256$, $\ell = 423$, $n = 288$

28. $m\angle M = 55$, $\ell = 6.3$, $n = 6.7$

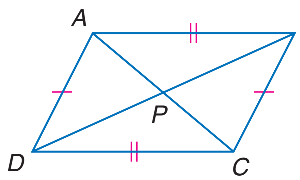
29. $m\angle M = 27$, $\ell = 5$, $n = 10$

30. $n = 17$, $m = 20$, $\ell = 14$

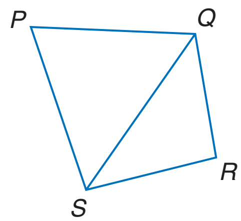
31. $\ell = 14$, $m = 15$, $n = 16$

32. $m\angle L = 51$, $\ell = 40$, $n = 35$

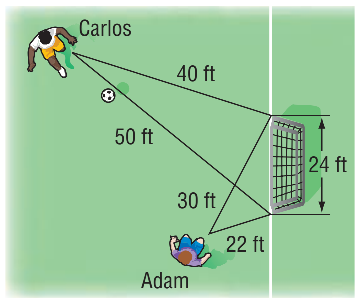
33. In quadrilateral $ABCD$, $AC = 188$, $BD = 214$, $m\angle BPC = 70$, and P is the midpoint of $\overline{AC}$ and $\overline{BD}$. Find the perimeter of $ABCD$.



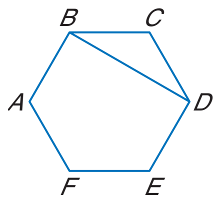
34. In quadrilateral $PQRS$, $PQ = 721$, $QR = 547$, $RS = 593$, $PS = 756$, and $m\angle P = 58$. Find QS, $m\angle PQS$, and $m\angle R$.



35. SOCCER Carlos and Adam are playing soccer. Carlos is standing 40 feet from one post of the goal and 50 feet from the other post. Adam is standing 30 feet from one post of the goal and 22 feet from the other post. If the goal is 24 feet wide, which player has a greater angle to make a shot on goal?



36. Each side of regular hexagon $ABCDEF$ is 18 feet long. What is the length of the diagonal $\overline{BD}$? Explain your reasoning.



EXTRA PRACTICE
See pages 816, 835.

Math Online
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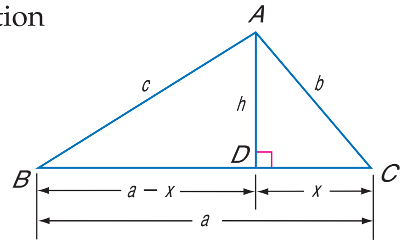
37. PROOF Justify each statement for the derivation of the Law of Cosines.

Given: $\overline{AD}$ is an altitude of $\triangle ABC$.

Prove: $c^2 = a^2 + b^2 - 2ab \cos C$



Proof:

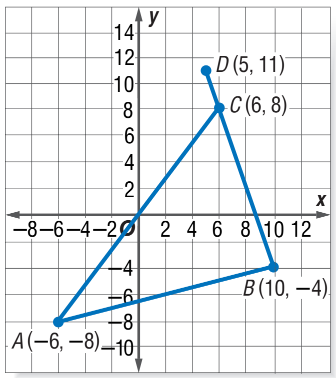
Statement	Reasons
a. $c^2 = (a - x)^2 + h^2$	a. ___?___
b. $c^2 = a^2 - 2ax + x^2 + h^2$	b. ___?___
c. $x^2 + h^2 = b^2$	c. ___?___
d. $c^2 = a^2 - 2ax + b^2$	d. ___?___
e. $\cos C = \frac{x}{b}$	e. ___?___
f. $b \cos C = x$	f. ___?___
g. $c^2 = a^2 - 2a(b \cos C) + b^2$	g. ___?___
h. $c^2 = a^2 + b^2 - 2ab \cos C$	h. ___?___

H.O.T. Problems

38. OPEN ENDED Draw and label one acute and one obtuse triangle, illustrating when you can use the Law of Cosines to find the missing measures.

39. REASONING Find a counterexample for the following statement.
The Law of Cosines can be used to find the length of a missing side in any triangle.

40. CHALLENGE Graph $A(-6, -8)$, $B(10, -4)$, $C(6, 8)$, and $D(5, 11)$ on the coordinate plane. Find the measure of interior angle ABC and the measure of exterior angle DCA.



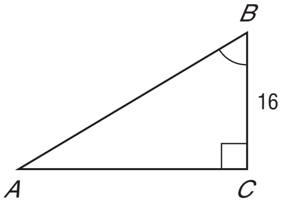
41. Which One Doesn't Belong? Analyze the four terms and determine which does not belong with the others.

Pythagorean triple	Pythagorean Theorem
Law of Cosines	cosine

42. *Writing in Math* Refer to the information about the Friedrichstrasses Skyscraper Competition on page 479. Describe how triangles were used in van der Rohe's design. Explain why the Law of Cosines could not be used to solve the triangle.

STANDARDIZED TEST PRACTICE

43. In the figure below, $\cos B = 0.8$.



What is the length of $\overline{AB}$?

A 12.8

B 16.8

C 20.0

D 28.8

44. REVIEW Which of the following shows $2x^2 - 24xy - 72y^2$ factored completely?

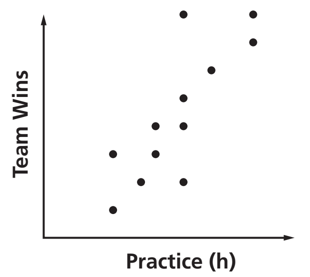
F $(2x - 18y)(x + 4y)$

G $2(x - 6y)(x + 6y)$

H $(2x - 8y)(x - 9)$

J $2(x - 6y)(x + 18y)$

45. REVIEW The scatter plot shows the responses of swim coaches to a survey about the hours of swim team practice and the number of team wins.



Which statement *best* describes the relationship between the two quantities?

A As the number of practice hours increases, the number of team wins increases.

B As the number of practice hours increases, the number of team wins decreases.

C As the number of practice hours increases, the number of team wins at first decreases, then increases.

D There is no relationship between the number of practice hours and the number of team wins.

Spiral Review

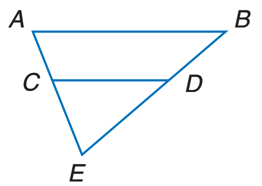
Find each measure using the given measures from $\triangle XYZ$. Round angle measure to the nearest degree and side measure to the nearest tenth. (Lesson 8-6)

46. If $y = 4.7$, $m\angle X = 22$, and $m\angle Y = 49$, find x.

47. If $y = 10$, $x = 14$, and $m\angle X = 50$, find $m\angle Y$.

48. SURVEYING A surveyor is 100 meters from a building and finds that the angle of elevation to the top of the building is 23°. If the surveyor's eye level is 1.55 meters above the ground, find the height of the building. (Lesson 8-5)

For Exercises 49–51, determine whether $\overline{AB} \parallel \overline{CD}$. (Lesson 7-4)



49. $AC = 8.4$, $BD = 6.3$, $DE = 4.5$, and $CE = 6$

50. $AC = 7$, $BD = 10.5$, $BE = 22.5$, and $AE = 15$

51. $AB = 8$, $AE = 9$, $CD = 4$, and $CE = 4$

COORDINATE GEOMETRY The vertices of $\triangle XYZ$ are $X(8, 0)$, $Y(-4, 8)$, and $Z(0, 12)$. Find the coordinates of the points of concurrency of $\triangle XYZ$ to the nearest tenth. (Lesson 5-1)

52. orthocenter

53. centroid

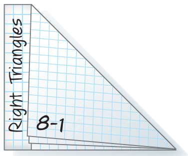
54. circumcenter

Chapter 8 Study Guide and Review

Download Vocabulary Review from geometryonline.com

FOLDABLES Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Geometric Mean (Lesson 8-1)

- For two positive numbers a and b, the geometric mean is the positive number x where the proportion $a : x = x : b$ is true. This proportion can be written using fractions as $\frac{a}{x} = \frac{x}{b}$ or with cross products as $x^2 = ab$ or $x = \sqrt{ab}$.

Pythagorean Theorem (Lesson 8-2)

- In a right triangle, the sum of the squares of the measures of the legs equals the square of the hypotenuse.

Special Right Triangles (Lesson 8-3)

- The measures of the sides of a 45°–45°–90° triangle are x, x, and $x\sqrt{2}$.
- The measures of the sides of a 30°–60°–90° triangle are x, $x\sqrt{3}$, and $2x$.

Trigonometry (Lesson 8-4)

- Trigonometric Ratios:

$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

Laws of Sines and Cosines (Lessons 8-6 and 8-7)

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively.

- Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cos A$
  $b^2 = a^2 + c^2 - 2ac \cos B$
  $c^2 = a^2 + b^2 - 2ab \cos C$

Key Vocabulary

angle of depression (p. 465)
angle of elevation (p. 464)
cosine (p. 456)
geometric mean (p. 432)
Pythagorean triple (p. 443)
sine (p. 456)
solving a triangle (p. 472)
tangent (p. 456)
trigonometric ratio (p. 456)
trigonometry (p. 456)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a *true* sentence.

1. To solve a triangle means to find the measures of all its sides and angles.
2. The Law of Sines can be applied if you know the measures of two sides and an angle opposite one of these sides of the triangle.
3. In any triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.
4. An angle of depression is the angle between the line of sight and the horizontal when an observer looks upward.
5. The geometric mean between two numbers is the positive square root of their product.
6. A 30°-60°-90° triangle is isosceles.
7. Looking at a city while flying in a plane is an example that uses an angle of elevation.
8. The numbers 3, 4, and 5 form a Pythagorean identity.

Lesson-by-Lesson Review

8-1 Geometric Mean (pp. 432–438)

Find the geometric mean between each pair of numbers.

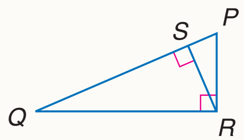
9. 4 and 16

10. 4 and 81

11. 20 and 35

12. 18 and 44

13. In PQR, $PS = 8$, and $QS = 14$. Find RS.



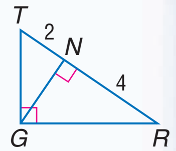
14. INDIRECT MEASUREMENT To estimate the height of the Space Needle in Seattle, Washington, James held a book up to his eyes so that the top and bottom of the building were in line with the bottom edge and binding of the cover. If James' eye level is 6 feet from the ground and he is standing 60 feet from the tower, how tall is the tower?

Example 1 **Find the geometric mean between 10 and 30.**

$\frac{10}{x} = \frac{x}{30}$	Definition of geometric mean
$x^2 = 300$	Cross products
$x = \sqrt{300}$ or $10\sqrt{3}$	Simplify.

Example 2 **Find NG in $\triangle TGR$.**

The measure of the altitude is the geometric mean between the measures of the two hypotenuse segments.

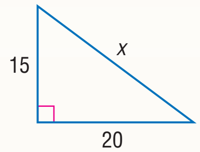


$\frac{TN}{GN} = \frac{GN}{RN}$	Definition of geometric mean
$\frac{2}{GN} = \frac{GN}{4}$	$TN = 2$, $RN = 4$
$8 = (GN)^2$	Cross products
$\sqrt{8}$ or $2\sqrt{2} = GN$	Take the square root of each side.

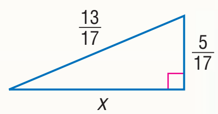
8-2 The Pythagorean Theorem and Its Converse (pp. 440–446)

Find x.

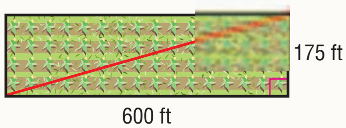
15.



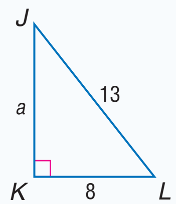
16.



17. FARMING A farmer wishes to create a maze in his corn field. He cuts a path 625 feet across the diagonal of the rectangular field. Did the farmer create two right triangles? Explain.



Example 3 **Use $\triangle JKL$ to find a.**



$a^2 + (LK)^2 = (JL)^2$	Pythagorean Theorem
$a^2 + 8^2 = 13^2$	$LK = 8$ and $JL = 13$
$a^2 + 64 = 169$	Simplify.
$a^2 = 105$	Subtract 64 from each side.
$a = \sqrt{105}$	Take the square root of each side.
$a \approx 10.2$	Use a calculator.

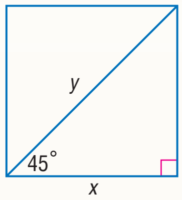
CHAPTER 8

Study Guide and Review

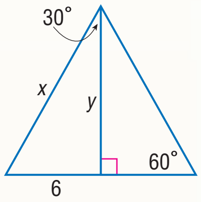
8-3 Special Right Triangles (pp. 448–454)

Find x and y.

18.



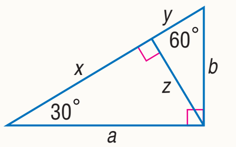
19.



For Exercises 20 and 21, use the figure.

20. If $y = 18$, find z and a.

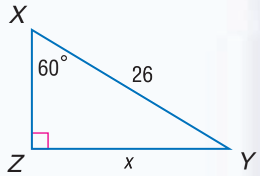
21. If $x = 14$, find a, z, b, and y.



22. **ORIGAMI** To create a bird, Michelle first folded a square piece of origami paper along one of the diagonals. If the diagonal measured 8 centimeters, find the length of one side of the square.

Example 4 **Find x.**

The shorter leg, $\overline{XZ}$, of $\triangle XYZ$ is half the measure of the hypotenuse $\overline{XY}$.

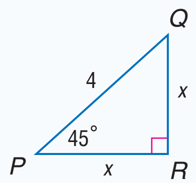


Therefore, $XZ = \frac{1}{2}(26)$ or 13. The longer leg is $\sqrt{3}$ times the measure of the shorter leg.

So, $x = 13\sqrt{3}$.

Example 5 **Find x.**

The hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of a leg.



$x\sqrt{2} = 4$

$x = \frac{4}{\sqrt{2}}$

$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ or $2\sqrt{2}$

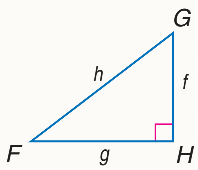
8-4 Trigonometry (pp. 456–462)

Use $\triangle FGH$ to find sin F, cos F, tan F, sin G, cos G, and tan G. Express each ratio as a fraction and as a decimal to the nearest hundredth.

23. $f = 9, g = 12, h = 15$

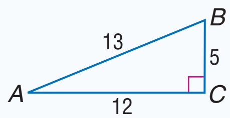
24. $f = 7, g = 24, h = 25$

25. $f = 9, g = 40, h = 41$



26. **SPACE FLIGHT** A space shuttle is directed towards the Moon but drifts 0.8° from its calculated path. If the distance from Earth to the Moon is 240,000 miles, how far has the space shuttle drifted from its path when it reaches the Moon?

Example 6 **Find sin A, cos A, and tan A. Express as a fraction and as a decimal.**



$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$

$= \frac{BC}{AB}$

$= \frac{5}{13}$ or about 0.38

$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$

$= \frac{AC}{AB}$

$= \frac{12}{13}$ or about 0.92

$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$

$= \frac{BC}{AC}$

$= \frac{5}{12}$ or about 0.42

Mixed Problem Solving
For mixed problem-solving practice, see page 835.

8-5 Angles of Elevation and Depression (pp. 464–470)

Determine the angles of elevation or depression in each situation.

27. Upon takeoff, an airplane must clear a 60-foot pole at the end of a runway 500 yards long.

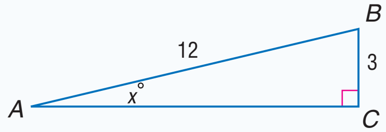
28. An escalator descends 100 feet for each horizontal distance of 240 feet.

29. A hot-air balloon ascends 50 feet for every 1000 feet traveled horizontally.

30. **EAGLES** An eagle, 1350 feet in the air, notices a rabbit on the ground. If the horizontal distance between the eagle and the rabbit is 700 feet, at what angle of depression must the eagle swoop down to catch the rabbit and fly in a straight path?

Example 7 **The ramp of a loading dock measures 12 feet and has a height of 3 feet. What is the angle of elevation?**

Make a drawing.



Let x represent $m\angle BAC$.

$\sin x° = \frac{BC}{AB}$ — $\sin x = \frac{\text{opposite leg}}{\text{hypotenuse}}$

$\sin x° = \frac{3}{12}$ — $BC = 3$ and $AB = 12$

$x = \sin^{-1}\left(\frac{3}{12}\right)$ — Find the inverse.

$x \approx 14.5$ — Use a calculator.

The angle of elevation for the ramp is about 14.5°.

8-6 The Law of Sines (pp. 471–477)

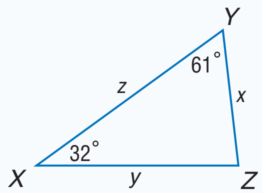
Find each measure using the given measures of $\triangle FGH$. Round angle measures to the nearest degree and side measures to the nearest tenth.

31. Find f if $g = 16$, $m\angle G = 48$, and $m\angle F = 82$.

32. Find $m\angle H$ if $h = 10.5$, $g = 13$, and $m\angle G = 65$.

33. **GARDENING** Elena is planning a triangular garden. She wants to build a fence around the garden to keep out the deer. The length of one side of the garden is 26 feet. If the angles at the end of this side are 78° and 44°, find the length of fence needed to enclose the garden.

Example 8 **Find x if $y = 15$. Round to the nearest tenth.**



To find x and z, use proportions involving $\sin Y$ and y.

$\frac{\sin Y}{y} = \frac{\sin X}{x}$ — Law of Sines

$\frac{\sin 61°}{15} = \frac{\sin 32°}{x}$ — Substitute.

$x \sin 61° = 15 \sin 32°$ — Cross Products

$x = \frac{15 \sin 32°}{\sin 61°}$ — Divide.

$x \approx 9.1$ — Use a calculator.

8-7 The Law of Cosines (pp. 479–485)

In $\triangle XYZ$, given the following measures, find the measures of the missing side.

34. $x = 7.6, y = 5.4, m\angle Z = 51$

35. $x = 21, m\angle Y = 73, z = 16$

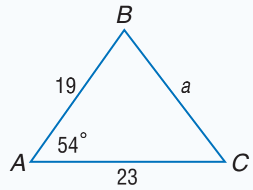
Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

36. $c = 18, b = 13, m\angle A = 64$

37. $b = 5.2, m\angle C = 53, c = 6.7$

38. ART Adelina is creating a piece of art that is in the shape of a parallelogram. Its dimensions are 35 inches by 28 inches and one angle is 80°. Find the lengths of both diagonals.

Example 9 Find a.



$a^2 = b^2 + c^2 - 2bc \cos A$ Law of Cosines

$a^2 = 23^2 + 19^2 - 2(23)(19) \cos 54°$ $b = 23$, $c = 19$, and $m\angle A = 54$

$a^2 = 890 - 874 \cos 54°$ Simplify.

$a = \sqrt{890 - 874 \cos 54°}$ Take the square root of each side.

$a \approx 19.4$ Use a calculator.

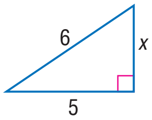
Practice Test

Find the geometric mean between each pair of numbers.

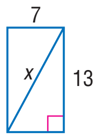
1. 7 and 63 **2.** 6 and 24 **3.** 10 and 50

Find the missing measures.

4.



5.



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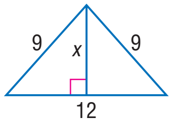

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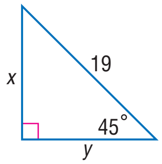

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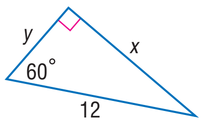
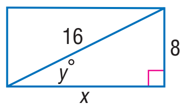
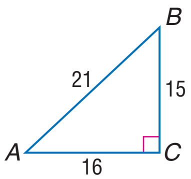


9.



Use the figure to find each trigonometric ratio. Express answers as a fraction.



10. cos B

11. tan A

12. sin A

Find each measure using the given measures from $\triangle FGH$. Round to the nearest tenth.

13. Find g if $m\angle F = 59$, $f = 13$, and $m\angle G = 71$.

14. Find $m\angle H$ if $m\angle F = 52$, $f = 10$, and $h = 12.5$.

15. Find f if $g = 15$, $h = 13$, and $m\angle F = 48$.

16. Find h if $f = 13.7$, $g = 16.8$, and $m\angle H = 71$.

Solve each triangle. Round each angle measure to the nearest degree and each side measure to the nearest tenth.

17. $a = 15$, $b = 17$, $m\angle C = 45$

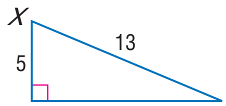
18. $a = 12.2$, $b = 10.9$, $m\angle B = 48$

19. $a = 19$, $b = 23.2$, $c = 21$

20. TRAVEL From an airplane, Janara looked down to see a city. If she looked down at an angle of 9° and the airplane was half a mile above the ground, what was the horizontal distance to the city?

21. CIVIL ENGINEERING A section of freeway has a steady incline of 10°. If the horizontal distance from the beginning of the incline to the end is 5 miles, how high does the incline reach?

22. MULTIPLE CHOICE Find tan X.

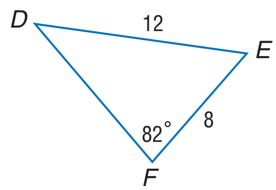


A $\frac{5}{12}$ **C** $\frac{17}{12}$

B $\frac{12}{13}$ **D** $\frac{12}{5}$

23. COMMUNICATIONS To secure a 500-foot radio tower against high winds, guy wires are attached to the tower 5 feet from the top. The wires form a 15° angle with the tower. Find the distance from the centerline of the tower to the anchor point of the wires.

24. Solve $\triangle DEF$.



25. MULTIPLE CHOICE The top of the Boone Island Lighthouse in Boone Island, Maine, is 137 feet above sea level. The angle of depression from the light on the top of the tower to a passing ferry is 37°. How many feet from the foot of the lighthouse is the ferry?

F 181.8 ft **H** 109.4 ft

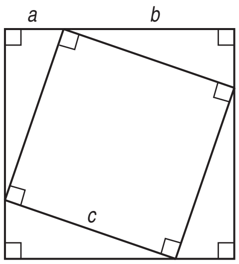
G 171.5 ft **J** 103.2 ft

Chapter 8 Standardized Test Practice

Cumulative, Chapters 1–8

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A diagram from a proof of the Pythagorean Theorem is pictured below. Which statement would be used in the proof of the Pythagorean Theorem?



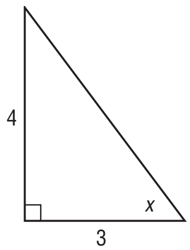
A The area of the larger square equals $(a + b)^2$.

B The area of the inner square is equal to half of the area of the larger square.

C The area of the larger square is equal to the sum of the areas of the smaller square and the four congruent triangles.

D The four right triangles are similar.

2. In the figure below, if $\tan x = \frac{4}{3}$, what are $\cos x$ and $\sin x$?



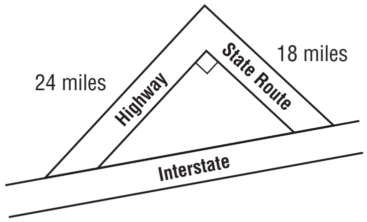
F $\cos x = \frac{3}{4}, \sin x = \frac{4}{5}$

G $\cos x = \frac{3}{4}, \sin x = \frac{5}{4}$

H $\cos x = \frac{3}{5}, \sin x = \frac{4}{5}$

J $\cos x = \frac{3}{5}, \sin x = \frac{5}{4}$

3. A detour has been set up on the interstate due to a gas leak. The diagram below shows the detour route. How many extra miles will drivers have to travel due to the detour?



A 12 miles

B 30 miles

C 42 miles

D 80 miles

4. A right triangle has legs of length $\sqrt{39}$ and 8. What is the length of the hypotenuse?

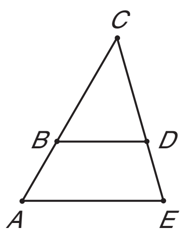
F 10 **H** 11

G $\sqrt{103}$ **J** $4\sqrt{26}$

TEST-TAKING TIP

Question 4 If a standardized test question involves trigonometric ratios, draw a diagram that represents the problem. Use a calculator (if allowed) or the table of trigonometric values provided to help you find the answer.

5. Given: $\overline{BD} \parallel \overline{AE}$



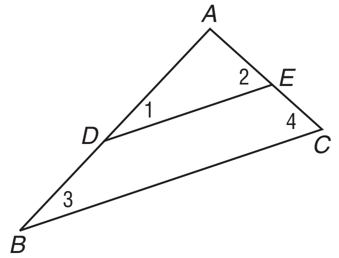
What theorem or postulate can be used to prove $\triangle ACE \sim \triangle BCD$?

A SSS **C** ASA

B SAS **D** AA

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

6. In $\triangle ABC$, D is the midpoint of $\overline{AB}$, and E is the midpoint of $\overline{AC}$.



Which of the following is *not* true?

F $\angle 1 \cong \angle 4$

G $\triangle ABC \sim \triangle ADE$

H $\overline{DE} \parallel \overline{BC}$

J $\frac{AD}{DB} = \frac{AE}{EC}$

7. If the sum of the measures of the interior angles of a polygon is 900, how many sides does the polygon have?

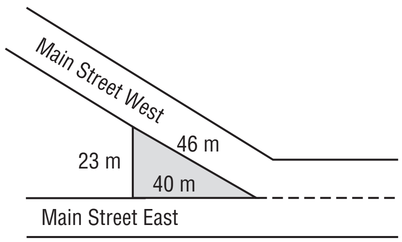
A 5

B 7

C 8

D 10

8. GRIDDABLE A city planner designs a triangular traffic median on Main Street to provide more green space in the downtown area. The planner builds a model so that the section of the median facing Main Street East measures 20 centimeters. What is the perimeter, in centimeters, of the model of the traffic median?



9. ALGEBRA Find $(x^2 + 2x - 24) \div (x - 4)$.

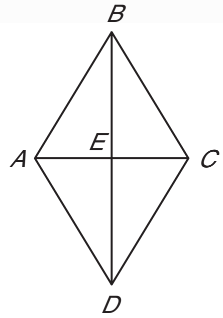
F $x - 8$

G $x + 8$

H $x - 6$

J $x + 6$

10. Rhombus $ABCD$ is shown. Which pair of triangles can be aestablished to be congruent to prove that $\overline{AC}$ bisects $\overline{BD}$?



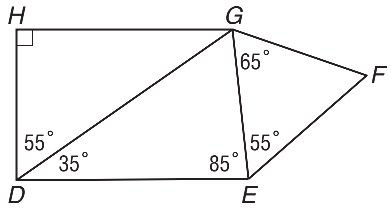
A $\triangle ABD$ and $\triangle CBD$

B $\triangle ACD$ and $\triangle ACB$

C $\triangle AEB$ and $\triangle BEC$

D $\triangle AEB$ and $\triangle CED$

11. What is the shortest side of quadrilateral $DEFG$?



F $\overline{GF}$

G $\overline{FE}$

H $\overline{DG}$

J $\overline{DE}$

Pre-AP

Record your answer on a sheet of paper. Show your work.

12. An extension ladder leans against the side of a house while gutters are being cleaned. The base of the ladder is 12 feet from the house, and the top of the ladder rests 16 feet up the side of the house.

a. Draw a figure representing this situation. What is the length of the ladder?

b. For safety, a ladder should have a climbing angle of no less than 75°. Is the climbing angle of this ladder safe?

c. If not, what distance from the house should the ladder be placed so that it still rests 16 feet up the side of the house at a 75° climbing angle and to what new length will the ladder need to be adjusted?

NEED EXTRA HELP?												
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