# UNIT 4

## Two- and Three-Dimensional Measurement

### Focus

**Calculate measures in two- and three-dimensions and use the properties of circles.**

### CHAPTER 10
### Circles

**BIG Idea** Prove and use theorems involving the properties of circles and the relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.

### CHAPTER 11
### Areas of Polygons and Circles

**BIG Idea** Students derive formulas and solve problems involving the areas of circles and polygons.

### CHAPTER 12
### Extending Surface Area

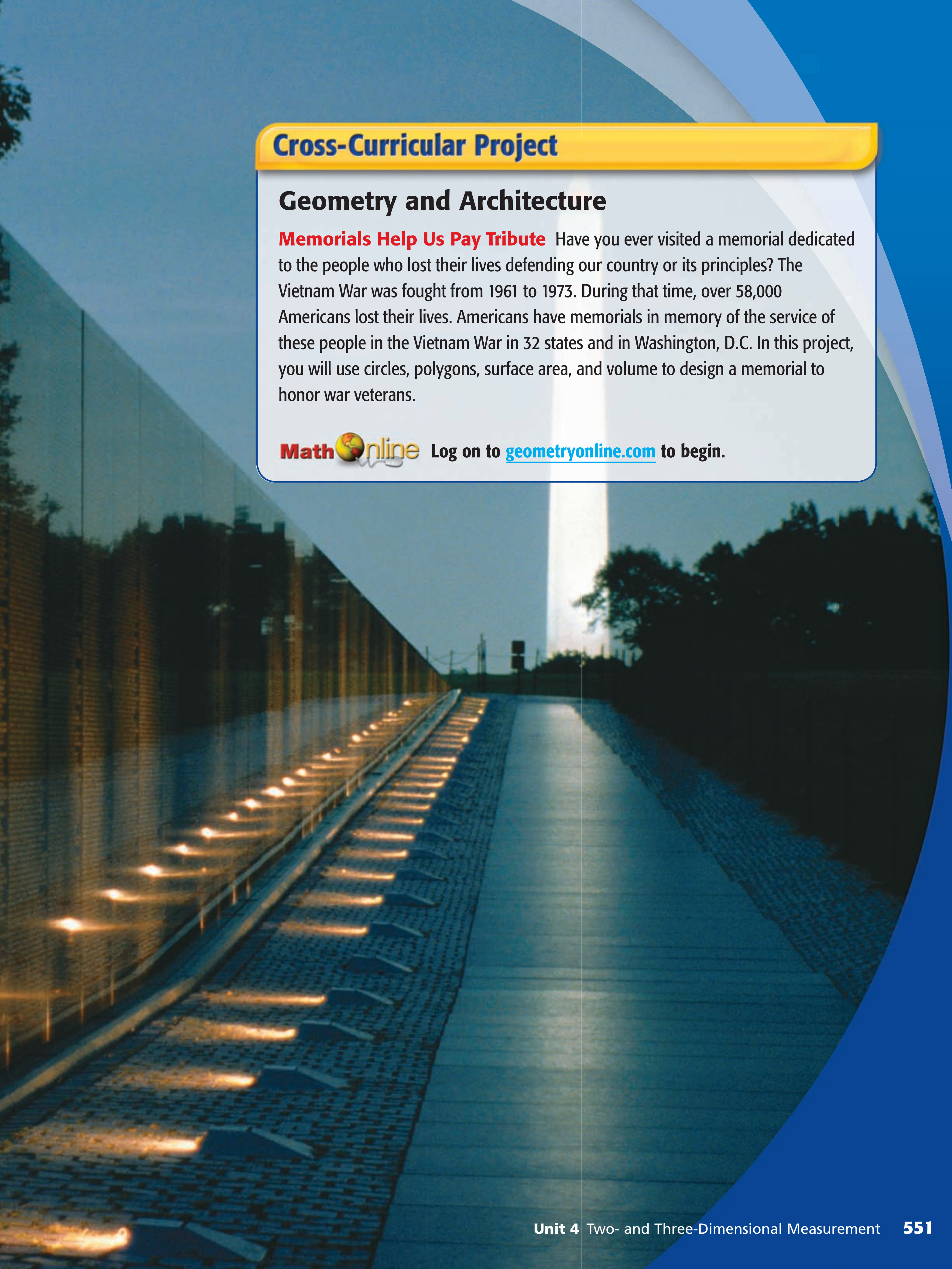
**BIG Idea** Derive formulas and solve problems involving the lateral area and surface area of solids.

### CHAPTER 13
### Extending Volume

**BIG Idea** Derive formulas and solve problems involving the volumes of three-dimensional figures.

**BIG Idea** Determine how changes in dimensions affect the volume of solids.

**BIG Idea** Investigate the effect of rigid motions on figures in the space.



## Cross-Curricular Project

# Geometry and Architecture

**Memorials Help Us Pay Tribute** Have you ever visited a memorial dedicated to the people who lost their lives defending our country or its principles? The Vietnam War was fought from 1961 to 1973. During that time, over 58,000 Americans lost their lives. Americans have memorials in memory of the service of these people in the Vietnam War in 32 states and in Washington, D.C. In this project, you will use circles, polygons, surface area, and volume to design a memorial to honor war veterans.

Math Online **Log on to geometryonline.com to begin.**

CHAPTER 10

# Circles

## BIG Ideas

- Identify parts of a circle and solve problems involving circumference.
- Find arc and angle measures in a circle.
- Find measures of segments in a circle.
- Write the equation of a circle.

## Key Vocabulary

chord (p. 554)
circumference (p. 556)
arc (p. 564)
tangent (p. 588)
secant (p. 599)

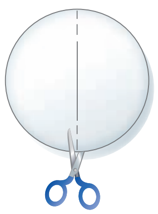
## Real-World Link

**Ferris Wheels** Modeled after the very first Ferris wheel built for the 1893 World Colombian Exposition, the Navy Pier Ferris wheel in Chicago, Illinois, is 150 feet high. It has 40 gondolas that each seat six passengers, and its 40 spokes span a diameter of 140 feet.

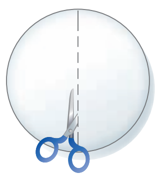
## FOLDABLES™ Study Organizer

**Circles** Make this Foldable to help you organize your notes. Begin with five sheets of plain 8½″ × 11″ paper, and cut out five large circles that are the same size.

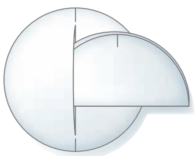
1. **Fold** two of the circles in half and cut one-inch slits at each end of the folds.



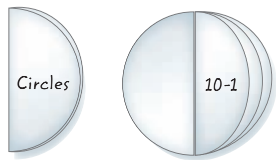
2. **Fold** the remaining three circles in half and cut a slit in the middle of the fold.



3. **Slide** the two circles with slits on the ends through the large slit of the other circles.



4. **Fold** to make a booklet. Label the cover with the title of the chapter and each sheet with a lesson number.






Joseph Sohm ChromoSohm Inc./CORBIS

# GET READY for Chapter 10

**Diagnose Readiness** You have two options for checking Prerequisite Skills.

## Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

## Option 1

Take the Quick Check below. Refer to the Quick Review for help.

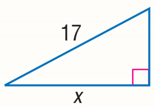
## QUICKCheck

**Solve each equation for the given variable.** (Prerequisite Skill)

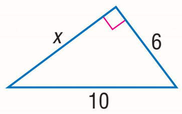
1. $\frac{4}{9}p = 72$ for $p$
2. $6.3p = 15.75$
3. $3x + 12 = 8x$ for $x$
4. $7(x + 2) = 3(x - 6)$
5. The circumference of a circle is given by the formula $C = 2\pi r$. Solve for $r$.

**Find $x$ to the nearest tenth unit.** (Lesson 8-2)

6.




7.




8. The lengths of the legs of an isosceles right triangle are 72 inches. Find the length of the hypotenuse. (Lesson 8-2)

**Solve each equation by using the Quadratic Formula. Round to the nearest tenth.** (Prerequisite Skill)

9. $x^2 - 4x = 10$
10. $3x^2 - 2x - 4 = 0$
11. $x^2 = x + 15$
12. **PHYSICS** A rocket is launched vertically up in the air from ground level. The distance in feet from the ground $d$ after $t$ seconds is given by the equation $d = 96t - 16t^2$. Find the values of $t$ when $d = 102$ feet. Round to the nearest tenth. (Prerequisite Skill)

## QUICKReview

### EXAMPLE 1

**Solve the equation $5 - y = 13(y + 2)$ for $y$.**

$5 - y = 13(y + 2)$

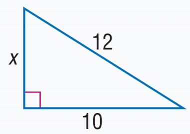
$5 - y = 13y + 26$ Distributive Property

$-21 = 14y$ Combine like terms.

$-1.5 = y$ Divide.

### EXAMPLE 2

**Find $x$. Round to the nearest tenth if necessary.**




$a^2 + b^2 = c^2$ Pythagorean Theorem

$x^2 + 10^2 = 12^2$ Substitution

$x^2 + 100 = 144$ Simplify.

$x^2 = 44$ Subtract 100 from each side.

$x = \sqrt{44}$ Take the square root of each side.

$x \approx 6.6$ Use a calculator.

### EXAMPLE 3

**Solve $x^2 + 3x - 10 = 0$ by using the Quadratic Formula. Round to the nearest tenth.**

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2(1)}$ $a = 1, b = 3, c = -10$

$= \frac{-3 \pm \sqrt{9 + 40}}{2}$ Simplify.

$= \frac{-3 \pm 7}{2}$ Simplify.

$x = \frac{-3 + 7}{2}$ or 2 $x = \frac{-3 - 7}{2}$ or $-5$

# 10-1 Circles and Circumference

## Main Ideas

- Identify and use parts of circles.
- Solve problems involving the circumference of a circle.

## New Vocabulary

circle
center
chord
radius
diameter
circumference
pi ($\pi$)

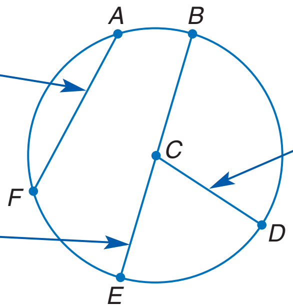
## GET READY for the Lesson

The largest carousel in the world still in operation is located in Spring Green, Wisconsin. It weighs 35 tons and contains 260 animals, none of which is a horse! The rim of the carousel base is a circle. The width, or diameter, of the circle is 80 feet. The distance that an animal on the outer edge travels can be determined by special segments in a circle.

**Parts of Circles** A **circle** is the locus of all points in a plane equidistant from a given point called the **center** of the circle. A circle is usually named by its center point. The figure below shows circle *C*, which can be written as $\odot C$. Several special segments in circle *C* are also shown.

Any segment with both endpoints that are on the circle is a **chord** of the circle. $\overline{AF}$ and $\overline{BE}$ are chords.

A chord that passes through the center is a **diameter** of the circle. $\overline{BE}$ is a diameter.




Any segment with endpoints that are the center and a point on the circle is a **radius**. $\overline{CD}$, $\overline{CB}$, and $\overline{CE}$ are radii of the circle.

The plural of radius is *radii,* pronounced RAY-dee-eye. The term *radius* can mean a segment or the measure of that segment. This is also true of the term *diameter*.

Note that diameter $\overline{BE}$ is made up of collinear radii $\overline{CB}$ and $\overline{CE}$.

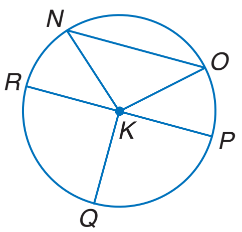
## EXAMPLE 1 Identify Parts of a Circle

**a. Name the circle.**

The circle has its center at *K*, so it is named circle *K*, or $\odot K$.

**b. Name a radius of the circle.**

Five radii are shown: $\overline{KN}$, $\overline{KO}$, $\overline{KP}$, $\overline{KQ}$, and $\overline{KR}$.



## Study Tip

**Centers of Circles** In this text, the center of the circle will often be shown in the figure with a dot.

**c. Name a chord of the circle.**

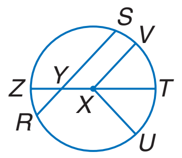
Two chords are shown: $\overline{NO}$ and $\overline{RP}$.

**d. Name a diameter of the circle.**

$\overline{RP}$ is the only chord that goes through the center, so $\overline{RP}$ is a diameter.

**CHECK Your Progress**

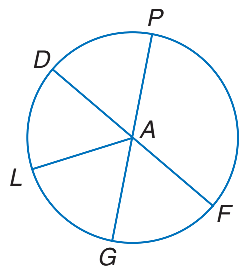
**1.** Name the circle, a radius, a chord, and a diameter of the circle.




By definition, the distance from the center to any point on a circle is always the same. Therefore, all radii $r$ are congruent. A diameter $d$ is composed of two radii, so all diameters are congruent. Thus, $d = 2r$ and $r = \frac{d}{2}$ or $\frac{1}{2}d$.

## EXAMPLE Find Radius and Diameter

**2** **Circle $A$ has diameters $\overline{DF}$ and $\overline{PG}$.**




**a. If $DF = 10$, find $DA$.**

$r = \frac{1}{2}d$ — Formula for radius

$= \frac{1}{2}(10)$ or 5 — Substitute and simplify.

**b. If $AG = 12$, find $LA$.**

Since all radii are congruent, $LA = AG$. So, $LA = 12$.

**CHECK Your Progress**

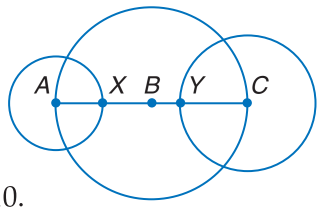
**2A.** If $PA = 7$, find $PG$.

**2B.** If $PG = 15$, find $DF$.

The segment connecting the centers of the two intersecting circles contains a radius of each circle.

## EXAMPLE Find Measures in Intersecting Circles

**3** **The diameters of $\odot A$, $\odot B$, and $\odot C$ are 10 inches, 20 inches, and 14 inches, respectively. Find $XB$.**




Since the diameter of $\odot A$ is 10, $AX = 5$.

Since the diameter of $\odot B$ is 20, $AB = 10$ and $BC = 10$.

$\overline{XB}$ is part of radius $\overline{AB}$.

$AX + XB = AB$ — Segment Addition Postulate

$5 + XB = 10$ — Substitution

$XB = 5$ — Subtract 5 from each side.

**CHECK Your Progress**

**3.** Find $BY$.

## Study Tip

**Congruent Circles**

The circles shown in Example 3 have different radii. They are *not* congruent circles. For two circles to be *congruent circles,* they must have congruent radii or congruent diameters.

**Circumference** The circumference of a circle is the distance around the circle. Circumference is most often represented by the letter $C$.

## GEOMETRY LAB

### Circumference Ratio

A special relationship exists between the circumference of a circle and its diameter.

**GATHER DATA AND ANALYZE**

**Collect ten round objects.**

1. Measure the circumference and diameter of each object in millimeters. Record the measures in a table.
2. Compute the value of $\frac{C}{d}$ to the nearest hundredth for each object. Record the result in the fourth column of the table.

| Object | $C$ | $d$ | $\frac{C}{d}$ |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| ⋮ | | | |
| 10 | | | |

3. **MAKE A CONJECTURE** What seems to be the relationship between the circumference and the diameter of the circle?

The Geometry Lab suggests that the circumference of any circle can be found by multiplying the diameter by a number slightly larger than 3. By definition, the ratio $\frac{C}{d}$ is an irrational number called **pi**, symbolized by the Greek letter $\pi$. Two formulas for the circumference can be derived using this definition.

$\frac{C}{d} = \pi$ Definition of pi

$C = \pi d$ Multiply each side by $d$.

$C = \pi d$

$C = \pi(2r)$ $d = 2r$

$C = 2\pi r$ Simplify.

**Study Tip**

**Radii and Diameters**

There are an infinite number of radii in each circle. Likewise, there are an infinite number of diameters.

**KEY CONCEPT** *Circumference*

For a circumference of $C$ units and a diameter of $d$ units or a radius of $r$ units, $C = \pi d$ or $C = 2\pi r$.

**EXAMPLE 4** Find Circumference, Diameter, and Radius

**a. Find $C$ if $r = 7$ centimeters.**

$C = 2\pi r$ Circumference formula

$= 2\pi(7)$ Substitution

$= 14\pi$ or about 43.98 cm

**b. Find $C$ if $d = 12.5$ inches.**

$C = \pi d$ Circumference formula

$= \pi(12.5)$ Substitution

$= 12.5\pi$ or 39.27 in.



Aaron Haupt

**c. Find $d$ and $r$ to the nearest hundredth if $C = 136.9$ meters.**

$C = \pi d$ Circumference formula

$136.9 = \pi d$ Substitution

$\frac{136.9}{\pi} = d$ Divide each side by $\pi$.

$43.58 \text{ m} \approx d$ Use a calculator.

$r = \frac{1}{2}d$ Radius formula

$\approx \frac{1}{2}(43.58)$ $d \approx 43.58$

$\approx 21.79 \text{ m}$ Use a calculator.

**CHECK Your Progress**

**4A.** Find C if $r = 12$ inches.

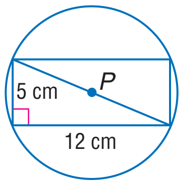
**4B.** Find C if $d = 7.25$ meters.

**4C.** Find $d$ and $r$ to the nearest hundredth if $C = 77.8$ centimeters.

You can also use other geometric figures to help you find the circumference of a circle.

## EXAMPLE Use Other Figures to Find Circumference

**5 Find the exact circumference of $\odot P$.**




The diameter of the circle is the same as the hypotenuse of the right triangle.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$5^2 + 12^2 = c^2$ Substitution

$169 = c^2$ Simplify.

$13 = c$ Take the square root of each side.

So the diameter of the circle is 13 centimeters. To find the circumference, substitute 13 for $d$ in $C = \pi d$. The *exact* circumference is $13\pi$.

**CHECK Your Progress**

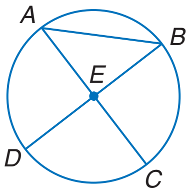
**5.** A square with a side length of 8 inches is inscribed in $\odot N$. Find the exact circumference of $\odot N$.

**Online Personal Tutor at** geometryonline.com

## CHECK Your Understanding

Examples 1, 2 (pp. 554–555)

**For Exercises 1–6, refer to the circle at the right.**




**1.** Name the circle.

**2.** Name a radius.

**3.** Name a chord.

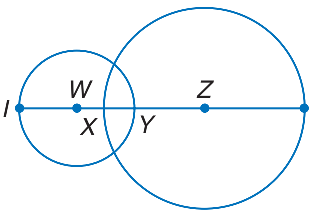
**4.** Name a diameter.

**5.** Suppose $BD = 12$ millimeters. Find the radius of the circle.

**6.** Suppose $CE = 5.2$ inches. What is the diameter of the circle?

Example 3 (p. 555)

**Circle $W$ has a radius of 4 units, $\odot Z$ has a radius of 7 units, and $XY = 2$. Find each measure.**




**7.** $YZ$

**8.** $IX$

**9.** $IC$

**Example 4** (p. 556)

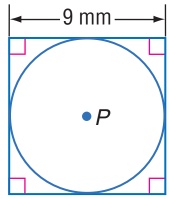
**The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.**

**10.** $r = 5$ m, $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

**11.** $C = 2368$ ft, $d = \underline{\ ?\ }$, $r = \underline{\ ?\ }$

**Example 5** (p. 557)

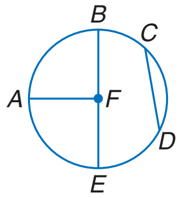
**12.** Find the exact circumference of the circle.




## Exercises

| HOMEWORK HELP | |
|---|---|
| For Exercises | See Examples |
| 13–22 | 1 |
| 23–28 | 2 |
| 29–34 | 3 |
| 35–42 | 4 |
| 43–46 | 5 |

**For Exercises 13–17, refer to the circle at the right.**




**13.** Name the circle.

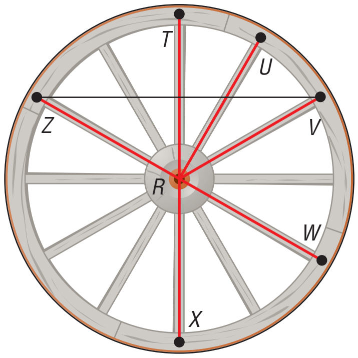
**14.** Name a radius.

**15.** Name a chord.

**16.** Name a diameter.

**17.** Name a radius not contained in a diameter.

**HISTORY** **For Exercises 18–28, refer to the model of the Conestoga wagon wheel.**




**18.** Name the circle.

**19.** Name a radius of the circle.

**20.** Name a chord of the circle.

**21.** Name a diameter of the circle.

**22.** Name a radius not contained in a diameter.

**23.** Suppose the radius of the circle is 2 feet. Find the diameter.

**24.** The larger wheel of a wagon was often 5 or more feet tall. What is the radius of a 5-foot wheel?

**25.** If $TX = 120$ centimeters, find $TR$.

**26.** If $RZ = 32$ inches, find $ZW$.

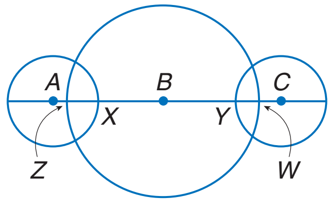
**27.** If $UR = 18$ inches, find $RV$.

**28.** If $XT = 1.2$ meters, find $UR$.

**Real-World Link**

The Conestoga Wagon, used from about 1717 to the 1820s, began as a farm wagon that was adapted for use in the hilly countryside of Pennsylvania. The wheels were larger to enable the wagon to go over stones and roots in the road.

**The diameters of $\odot A$, $\odot B$, and $\odot C$ are 10, 30, and 10 units, respectively. Find each measure if $\overline{AZ} \cong \overline{CW}$ and $CW = 2$.**




**29.** $AZ$ **30.** $ZX$ **31.** $BX$

**32.** $BY$ **33.** $YW$ **34.** $AC$

**The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.**

**35.** $r = 7$ mm, $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

**36.** $d = 26.8$ cm, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

**37.** $C = 26\pi$ mi, $d = \underline{\ ?\ }$, $r = \underline{\ ?\ }$

**38.** $C = 76.4$ m, $d = \underline{\ ?\ }$, $r = \underline{\ ?\ }$

**39.** $d = 12\frac{1}{2}$ yd, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

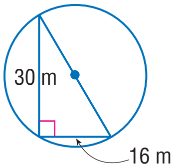
**40.** $r = 6\frac{3}{4}$ in., $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

**41.** $d = 2a$, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

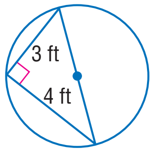
**42.** $r = \frac{a}{6}$, $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

**Find the exact circumference of each circle.**

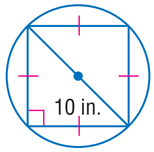
**43.**



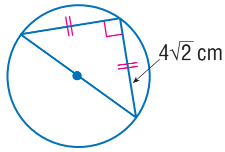

**44.**



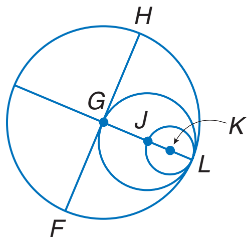

**45.**




**46.**




**Circles *G*, *J*, and *K* all intersect at *L*. If *GH* = 10, find each measure.**




**47.** *FG*

**48.** *FH*

**49.** *GL*

**50.** *GJ*

**51.** *JL*

**52.** *JK*

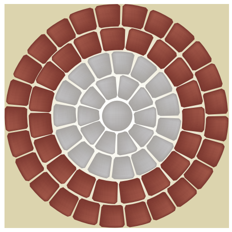
**Cross-Curricular Project**

**Math Online** Drawing a radius and circle will help you begin to design your memorial. Visit geometryonline.com to continue work on your project.

**53. PROBABILITY** Find the probability that a segment whose endpoints are the center of the circle and a point on the circle is a radius. Explain.

**54. PROBABILITY** Find the probability that a chord that does not contain the center of a circle is the longest chord of the circle.

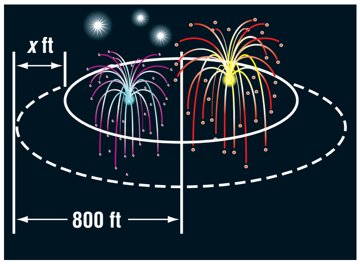
**PATIO For Exercises 55 and 56, use the following information.**
Mr. Hintz is going to build a patio as shown at the right.



**55.** If the radius of the entire patio is six feet, what is the approximate circumference?

**56.** If Mr. Hintz wants the inner circle to have a circumference of approximately 19 feet, what should the radius of the circle be to the nearest foot?

**FIREWORKS For Exercises 57–59, use the following information.**
Every July 4th, Boston puts on a gala with the Boston Pops Orchestra, followed by a huge fireworks display. The fireworks are shot from a barge in the river. There is an explosion circle inside which all of the fireworks will explode. Spectators sit outside a safety circle that is 800 feet from the center of the fireworks display.




**EXTRA PRACTICE**
See pages 819, 837.
**Math Online**
Self-Check Quiz at geometryonline.com

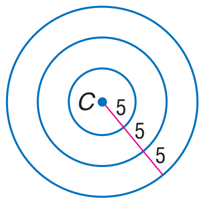
**57.** Find the approximate circumference of the safety circle.

**58.** If the safety circle is 200 to 300 feet farther from the center than the explosion circle, find the range of values for the radius of the explosion circle.

**59.** Find the least and maximum circumference of the explosion circle to the nearest foot.

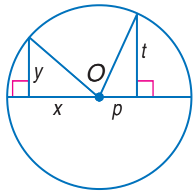
**H.O.T. Problems**

**60. REASONING** Circles that have the same center, but different radii, are called **concentric circles**. Use the figure at the right to find the exact circumference of each circle. List the circumferences in order from least to greatest.




**61. OPEN ENDED** Draw a circle with circumference between 8 and 12 centimeters. What is the radius of the circle? Explain.

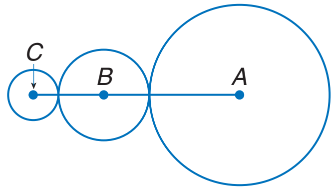
**62. CHALLENGE** In the figure, $O$ is the center of the circle, and $x^2 + y^2 + p^2 + t^2 = 288$. What is the exact circumference of $\odot O$?




**63. Which One Doesn't Belong?** A circle has diameter $d$, radius $r$, circumference $C$, and area $A$. Which ratio does *not* belong?

| $\frac{C}{2r}$ | $\frac{A}{r^2}$ | $\frac{C}{r}$ | $\frac{C}{d}$ |
|---|---|---|---|

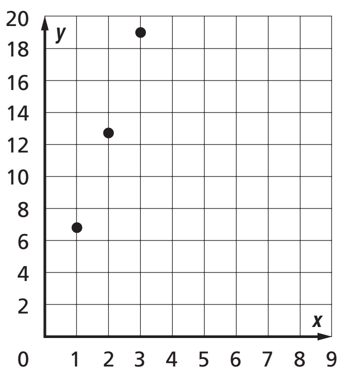
**64. CHALLENGE** In the figure, the radius of $\odot A$ is twice the radius of $\odot B$ and four times the radius of $\odot C$. If the sum of the circumferences of the three circles is $42\pi$, find the measure of $\overline{AC}$.




**65. *Writing in Math*** Use the information about carousels on page 554 to explain how far a carousel horse will travel in one rotation. Describe how the circumference of a circle relates to the distance traveled by the horse and whether a horse located one foot from the outside edge of the carousel travels a mile when it makes 22 rotations for each ride.

## A STANDARDIZED TEST PRACTICE

**66. REVIEW** Which of the following is *best* represented by the data in the graph?



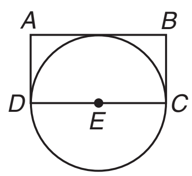

**A** comparing the length of a side of a square to the width of that square

**B** comparing the length of a side of a cube to the cube's surface area

**C** comparing a circle's radius to its circumference

**D** comparing a circle's radius to its diameter

**67.** Compare the circumference of $\odot E$ with the perimeter of rectangle $ABCD$. Which statement is true?




**F** The perimeter of $ABCD$ is greater than the circumference of circle $E$.

**G** The circumference of circle $E$ is greater than the perimeter of $ABCD$.

**H** The perimeter of $ABCD$ equals the circumference of circle $E$.

**J** There is not enough information to determine this comparison.

## Spiral Review

**Find the magnitude to the nearest tenth and direction to the nearest degree of each vector.** (Lesson 9-6)

**68.** $\overrightarrow{AB} = \langle 1, 4 \rangle$

**69.** $\vec{v} = \langle 4, 9 \rangle$

**70.** $\overrightarrow{AB}$ if $A(4, 2)$ and $B(7, 22)$

**71.** $\overrightarrow{CD}$ if $C(0, -20)$ and $D(40, 0)$

**Find the measure of the dilation image of $\overline{AB}$ for each scale factor $k$.** (Lesson 9-5)

**72.** $AB = 5, k = 6$

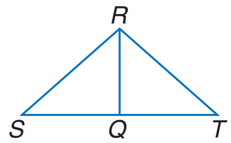
**73.** $AB = 16, k = 1.5$

**74.** $AB = \frac{2}{3}, k = -\frac{1}{2}$

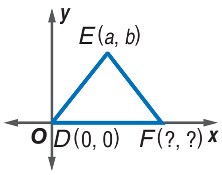
**75. PROOF** Write a two-column proof. (Lesson 5-2)

**Given:** $\overline{RQ}$ bisects $\angle SRT$.

**Prove:** $m\angle SQR > m\angle SRQ$




**76. COORDINATE GEOMETRY** Name the missing coordinates if $\triangle DEF$ is isosceles with vertex angle $E$. (Lesson 4-3)

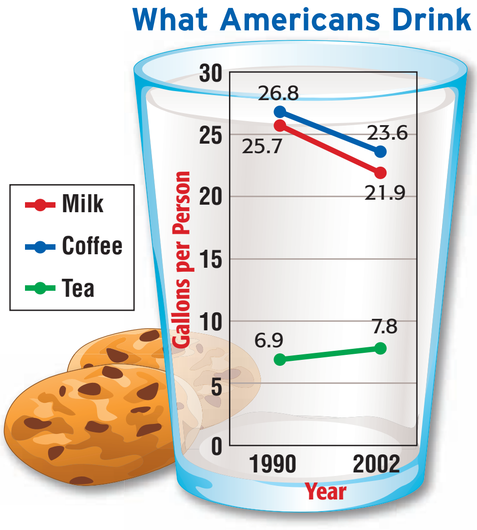



**POPULATION** For Exercises 77–79, refer to the graph. (Lesson 3-3)

**77.** Estimate the annual rate of change for gallons of tea consumed from 1990 to 2002.

**78.** If the trend continues in consumption of coffee, how many gallons of coffee will each American drink in 2010?

**79.** If the consumption of milk continues to decrease at the same rate, in what year will each American drink about 18 gallons of milk?

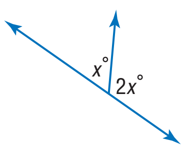



**Source:** *Statistical Abstract of the United States*

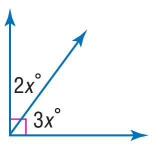
## GET READY for the Next Lesson

**PREREQUISITE SKILL** Find $x$. (Lesson 1-4)

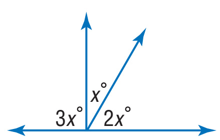
**80.**



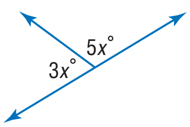

**81.**



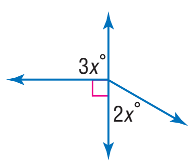

**82.**



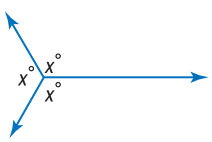

**83.**




**84.**




**85.**



# READING MATH

## Everyday Expressions Containing Math Words

Have you ever heard of someone "going around in circles"? Does the mathematical meaning of *circle* have anything to do with the meaning of the expression? Expressions used in everyday conversations often are related to the math meaning of the words they contain.

| Expression | Everyday Meaning | Math Meaning |
|---|---|---|
| an acute pain | sharp pain | an angle that measures less than 90 |
| the degree of her involvement | the measure or scope of an action | a unit of measure used in measuring angles and arcs |
| dietary supplements | something added to fulfill nutritional requirements | two supplementary angles have measures that have a sum of 180 |
| going to extremes | to the greatest possible extent | in $a:b = c:d$, the numbers $a$ and $d$ |

Notice that in all of these cases, the math meaning of the underlined word is related to the everyday meaning of the expression.

## Reading to Learn

**Describe the everyday meaning of each expression and how, if at all, it is related to the mathematical meaning of the word it contains.**

**1.** going around in circles

**2.** going off on a tangent

**3.** striking a chord

**4.** getting to the point

**5.** having no proof

**6.** How is the mathematical definition of *arc* related to an *arcade*?

**7.** **RESEARCH** Use the Internet or another resource to describe the everyday meaning and the related mathematical meaning of each term.

**a.** inscribed

**b.** intercepted

# 10-2 Measuring Angles and Arcs

## Main Ideas

- Recognize major arcs, minor arcs, semicircles, and central angles and their measures.
- Find arc length.

## New Vocabulary

central angle
arc
minor arc
major arc
semicircle

## GET READY for the Lesson

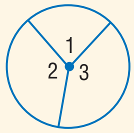
Most clocks on electronic devices are digital, showing the time as numerals. Analog clocks are often used in decorative furnishings and wrist watches. An analog clock has moving hands that indicate the hour, minute, and sometimes the second. This clock face is a circle. The three hands form three central angles of the circle.

**Angles and Arcs** In Chapter 1, you learned that a degree is $\frac{1}{360}$ of the circular rotation about a point. This means that the sum of the measures of the angles about the center of the clock above is 360. Each of the angles formed by the clock hands is called a central angle. A **central angle** has the center of the circle as its vertex, and its sides contain two radii of the circle.

### KEY CONCEPT — Sum of Central Angles

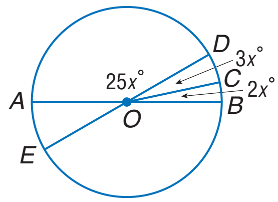
**Words** The sum of the measures of the central angles of a circle with no interior points in common is 360.

**Example:** $m\angle 1 + m\angle 2 + m\angle 3 = 360$




### EXAMPLE Measures of Central Angles

**1** **ALGEBRA** **Refer to $\odot O$.**




**Find $m\angle AOD$.**

$\angle AOD$ and $\angle DOB$ are a linear pair, and the angles of a linear pair are supplementary.

$m\angle AOD + m\angle DOB = 180$

$m\angle AOD + m\angle DOC + m\angle COB = 180$ Angle Sum Theorem

$25x + 3x + 2x = 180$ Substitution

$30x = 180$ Simplify.

$x = 6$ Divide each side by 30.

Use the value of $x$ to find $m\angle AOD$.

$m\angle AOD = 25x$ Given

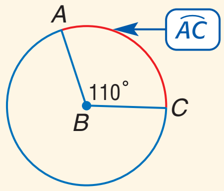
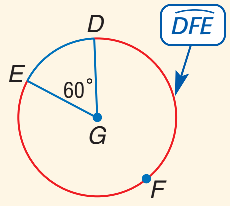
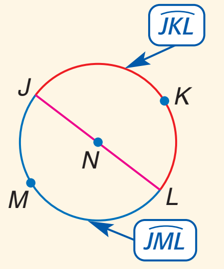
$= 25(6)$ or $150$ Substitution

**CHECK Your Progress**

**1.** Find $m\angle AOE$.

A central angle separates the circle into two parts, each of which is an **arc**. The measure of each arc is related to the measure of its central angle.

## KEY CONCEPT — Arcs of a Circle

| Type of Arc: | minor arc | major arc | semicircle |
|---|---|---|---|
| **Definition:** | an arc that measures less than 180° | an arc that measures greater than 180° | an arc that measures 180° |
| **Example:** |   |   |   |
| **Named:** | usually by the letters of the two endpoints $\widehat{AC}$ | by the letters of the two endpoints and another point on the arc $\widehat{DFE}$ | by the letters of the two endpoints and another point on the arc $\widehat{JML}$ and $\widehat{JKL}$ |
| **Arc Degree Measure Equals:** | the measure of the central angle $m\angle ABC = 110$, so $m\widehat{AC} = 110$ | 360 minus the measure of the minor arc with the same endpoints $m\widehat{DFE} = 360 - m\widehat{DE}$ $m\widehat{DFE} = 360 - 60$ or 300 | $360 \div 2$ or 180 $m\widehat{JML} = 180$ $m\widehat{JKL} = 180$ |

> **Study Tip**
>
> **Naming Arcs**
>
> Do not assume that because an arc is named by three letters that it is a semicircle or major arc. You can also correctly name a minor arc using three letters.

**Concepts in Motion**
**Animation**
geometryonline.com

Arcs with the same measure in the same circle or in congruent circles are congruent.

## THEOREM 10.1

In the same or in congruent circles, two arcs are congruent if and only if their corresponding central angles are congruent.

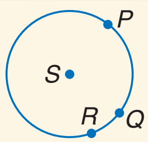
You will prove Theorem 10.1 in Exercise 50.

Arcs of a circle that have exactly one point in common are *adjacent arcs.* Like adjacent angles, the measures of adjacent arcs can be added.

## POSTULATE 10.1 — Arc Addition Postulate

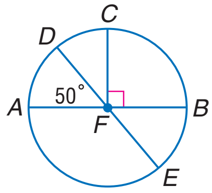
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

**Example:** In ⊙$S$, $m\widehat{PQ} + m\widehat{QR} = m\widehat{PQR}$.



## EXAMPLE Measures of Arcs

**2** **In $\odot F$, $m\angle DFA = 50$ and $\overline{CF} \perp \overline{FB}$. Find each measure.**




**a.** $m\widehat{BE}$

$\widehat{BE}$ is a minor arc, so $m\widehat{BE} = m\angle BFE$.

| | |
|---|---|
| $\angle BFE \cong \angle DFA$ | Vertical angles are congruent. |
| $m\angle BFE = m\angle DFA$ | Definition of congruent angles |
| $m\widehat{BE} = m\angle DFA$ | Transitive Property |
| $m\widehat{BE} = 50$ | Substitution |

**b.** $m\widehat{CBE}$

$\widehat{CBE}$ is composed of adjacent arcs, $\widehat{CB}$ and $\widehat{BE}$.

| | |
|---|---|
| $m\widehat{CB} = m\angle CFB$ | |
| $= 90$ | $\angle CFB$ is a right angle. |
| $m\widehat{CBE} = m\widehat{CB} + m\widehat{BE}$ | Arc Addition Postulate |
| $m\widehat{CBE} = 90 + 50$ or $140$ | Substitution |

**c.** $m\widehat{ACE}$

One way to find $m\widehat{ACE}$ is by using $\widehat{ACB}$ and $\widehat{BE}$. $\widehat{ACB}$ is a semicircle.

| | |
|---|---|
| $m\widehat{ACE} = m\widehat{ACB} + \widehat{BE}$ | Arc Addition Postulate |
| $m\widehat{ACE} = 180 + 50$ or $230$ | Substitution |

### CHECK Your Progress

**Find each measure.**

**2A.** $m\widehat{CD}$ **2B.** $m\widehat{DCB}$ **2C.** $m\widehat{CAE}$

Personal Tutor at geometryonline.com

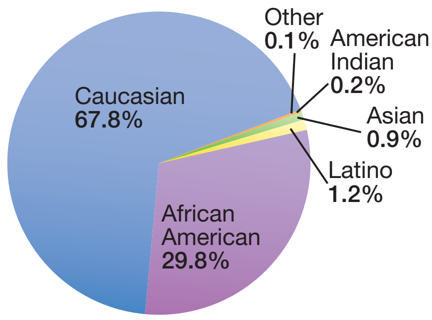
In a circle graph, the central angles divide a circle into wedges to represent data, often expressed as a percent. The size of the angle is proportional to the percent.

## Real-World EXAMPLE Circle Graphs

**3** **POPULATION** **Refer to the graphic. Find the measurement of the central angle for each category.** The sum of the percents is 100% and represents the whole. Use the percents to determine what part of the whole circle (360°) each central angle contains.

$$67.8\%(360°) = 244.08°$$
$$29.8\%(360°) = 107.28°$$
$$1.2\%(360°) = 4.32°$$
$$0.9\%(360°) = 3.24°$$
$$0.2\%(360°) = 0.72°$$
$$0.1\%(360°) = 0.36°$$

**South Carolina Population by Race**




**Source:** U.S. Census Bureau

**CHECK Your Progress**

**SPORTS** **Refer to the table, which shows the seven most popular sports for females by percentage of participation.**

**3A.** If you were to construct a circle graph of this information, how many degrees would be needed for each category?

**3B.** Do any of the sports have congruent arcs? Why or why not? What is the measurement of the arcs for volleyball and soccer together?

| Female Participation in Sports | |
|---|---|
| basketball | 20% |
| track & field | 18% |
| volleyball | 18% |
| softball (fast pitch) | 16% |
| soccer | 14% |
| cross country | 7% |
| tennis | 7% |

**Source:** NFHS

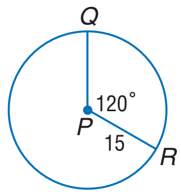
**Arc Length** Another way to measure an arc is by its length. An arc is part of the circle, so the length of an arc is a part of the circumference.

**Study Tip**

**Look Back**
To review **proportions**, see Lesson 7-1.

## EXAMPLE Arc Length

**4** **In $\odot P$, $PR = 15$ and $m\angle QPR = 120$. Find the length of $\widehat{QR}$.**




In $\odot P$, $r = 15$, so $C = 2\pi(15)$ or $30\pi$ and $m\widehat{QR} = m\angle QPR$ or 120. Write a proportion to compare each part to its whole.

degree measure of arc → $\frac{120}{360} = \frac{\ell}{30\pi}$ ← arc length
degree measure of whole circle → ← circumference

Now solve the proportion for $\ell$.

$\frac{120}{360} = \frac{\ell}{30\pi}$

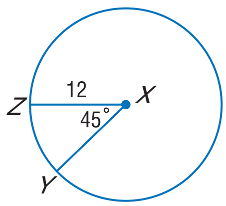
$\frac{120}{360}(30\pi) = \ell$ Multiply each side by $30\pi$.

$10\pi = \ell$ Simplify.

The length of $\widehat{QR}$ is $10\pi$ units or about 31.42 units.

**CHECK Your Progress**

**4.** Find the length of $\widehat{ZY}$.




The proportion used to find the arc length in Example 4 can be adapted to find the arc length in any circle.

**KEY CONCEPT** *Arc Length*

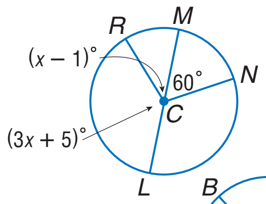
degree measure of arc → $\frac{A}{360} = \frac{\ell}{2\pi r}$ ← arc length
degree measure of whole circle → ← circumference

This can also be expressed as $\frac{A}{360} \cdot C = \ell$.

## CHECK Your Understanding

Example 1 (p. 563)

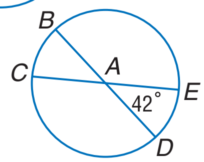
**ALGEBRA** **Find each measure.**




1. $m\angle NCL$
2. $m\angle RCL$
3. $m\angle RCM$
4. $m\angle RCN$

Example 2 (p. 565)

**In $\odot A$, $m\angle EAD = 42$. Find each measure.**

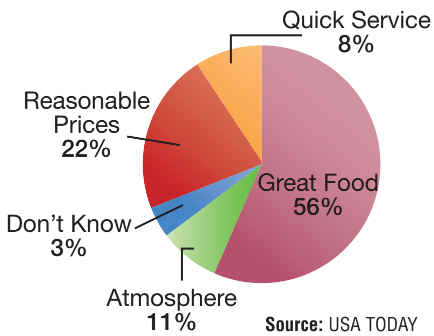



5. $m\widehat{BC}$
6. $m\widehat{CBE}$
7. $m\widehat{EDB}$
8. $m\widehat{CD}$

Example 3 (p. 565)

9. **RESTAURANTS** The graph shows the results of a survey taken by diners relating what is most important about the restaurants where they eat. Determine the measurement of each angle of the graph. Round to the nearest degree.

**What Diners Want**




**Source:** USA TODAY

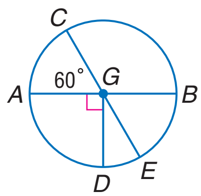
Example 4 (p. 566)

10. Points $T$ and $R$ lie on $\odot W$ so that $WR = 12$ and $m\angle TWR = 60$. Find the length of $\widehat{TR}$.

## Exercises

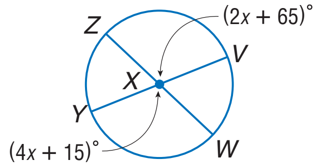
| HOMEWORK HELP | |
|---|---|
| For Exercises | See Examples |
| 11–20 | 1 |
| 21–28 | 2 |
| 29–31 | 3 |
| 32–35 | 4 |

**Find each measure.**



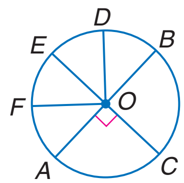

11. $m\angle CGB$
12. $m\angle BGE$
13. $m\angle AGD$
14. $m\angle DGE$
15. $m\angle CGD$
16. $m\angle AGE$

**ALGEBRA** **Find each measure.**




17. $m\angle ZXV$
18. $m\angle YXW$
19. $m\angle ZXY$
20. $m\angle VXW$

**In $\odot O$, $\overline{EC}$ and $\overline{AB}$ are diameters, and $\angle BOD \cong \angle DOE \cong \angle EOF \cong \angle FOA$. Find each measure.**




21. $m\widehat{BC}$
22. $m\widehat{AC}$
23. $m\widehat{AE}$
24. $m\widehat{EB}$
25. $m\widehat{ACB}$
26. $m\widehat{AD}$
27. $m\widehat{CBF}$
28. $m\widehat{ADC}$

**FOOD** **For Exercises 29–31, refer to the table and use the following information.**

A recent survey asked Americans how long food could be on the floor and still be safe to eat. The results are shown in the table.

**Dropped Food**

| Do you eat food dropped on the floor? | |
|---|---|
| Not safe to eat | 78% |
| Three-second rule* | 10% |
| Five-second rule* | 8% |
| Ten-second rule* | 4% |

**Source:** American Diabetic Association

* The length of time the food is on the floor.

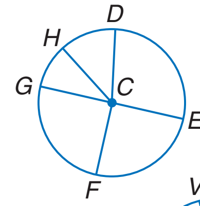
29. If you were to construct a circle graph of this information, how many degrees would be needed for each category?
30. Describe the kind of arc associated with each category.
31. Construct a circle graph for these data.

**Real-World Link**

In the Great Plains of the United States, farmers use center-pivot irrigation systems to water crops. New low-energy spray systems water circles of land that are thousands of feet in diameter with minimal water loss to evaporation from the spray.

**Source:** U.S. Geological Survey

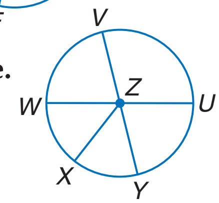
**The diameter of ⊙$C$ is 32 units long. Find the length of each arc for the given angle measure.**




**32.** $\overset{\frown}{DE}$ if $m\angle DE = 100$ **33.** $\overset{\frown}{DHE}$ if $m\angle DCE = 90$

**34.** $\overset{\frown}{HDF}$ if $m\angle HCF = 125$ **35.** $\overset{\frown}{HD}$ if $m\angle DCH = 45$

**ALGEBRA In ⊙$Z$, $\angle WZX \cong \angle XZY$, $m\angle VZU = 4x$, $m\angle UZY = 2x + 24$, and $\overline{VY}$ and $\overline{WU}$ are diameters. Find each measure.**




**36.** $m\overset{\frown}{UY}$ **37.** $m\overset{\frown}{WV}$ **38.** $m\overset{\frown}{WX}$ **39.** $m\overset{\frown}{XY}$

**40.** $m\overset{\frown}{WUY}$ **41.** $m\overset{\frown}{YVW}$ **42.** $m\overset{\frown}{XVY}$ **43.** $m\overset{\frown}{WUX}$

**Determine whether each statement is *sometimes, always,* or *never* true.**

**44.** The measure of a major arc is greater than 180.

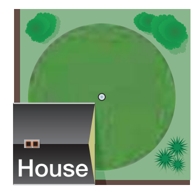
**45.** The central angle of a minor arc is an acute angle.

**46.** The sum of the measures of the central angles of a circle depends on the measure of the radius.

**47.** The semicircles of two congruent circles are congruent.

**48. CLOCKS** The hands of a clock form the same angle at various times of the day. For example, the angle formed at 2:00 is congruent to the angle formed at 10:00. If a clock has a diameter of 1 foot, what is the distance along the edge of the clock from the minute hand to the hour hand at 2:00?

**49. IRRIGATION** Some irrigation systems spray water in a circular pattern. You can adjust the nozzle to spray in certain directions. The nozzle in the diagram is set so it does not spray on the house. If the spray has a radius of 12 feet, what is the approximate length of the arc that the spray creates?




**EXTRA PRACTICE**

See pages 819, 837.

**Math Online**

Self-Check Quiz at geometryonline.com

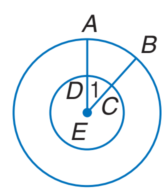
**H.O.T. Problems**

**50. PROOF** Write a proof of Theorem 10.1.

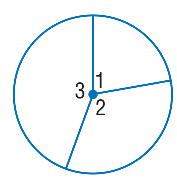
**51. REASONING** Compare and contrast *concentric* circles and *congruent* circles.

**52. OPEN ENDED** Draw a circle and locate three points on the circle. Name all of the arcs determined by the three points and use a protractor to find the measure of each arc.

**53. CHALLENGE** The circles at the right are concentric circles that both have point $E$ as their center. If $m\angle 1 = 42$, determine whether $\overset{\frown}{AB} \cong \overset{\frown}{CD}$. Explain.



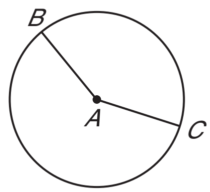

**54. CHALLENGE** Central angles 1, 2, and 3 have measures in the ratio 2:3:4. Find the measure of each angle.




**55. *Writing in Math*** Use the information about clocks on page 563 to explain what kinds of angles the hands on a clock form. Include the kind of angle formed by the hands of a clock, and describe several times of day when these angles are congruent.

## STANDARDIZED TEST PRACTICE

**56.** In the figure, $\overline{AB}$ is a radius of circle $A$, and $\widehat{BC}$ is a minor arc.




If $AB = 5$ inches and the length of $\widehat{BC}$ is $4\pi$ inches, what is $m\angle BAC$?

**A** 150° **C** 120°

**B** 144° **D** 72°

**57. REVIEW** Lupe received a 4% raise at her job. If she was earning $x$ dollars before, which expression represents how much money she is earning now?

**F** $x + 0.4x$

**G** $x + 0.4$

**H** $x + 0.04x$

**J** $x + 0.04$

## Spiral Review

**The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.** (Lesson 10-1)

**58.** $r = 10$, $d =$ __?__, $C =$ __?__

**59.** $d = 13$, $r =$ __?__, $C =$ __?__

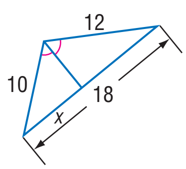
**60.** $C = 28\pi$, $d =$ __?__, $r =$ __?__

**61.** $C = 75.4$, $d =$ __?__, $r =$ __?__

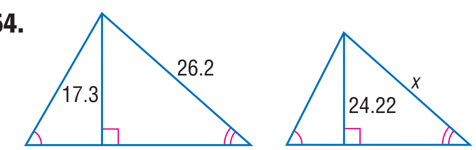
**62. SOCCER** Two soccer players kick the ball at the same time. One exerts a force of 72 newtons east. The other exerts a force of 45 newtons north. What is the magnitude to the nearest tenth and direction to the nearest degree of the resultant force on the soccer ball? (Lesson 9-6)

**ALGEBRA** **Find $x$.** (Lesson 7-5)

**63.**




**64.**




**Find the exact distance between each point and line or pair of lines.** (Lesson 3-6)

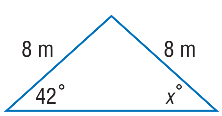
**65.** point $Q(6, -2)$ and the line with equation $y - 7 = 0$

**66.** parallel lines with equations $y = x + 3$ and $y = x - 4$

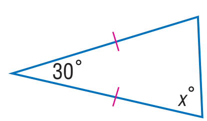
## GET READY for the Next Lesson

**PREREQUISITE SKILL** **Find $x$.** (Lesson 4-6)

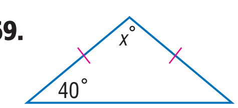
**67.**



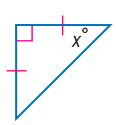

**68.**



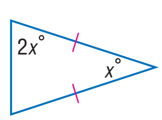

**69.**



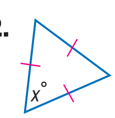

**70.**




**71.**




**72.**



# 10-3 Arcs and Chords

## Main Ideas

- Recognize and use relationships between arcs and chords.
- Recognize and use relationships between chords and diameters.

## New Vocabulary

inscribed
circumscribed

## GET READY for the Lesson

Waffle irons have grooves in each heated plate that result in the waffle pattern when the batter is cooked. One model of a Belgian waffle iron is round, and each groove is a chord of the circle.


**Arcs and Chords** The endpoints of a chord are also endpoints of an arc. If you trace the waffle pattern on patty paper and fold along the diameter, $\overline{AB}$ and $\overline{CD}$ match exactly, as well as $\widehat{AB}$ and $\widehat{CD}$. This suggests the following theorem.

## Reading Math

**If and only if**
Remember that the phrase *if and only if* means that the conclusion and the hypothesis can be switched and the statement is still true.

### THEOREM 10.2

In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

**Abbreviations:**

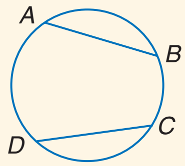
*In ⊙, 2 minor arcs are ≅, corr. chords are ≅.*

*In ⊙, 2 chords are ≅, corr. minor arcs are ≅.*

**Examples:**

If $\overline{AB} \cong \overline{CD}$, $\widehat{AB} \cong \widehat{CD}$.

If $\widehat{AB} \cong \widehat{CD}$, $\overline{AB} \cong \overline{CD}$.



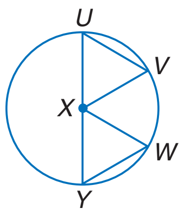

You will prove part 2 of Theorem 10.2 in Exercise 1.

## EXAMPLE Prove Theorem 10.2

### 1 Theorem 10.2 (part 1)

**Given:** $\odot X$, $\widehat{UV} \cong \widehat{YW}$

**Prove:** $\overline{UV} \cong \overline{YW}$



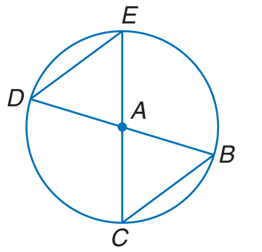

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\odot X$, $\widehat{UV} \cong \widehat{YW}$ | 1. Given |
| 2. $\angle UXV \cong \angle WXY$ | 2. If arcs are ≅, their corresponding central ∠s are ≅. |
| 3. $\overline{UX} \cong \overline{XV} \cong \overline{WX} \cong \overline{XY}$ | 3. All radii of a circle are congruent. |
| 4. $\triangle UXV \cong \triangle WXY$ | 4. SAS |
| 5. $\overline{UV} \cong \overline{YW}$ | 5. CPCTC |

## CHECK Your Progress

1. **Given:** $\odot A$, $\widehat{BC} \cong \widehat{DE}$

**Prove:** $\overline{BC} \cong \overline{DE}$

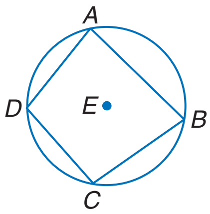



### Study Tip

**Circumcircle**

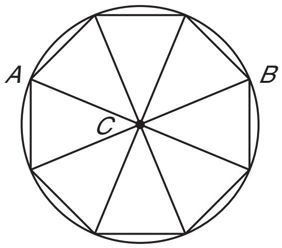
The *circumcircle* of a polygon is a circle that passes through all of the vertices of a polygon.

The chords of adjacent arcs can form a polygon. Quadrilateral *ABCD* is an **inscribed** polygon because all of its vertices lie on the circle. Circle *E* is **circumscribed** about the polygon because it contains all the vertices of the polygon.




## STANDARDIZED TEST EXAMPLE

**2** A regular octagon is inscribed in a circle as part of a stained glass art piece. If opposite vertices are connected by line segments, what is the measure of angle *ACB*?




**A** 108 **C** 135

**B** 120 **D** 150

All the central angles of a regular polygon are congruent. The measure of each angle of a rectangular octagon is 360 ÷ 8 or 45. Angle *ACB* is made up of three central angles, so its measure is 3(45) or 135. The correct answer is C.

**Real-World Link**

The Pentagon, in Washington, D.C., houses the Department of Defense. Construction was finished on January 15, 1943. About 23,000 employees work at the Pentagon.

**Source:** defenselink.mil

## CHECK Your Progress

2. A circle is circumscribed about a regular pentagon. What is the measure of the arc between each pair of consecutive vertices?

**F** 60

**G** 72

**H** 36

**J** 30

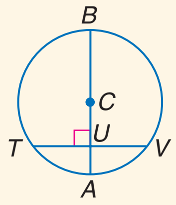
Online **Personal Tutor at** geometryonline.com

**Diameters and Chords** Diameters that are perpendicular to chords create special segment and arc relationships. Suppose you draw circle *C* and one of its chords $\overline{WX}$ on a piece of patty paper and fold the paper to construct the perpendicular bisector. You will find that the bisector also cuts $\widehat{WX}$ in half and passes through the center of the circle, making it contain a diameter.

## THEOREM 10.3

In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.

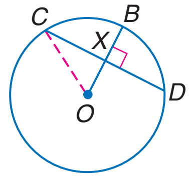
**Example:** If $\overline{BA} \perp \overline{TV}$, then $\overline{UT} \cong \overline{UV}$ and $\widehat{AT} \cong \widehat{AV}$.




You will prove Theorem 10.3 in Exercise 8.

## EXAMPLE 3 Radius Perpendicular to a Chord

**Circle $O$ has a radius of 13 inches. Radius $\overline{OB}$ is perpendicular to chord $\overline{CD}$, which is 24 inches long.**




**a. If $m\widehat{CD} = 134$, find $m\widehat{CB}$.**

$\overline{OB}$ bisects $\widehat{CD}$, so $m\widehat{CB} = \frac{1}{2}m\widehat{CD}$.

$m\widehat{CB} = \frac{1}{2}m\widehat{CD}$ — Definition of arc bisector

$= \frac{1}{2}(134)$ or 67 — $m\widehat{CD} = 134$

**b. Find $OX$.**

Draw radius $\overline{OC}$. $\triangle CXO$ is a right triangle.

$CO = 13$ — $r = 13$

$\overline{OB}$ bisects $\overline{CD}$. — A radius perpendicular to a chord bisects it.

$CX = \frac{1}{2}(CD)$ — Definition of segment bisector

$= \frac{1}{2}(24)$ or 12 — $CD = 24$

Use the Pythagorean Theorem to find $OX$.

$(CX)^2 + (OX)^2 = (CO)^2$ — Pythagorean Theorem

$12^2 + (OX)^2 = 13^2$ — $CX = 12$, $CO = 13$

$144 + (OX)^2 = 169$ — Simplify.

$(OX)^2 = 25$ — Subtract 144 from each side.

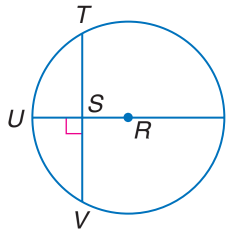
$OX = 5$ — Take the square root of each side.

### CHECK Your Progress

**Circle $R$ has a radius of 16 centimeters. Radius $\overline{RU}$ is perpendicular to chord $\overline{TV}$, which is 22 centimeters long.**

**3A.** If $m\widehat{TV} = 110$, find $m\widehat{UV}$.

**3B.** Find $RS$.



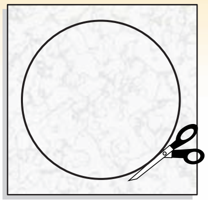

In the next lab, you will discover another property of congruent chords.

# GEOMETRY LAB

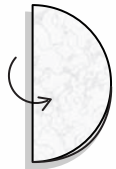
## Congruent Chords and Distance

### MODEL

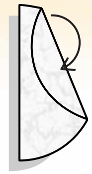
**Step 1** Use a compass to draw a large circle on patty paper. Cut out the circle.



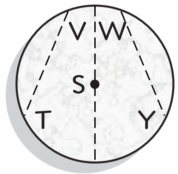
**Step 2** Fold the circle in half.



**Step 3** Without opening the circle, fold the edge of the circle so it does not intersect the first fold.



**Step 4** Unfold the circle and label as shown.




**Step 5** Fold the circle, laying point $V$ onto $T$ to bisect the chord. Open the circle and fold again to bisect $\overline{WY}$. Label the intersection points $U$ and X as shown.


### ANALYZE

**1.** What is the relationship between $\overline{SU}$ and $\overline{VT}$? $\overline{SX}$ and $\overline{WY}$?

**2.** Use a centimeter ruler to measure $\overline{VT}$, $\overline{WY}$, $\overline{SU}$, and $\overline{SX}$. What do you find?

**3. Make a conjecture** about the distance that two chords are from the center when they are congruent.

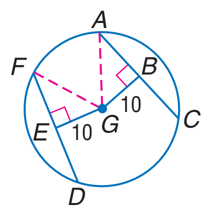
## THEOREM 10.4

In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

You will prove Theorem 10.4 in Exercises 34 and 35.

## EXAMPLE Chords Equidistant from Center

**4** **Chords $\overline{AC}$ and $\overline{DF}$ are equidistant from the center. If the radius of $\odot G$ is 26, find $AC$ and $DE$.**




$\overline{AC}$ and $\overline{DF}$ are equidistant from $G$, so $\overline{AC} \cong \overline{DF}$.

Draw $\overline{AG}$ and $\overline{GF}$ to form two right triangles.

$(AB)^2 + (BG)^2 = (AG)^2$ Pythagorean Theorem

$(AB)^2 + 10^2 = 26^2$ $BG = 10, AG = 26$

$(AB)^2 + 100 = 676$ Simplify.

$(AB)^2 = 576$ Subtract 100 from each side.

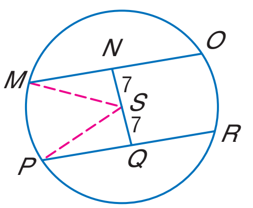
$AB = 24$ Take the square root of each side.

$AB = \frac{1}{2}(AC)$, so $AC = 2(24)$ or 48.

$\overline{AC} \cong \overline{DF}$, so $DF$ also equals 48. $DE = \frac{1}{2}DF$, so $DE = \frac{1}{2}(48)$ or 24.

## CHECK Your Progress

4. Chords $\overline{MO}$ and $\overline{PR}$ are equidistant from the center. If the radius of $\odot S$ is 15, find $MO$ and $PQ$.



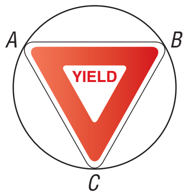

## CHECK Your Understanding

**Example 1 (p. 570)**

1. **PROOF** (Part 2 of Theorem 10.2) Given $\odot X$ and $\overline{UV} \cong \overline{YW}$, prove $\widehat{UV} \cong \widehat{YW}$. (Use the figure from part 1 of Theorem 10.2.)

**Example 2 (p. 571)**

2. **STANDARDIZED TEST PRACTICE** A yield sign, an equilateral triangle, isinscribed in a circle. What is the measure of $\widehat{ABC}$?



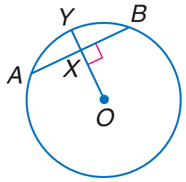

**A** 60 **C** 180

**B** 120 **D** 240

**Example 3 (p. 572)**

**Circle $O$ has a radius of 10, $AB = 10$, and $m\widehat{AB} = 60$. Find each measure.**

3. $m\widehat{AY}$ 4. $AX$ 5. $OX$

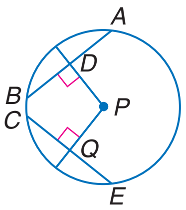



Exercises 3–5

**Example 4 (p. 573)**

**In $\odot P$, $PD = 10$, $PQ = 10$, and $QE = 20$. Find each measure.**

6. $AB$ 7. $PE$




Exercises 6–7

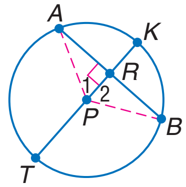
## Exercises

| HOMEWORK HELP | |
|---|---|
| For Exercises | See Examples |
| 8 | 1 |
| 9–11 | 2 |
| 12–19 | 3 |
| 20–27 | 4 |

8. **PROOF** **Write a two-column proof of Theorem 10.3.**

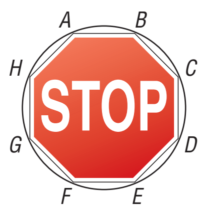
**Given:** $\odot P$, $\overline{AB} \perp \overline{TK}$

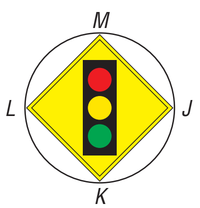
**Prove:** $\overline{AR} \cong \overline{BR}$, $\widehat{AK} \cong \widehat{BK}$

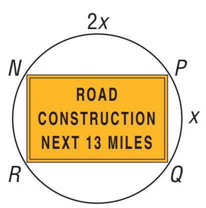



**TRAFFIC SIGNS** **Determine the measure for each arc of the circle circumscribed about the traffic sign.**

9. regular octagon 10. square 11. rectangle



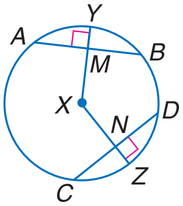

**In $\odot X$, $AB = 30$, $CD = 30$, and $m\widehat{CZ} = 40$. Find each measure.**

12. $AM$ 13. $MB$

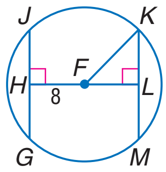
14. $CN$ 15. $ND$

16. $m\widehat{DZ}$ 17. $m\widehat{CD}$

18. $m\widehat{AB}$ 19. $m\widehat{YB}$



**In ⊙*F*, $\overline{FH} \cong \overline{FL}$ and *FK* = 17. Find each measure.**



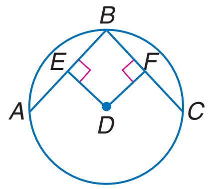

**20.** *LK*

**21.** *KM*

**22.** *JG*

**23.** *JH*

**In ⊙*D*, *CF* = 8, *DE* = *FD*, and *DC* = 10. Find each measure.**



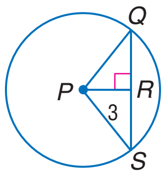

**24.** *FB*

**25.** *BC*

**26.** *AB*

**27.** *ED*

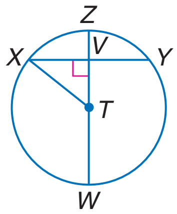
**The radius of ⊙*P* is 5 and *PR* = 3. Find each measure.**




**28.** *QR*

**29.** *QS*

**In ⊙*T*, *ZV* = 1, and *TW* = 13. Find each measure.**



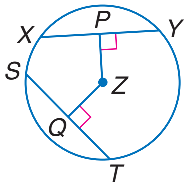

**30.** *XV*

**31.** *XY*

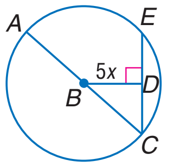
**Real-World Link**

Many everyday sayings or expressions have a historical origin. The square peg comment is attributed to Sydney Smith (1771–1845), a witty British lecturer, who said "Trying to get those two together is like trying to put a square peg in a round hole."

**32. ALGEBRA** In ⊙*Z*, *PZ* = *ZQ*, *XY* = 4*a* − 5, and *ST* = −5*a* + 13. Find *SQ*.




**33. ALGEBRA** In ⊙*B*, the diameter is 20 units long, and *m*∠*ACE* = 45. Find *x*.

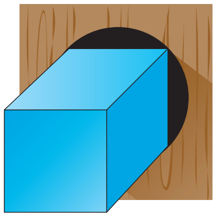



**PROOF** **Write a proof for each part of Theorem 10.4.**

**34.** In a circle, if two chords are equidistant from the center, then they are congruent.

**35.** In a circle, if two chords are congruent, then they are equidistant from the center.

**36. SAYINGS** An old adage states that "You can't fit a square peg in a round hole." Actually, you can; it just won't fill the hole. If a hole is 4 inches in diameter, what is the approximate width of the largest square peg that fits in the round hole?



**For Exercises 37–39, draw and label a figure. Then solve.**

**37.** The radius of a circle is 34 meters long, and a chord of the circle is 60 meters long. How far is the chord from the center of the circle?

**38.** The diameter of a circle is 60 inches, and a chord of the circle is 48 inches long. How far is the chord from the center of the circle?

**39.** A chord of a circle is 48 centimeters long and is 10 centimeters from the center of the circle. Find the radius.

## Study Tip

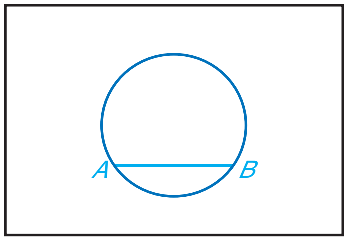
**Finding the Center of a Circle**

The process Mr. Ortega used can be done by construction and is often called *locating the center of a circle.*

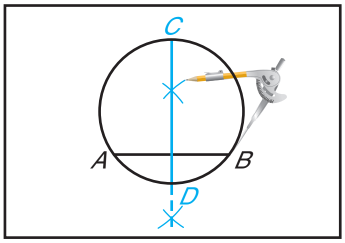
**40. CARPENTRY** Mr. Ortega wants to drill a hole in the center of a round picnic table for an umbrella pole. To locate the center of the circle, he draws two chords of the circle and uses a ruler to find the midpoint for each chord. Then he uses a framing square to draw a line perpendicular to each chord at its midpoint. Explain how this process locates the center of the tabletop.

**CONSTRUCTION** **Consider the following construction for Exercises 41–43.**

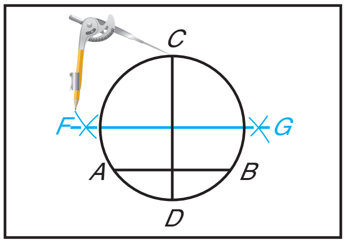
**Step 1** Trace the bottom of a circular object and draw a chord $\overline{AB}$.



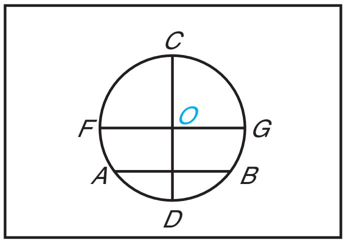

**Step 2** Construct the perpendicular bisector of $\overline{AB}$. Label it $\overline{CD}$.




**Step 3** Construct the perpendicular bisector of $\overline{CD}$. Label it $\overline{FG}$.




**Step 4** Label the point of intersection of the two perpendicular bisectors *O*.



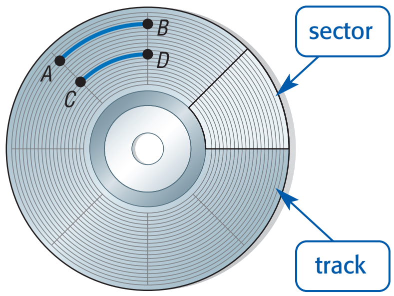

**41.** Repeat this construction using a different circular object.

**42.** Use an indirect proof to show that $\overline{CD}$ passes through the center of the circle by assuming that the center of the circle is *not* on $\overline{CD}$.

**43.** Prove that *O* is the center of the circle.

**COMPUTERS** **For Exercises 44 and 45, use the following information.**

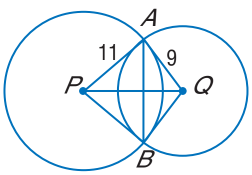
A computer hard drive contains platters divided into tracks, which are defined by concentric circles, and sectors, defined by radii of the circles.




**44.** In the diagram at the right, what is the relationship between $m\widehat{AB}$ and $m\widehat{CD}$?

**45.** Are $\widehat{AB}$ and $\widehat{CD}$ congruent? Explain.

**46.** The common chord $\overline{AB}$ between $\odot P$ and $\odot Q$ is perpendicular to the segment connecting the centers of the circles. If $AB = 10$, what is the length of $\overline{PQ}$? Explain your reasoning.




**EXTRA PRACTICE**
See pages 820, 837.

**Math Online**
Self-Check Quiz at geometryonline.com

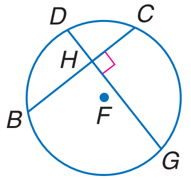


Aaron Haupt

## H.O.T. Problems

**47. OPEN ENDED** Construct a circle and inscribe any polygon. Draw the radii and use a protractor to determine whether any sides of the polygon are congruent. Describe a situation in which this would be important.

**48. FIND THE ERROR** Lucinda and Tokei are writing conclusions about the chords in ⊙$F$. Who is correct? Explain your reasoning.




Lucinda
Because $\overline{DG} \perp \overline{BC}$, $\angle DHB \cong \angle DHC \cong \angle CHG \cong \angle BHG$, and $\overline{DG}$ bisects $\overline{BC}$.

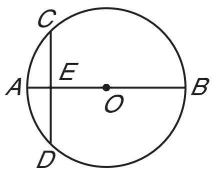
Tokei
$\overline{DG} \perp \overline{BC}$, but $\overline{DG}$ does not bisect $\overline{BC}$ because it is not a diameter.

**49. CHALLENGE** A diameter of ⊙$P$ has endpoints $A$ and $B$. Radius $\overline{PQ} \perp \overline{AB}$. Chord $\overline{DE}$ bisects $\overline{PQ}$ and is parallel to $\overline{AB}$. Does $DE = \frac{1}{2}(AB)$? Explain.

**50. *Writing in Math*** Refer to the information about Belgian waffles on page 570. Explain how the grooves in a Belgian waffle iron model segments in a circle. Include a description of how you might find the length of a groove without directly measuring it.

## STANDARDIZED TEST PRACTICE

**51.** $\overline{AB}$ is a diameter of circle $O$ and intersects chord $\overline{CD}$ at point $E$.




If $AE = 2$ and $OB = 10$, what is the length of $\overline{CD}$?

**A** 4  **C** 8
**B** 6  **D** 9

**52. REVIEW**
Solve: $-4y > 18 - 2(y + 8)$
Step 1: $-4y > 18 - 2y - 16$
Step 2: $-4y > -2y + 2$
Step 3: $-2y > 2$
Step 4: $y > -1$

Which is the first *incorrect* step in the solution shown above?

**F** Step 1  **H** Step 3
**G** Step 2  **J** Step 4

## Spiral Review

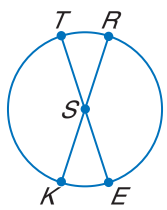
**In ⊙$S$, $m\angle TSR = 42$. Find each measure.** (Lesson 10-2)

**53.** $m\widehat{KT}$  **54.** $\widehat{ERT}$  **55.** $\widehat{KRT}$

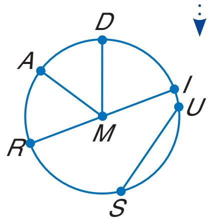
**Refer to ⊙$M$.** (Lesson 10-1)

**56.** Name a chord that is not a diameter.

**57.** If $MD = 7$, find $RI$.




Exercises 53–55




Exercises 56–57

## GET READY for the Next Lesson

**PREREQUISITE SKILL** **Solve each equation.** (Pages 781 and 782)

**58.** $\frac{1}{2}x = 120$  **59.** $2x = \frac{1}{2}(45 + 35)$  **60.** $3x = \frac{1}{2}(120 - 60)$  **61.** $45 = \frac{1}{2}(4x + 30)$

# 10-4 Inscribed Angles

## Main Ideas

- Find measures of inscribed angles.
- Find measures of angles of inscribed polygons.

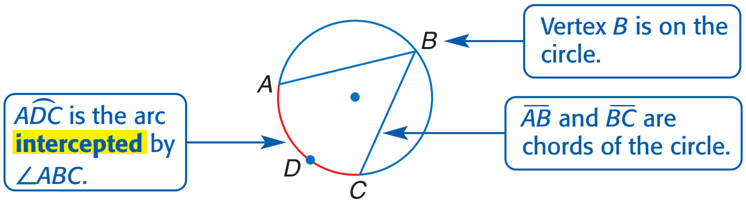
## New Vocabulary

intercepted

## GET READY for the Lesson

A socket is a tool that comes in varying diameters. It is used to tighten or unscrew nuts or bolts. The "hole" in the socket is a hexagon cast in a metal cylinder.

**Inscribed Angles** In Lesson 10-3, you learned that a polygon that has its vertices on a circle is called an inscribed polygon. Likewise, an *inscribed angle* is an angle that has its vertex on the circle and its sides contained in chords of the circle.

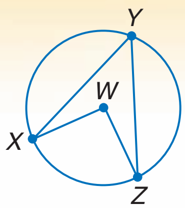



## GEOMETRY LAB

### Measure of Inscribed Angles

**MODEL**

- Use a compass to draw $\odot W$.
- Draw an inscribed angle and label it $XYZ$.
- Draw $\overline{WX}$ and $\overline{WZ}$.



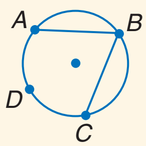

**ANALYZE**

1. Measure $\angle XYZ$ and $\angle XWZ$.
2. Find $m\overset{\frown}{XZ}$ and compare it with $m\angle XYZ$.
3. **Make a conjecture** about the relationship of the measure of an inscribed angle and the measure of its intercepted arc.

**THEOREM 10.5** *Inscribed Angle Theorem*

If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle).

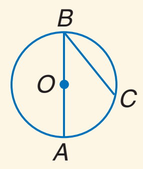
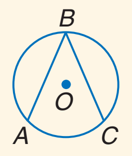
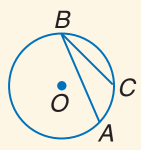
**Example:** $m\angle ABC = \frac{1}{2}(m\overset{\frown}{ADC})$ or $2(m\angle ABC) = m\overset{\frown}{ADC}$



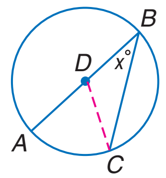



Aaron Haupt

To prove Theorem 10.5, you must consider three cases.

| | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| **Model of Angle Inscribed in ⊙*O*** |  |  |  |
| **Location of center of circle** | on a side of the angle | in the interior of the angle | in the exterior of the angle |

## PROOF Theorem 10.5 (Case 1)




**Given:** $\angle ABC$ inscribed in $\odot D$ and $\overline{AB}$ is a diameter.

**Prove:** $m\angle ABC = \frac{1}{2}m\widehat{AC}$

Draw $\overline{DC}$ and let $m\angle B = x$.

**Proof:**

Since $\overline{DB}$ and $\overline{DC}$ are congruent radii, $\triangle BDC$ is isosceles and $\angle B \cong \angle C$. Thus, $m\angle B = m\angle C = x$. By the Exterior Angle Theorem, $m\angle ADC = m\angle B + m\angle C$. So $m\angle ADC = 2x$. From the definition of arc measure, we know that $m\widehat{AC} = m\angle ADC$ or $2x$. Comparing $m\widehat{AC}$ and $m\angle ABC$, we see that $m\widehat{AC} = 2(m\angle ABC)$ or that $m\angle ABC = \frac{1}{2}m\widehat{AC}$.

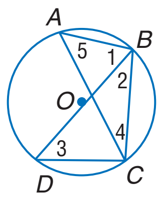
You will prove Cases 2 and 3 of Theorem 10.5 in Exercises 33 and 34.

## EXAMPLE Measures of Inscribed Angles

**Study Tip**

**Using Variables**

You can also assign a variable to an unknown measure. So, if you let $m\widehat{AD} = x$, the second equation becomes $140 + 100 + x + x = 360$, or $240 + 2x = 360$. This last equation may seem simpler to solve.




**1** **In $\odot O$, $m\widehat{AB} = 140$, $m\widehat{BC} = 100$, and $m\widehat{AD} = m\widehat{DC}$. Find the measures of $\angle 1$, $\angle 2$, and $\angle 3$.**

First determine $m\widehat{DC}$ and $m\widehat{AD}$.

| | |
|---|---|
| $m\widehat{AB} + m\widehat{BC} + m\widehat{DC} + m\widehat{AD} = 360$ | Arc Addition Theorem |
| $140 + 100 + m\widehat{DC} + m\widehat{DC} = 360$ | $m\widehat{AB} = 140$, $m\widehat{BC} = 100$, $m\widehat{DC} = m\widehat{AD}$ |
| $240 + 2(m\widehat{DC}) = 360$ | Simplify. |
| $2(m\widehat{DC}) = 120$ | Subtract 240 from each side. |
| $m\widehat{DC} = 60$ | Divide each side by 2. |

So, $m\widehat{DC} = 60$ and $m\widehat{AD} = 60$.

$m\angle 1 = \frac{1}{2}m\widehat{AD}$
$= \frac{1}{2}(60)$ or 30

$m\angle 2 = \frac{1}{2}m\widehat{DC}$
$= \frac{1}{2}(60)$ or 30

$m\angle 3 = \frac{1}{2}m\widehat{BC}$
$= \frac{1}{2}(100)$ or 50

**CHECK Your Progress**

**1A.** Find $m\angle 4$.

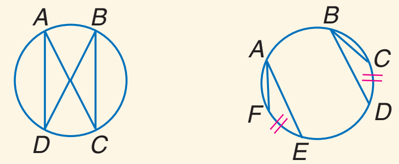
**1B.** Find $m\angle 5$.

In Example 1, note that ∠3 and ∠5 intercept the same arc and are congruent.

**THEOREM 10.6**

If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.

**Abbreviations:**
*Inscribed ∠s of ≅ arcs are ≅.*
*Inscribed ∠s of same arc are ≅.*




∠DAC ≅ ∠DBC    ∠FAE ≅ ∠CBD

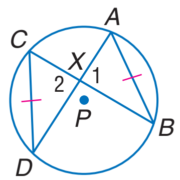
You will prove Theorem 10.6 in Exercise 35.

## EXAMPLE 2 Proof with Inscribed Angles

**Given:** ⊙P with $\overline{CD} \cong \overline{AB}$

**Prove:** $\triangle AXB \cong \triangle CXD$

**Proof:**

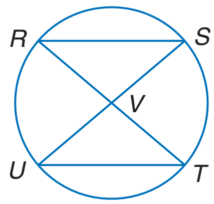



| Statements | Reasons |
|---|---|
| **1.** $\angle DAB$ intercepts $\widehat{DB}$. $\angle DCB$ intercepts $\widehat{DB}$. | **1.** Definition of intercepted arc |
| **2.** $\angle DAB \cong \angle DCB$ | **2.** Inscribed ∠s of same arc are ≅. |
| **3.** $\angle 1 \cong \angle 2$ | **3.** Vertical ∠s are ≅. |
| **4.** $\overline{CD} \cong \overline{AB}$ | **4.** Given |
| **5.** $\triangle AXB \cong \triangle CXD$ | **5.** AAS |

**CHECK Your Progress**

**2. Given:** $\overline{RT}$ bisects $\overline{SU}$; $\overline{RV} \cong \overline{SV}$

**Prove:** $\triangle RVS \cong \triangle UVT$




**Study Tip**

**Eliminate the Possibilities**

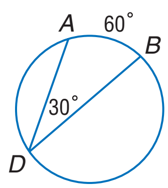
Think about what would be true if *D* was on minor arc $\widehat{AB}$. Then $\angle ADB$ would intercept the major arc. Thus, $m\angle ADB$ would be half of 300, or 150. This is not the desired angle measure in the problem, so you can eliminate the possibility that *D* can lie on $\widehat{AB}$.

You can also use the measure of an inscribed angle to determine probability of a point lying on an arc.

## EXAMPLE 3 Inscribed Arcs and Probability

**PROBABILITY Points *A* and *B* are on a circle so that $m\widehat{AB} = 60$. Suppose point *D* is randomly located on the same circle so that it does not coincide with *A* or *B*. What is the probability that $m\angle ADB = 30$?**

Since the angle measure is half the arc measure, inscribed $\angle ADB$ must intercept $\widehat{AB}$, so *D* must lie on major arc *AB*. Draw a figure and label any information you know.




$$m\widehat{BDA} = 360 - m\widehat{AB}$$
$$= 360 - 60 \text{ or } 300$$

Since $\angle ADB$ must intercept $\widehat{AB}$, the probability that $m\angle ADB = 30$ is the same as the probability of $D$ being contained in $\widehat{BDA}$.

The probability that $D$ is located on $\widehat{ADB}$ is $\frac{300}{360}$ or $\frac{5}{6}$. So, the probability that $m\angle ADB = 30$ is also $\frac{5}{6}$.

**CHECK Your Progress**

**3.** Points $X$ and $Y$ are on a circle so that $m\widehat{XY} = 90$. Suppose point $Z$ is randomly located on the same circle so that it does not coincide with $X$ or $Y$. What is the probability that $m\angle XZY = 45$?

**Study Tip**

**Inscribed Polygons**

Remember that for a polygon to be an inscribed polygon, *all* of its vertices must lie on the circle.

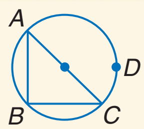
## Angles of Inscribed Polygons

An inscribed triangle with a side that is a diameter is a special type of triangle.

**THEOREM 10.7**

If the inscribed angle of a triangle intercepts a semicircle, the angle is a right angle.

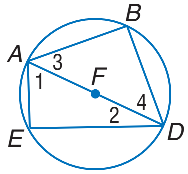
**Example:** $\widehat{ADC}$ is a semicircle, so $m\angle ABC = 90$.




You will prove Theorem 10.7 in Exercise 36.

**EXAMPLE** **Angles of an Inscribed Triangle**

**4** **ALGEBRA** **Triangles $ABD$ and $ADE$ are inscribed in $\odot F$ with $\widehat{AB} \cong \widehat{BD}$. Find the measures of $\angle 1$ and $\angle 2$ if $m\angle 1 = 12x - 8$ and $m\angle 2 = 3x + 8$.**




$AED$ is a right angle because $\widehat{AED}$ is a semicircle.

| | |
|---|---|
| $m\angle 1 + m\angle 2 + m\angle AED = 180$ | Angle Sum Theorem |
| $(12x - 8) + (3x + 8) + 90 = 180$ | $m\angle 1 = 12x - 8$, $m\angle 2 = 3x + 8$, $m\angle AED = 90$ |
| $15x + 90 = 180$ | Simplify. |
| $15x = 90$ | Subtract 90 from each side. |
| $x = 6$ | Divide each side by 15. |

Use the value of $x$ to find the measures of $\angle 1$ and $\angle 2$.

| | | | |
|---|---|---|---|
| $m\angle 1 = 12x - 8$ | Given | $m\angle 2 = 3x + 8$ | Given |
| $= 12(6) - 8$ | $x = 6$ | $= 3(6) + 8$ | $x = 6$ |
| $= 64$ | Simplify. | $= 26$ | Simplify. |

**CHECK** $90 + 64 + 26 = 180$

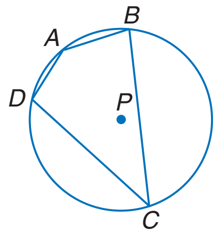
$180 = 180$ ✓

**CHECK Your Progress**

**4A.** Find $m\angle 3$.

**4B.** Find $m\angle 4$.

## EXAMPLE Angles of an Inscribed Quadrilateral




**5** **Quadrilateral *ABCD* is inscribed in ⊙*P*. If $m\angle B = 80$ and $m\angle C = 40$, find $m\angle A$ and $m\angle D$.**

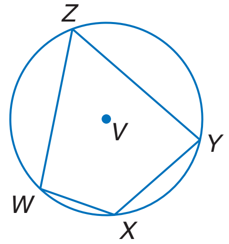
To find $m\angle A$, we need to know $m\widehat{BCD}$.

To find $m\widehat{BCD}$, first find $m\widehat{DAB}$.

| | |
|---|---|
| $m\widehat{DAB} = 2(m\angle C)$ | Inscribed Angle Theorem |
| $= 2(40)$ or $80$ | $m\angle C = 40$ |
| $m\widehat{BCD} + m\widehat{DAB} = 360$ | Sum of angles in circle $= 360$ |
| $m\widehat{BCD} + 80 = 360$ | $m\widehat{DAB} = 80$ |
| $m\widehat{BCD} = 280$ | Subtract 80 from each side. |
| $m\widehat{BCD} = 2(m\angle A)$ | Inscribed Angle Theorem |
| $280 = 2(m\angle A)$ | Substitution |
| $140 = m\angle A$ | Divide each side by 2. |

To find $m\angle D$, we need to know $m\widehat{ABC}$, but first we must find $m\widehat{ADC}$.

| | |
|---|---|
| $m\widehat{ADC} = 2(m\angle B)$ | Inscribed Angle Theorem |
| $m\widehat{ADC} = 2(80)$ or $160$ | $m\angle B = 80$ |
| $m\widehat{ABC} + m\widehat{ADC} = 360$ | Sum of angles in circle $= 360$ |
| $m\widehat{ABC} + 160 = 360$ | $m\widehat{ADC} = 160$ |
| $m\widehat{ABC} = 200$ | Subtract 160 from each side. |
| $m\widehat{ABC} = 2(m\angle D)$ | Inscribed Angle Theorem |
| $200 = 2(m\angle D)$ | Substitution |
| $100 = m\angle D$ | Divide each side by 2. |




### CHECK Your Progress

**5.** Quadrilateral *WXYZ* is inscribed in ⊙*V*. If $m\angle W = 95$ and $m\angle Z = 60$, find $m\angle X$ and $m\angle Y$.

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In Example 5, note that the opposite angles of the quadrilateral are supplementary.

**Study Tip**

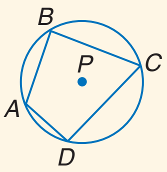
**Quadrilaterals**

Theorem 10.8 can be verified by considering that the arcs intercepted by opposite angles of an inscribed quadrilateral form a circle.

### THEOREM 10.8

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

**Example:** Quadrilateral *ABCD* is inscribed in ⊙*P*.
$\angle A$ and $\angle C$ are supplementary.
$\angle B$ and $\angle D$ are supplementary.

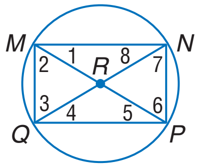



You will prove Theorem 10.8 in Exercise 37.

## CHECK Your Understanding

Example 1 (p. 579)

**1.** In $\odot R$, $m\overset{\frown}{MN} = 120$ and $m\overset{\frown}{MQ} = 60$. Find the measure of each numbered angle.



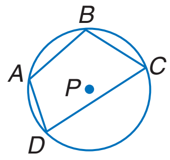

Example 2 (p. 580)

**2. PROOF** Write a paragraph proof.

**Given:** Quadrilateral $ABCD$ is inscribed in $\odot P$.

$$m\angle C = \frac{1}{2}m\angle B$$

**Prove:** $m\overset{\frown}{CDA} = 2(m\overset{\frown}{DAB})$

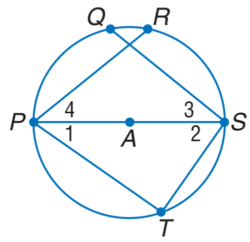



Example 3 (p. 580)

**3. PROBABILITY** Points $X$ and $Y$ are endpoints of a diameter of $\odot W$. Point $Z$ is another point on the circle. Find the probability that $\angle XZY$ is a right angle.

Example 4 (p. 581)

**4. ALGEBRA** In $\odot A$ at the right, $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$. Find the measure of each numbered angle if $m\angle 1 = 6x + 11$, $m\angle 2 = 9x + 19$, $m\angle 3 = 4y - 25$, and $m\angle 4 = 3y - 9$.




Example 5 (p. 582)

**5.** Quadrilateral $VWXY$ is inscribed in $\odot C$. If $m\angle X = 28$ and $m\angle W = 110$, find $m\angle V$ and $m\angle Y$.

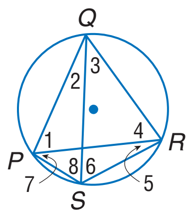
## Exercises

| For Exercises | See Examples |
|---|---|
| 6–8 | 1 |
| 9–10 | 2 |
| 11–14 | 3 |
| 15–19 | 4 |
| 20–23 | 5 |

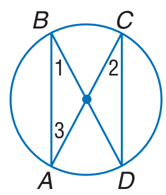
HOMEWORK HELP

**Find the measure of each numbered angle for each figure.**

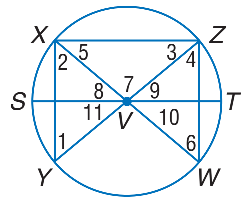
**6.** $\overline{PQ} \cong \overline{RQ}$, $m\overset{\frown}{PS} = 45$, and $m\overset{\frown}{SR} = 75$




**7.** $m\angle BDC = 25$, $m\overset{\frown}{AB} = 120$, and $m\overset{\frown}{CD} = 130$

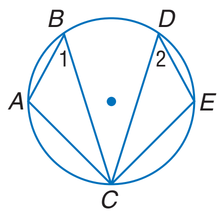



**8.** $m\overset{\frown}{XZ} = 100$, $\overline{XY} \perp \overline{ST}$, and $\overline{ZW} \perp \overline{ST}$

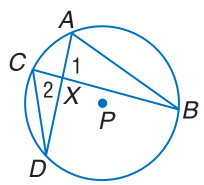



**PROOF** Write a two-column proof.

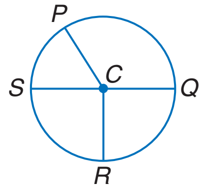
**9. Given:** $\overset{\frown}{AB} \cong \overset{\frown}{DE}$, $\overset{\frown}{AC} \cong \overset{\frown}{CE}$
**Prove:** $\triangle ABC \cong \triangle EDC$




**10. Given:** $\odot P$
**Prove:** $\triangle AXB \sim \triangle CXD$



**PROBABILITY** **Use the following information for Exercises 11–14.**
In $\odot C$, point $T$ is randomly selected so that it does not coincide with points $P$, $Q$, $R$, or $S$. $\overline{SQ}$ is a diameter of $\odot C$.




**11.** Find the probability that $m\angle PTS = 20$ if $m\widehat{PS} = 40$.

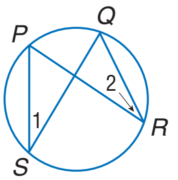
**12.** Find the probability that $m\angle PTR = 55$ if $m\widehat{PSR} = 110$.

**13.** Find the probability that $m\angle STQ = 90$.

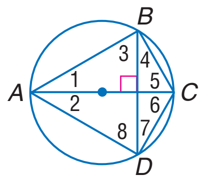
**14.** Find the probability that $m\angle PTQ = 180$.

**ALGEBRA** **Find the measure of each numbered angle for each figure.**

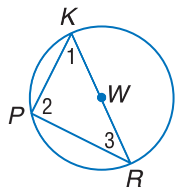
**15.** $m\angle 1 = x$, $m\angle 2 = 2x - 30$



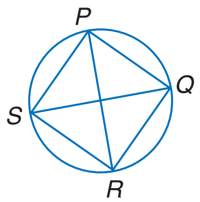

**16.** $m\widehat{AB} = 120$



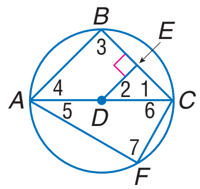

**17.** $m\angle R = \frac{1}{3}x + 5$, $m\angle K = \frac{1}{2}x$



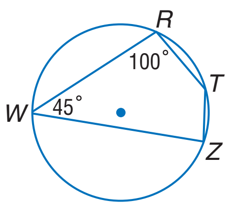

**18.** $PQRS$ is a rhombus inscribed in a circle. Find $m\angle QRP$ and $m\widehat{SP}$.



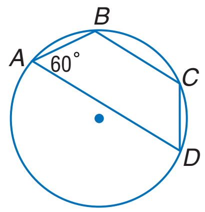

**19.** In $\odot D$, $\overline{DE} \cong \overline{EC}$, $m\widehat{CF} = 60$, and $\overline{DE} \perp \overline{EC}$. Find $m\angle 4$, $m\angle 5$, and $m\widehat{AF}$.




**20.** Quadrilateral $WRTZ$ is inscribed in a circle. Find $m\angle T$ and $m\angle Z$.



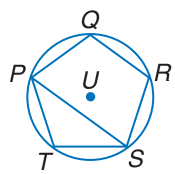

**21.** Trapezoid $ABCD$ is inscribed in a circle. Find $m\angle B$, $m\angle C$, and $m\angle D$.




**22.** Rectangle $PDQT$ is inscribed in a circle. What can you conclude about $\overline{PQ}$?

**23.** Square $EDFG$ is inscribed in a circle. What can you conclude about $\overline{EF}$?

**Regular pentagon $PQRST$ is inscribed in $\odot U$. Find each measure.**



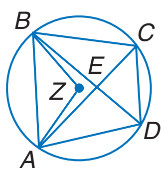

**24.** $m\widehat{QR}$

**25.** $m\angle PSR$

**26.** $m\angle PQR$

**27.** $m\widehat{PTS}$

**Quadrilateral $ABCD$ is inscribed in $\odot E$ such that $m\angle BZA = 104$, $m\widehat{CB} = 94$, and $\overline{AB} \parallel \overline{DC}$. Find each measure.**




**28.** $m\widehat{BA}$

**29.** $m\widehat{ADC}$

**30.** $m\angle BDA$

**31.** $m\angle ZAC$

**Real-World Link**

Many companies that sell school rings also offer schools and individuals the option to design their own ring.

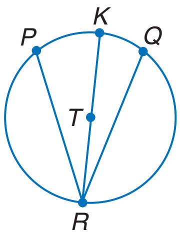
**32. SCHOOL RINGS** Some designs of class rings involve adding gold or silver to the surface of the round stone. The design at the right includes two inscribed angles. If $m\angle ABC = 50$ and $m\widehat{DBF} = 128$, find $m\widehat{AC}$ and $m\angle DEF$.


**PROOF** **Write the indicated type of proof for each theorem.**

**33.** two-column proof: Case 2 of Theorem 10.5

**Given:** $T$ lies inside $\angle PRQ$.
$\overline{RK}$ is a diameter.

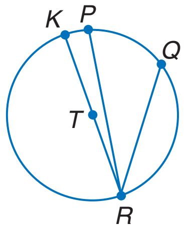
**Prove:** $m\angle PRQ = \frac{1}{2}m\widehat{PQ}$




**34.** two-column proof: Case 3 of Theorem 10.5

**Given:** $T$ lies outside $\angle PRQ$.
$\overline{RK}$ is a diameter.

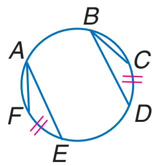
**Prove:** $m\angle PRQ = \frac{1}{2}m\widehat{PQ}$




**35.** two-column proof: Theorem 10.6

**Given:** $\widehat{DC} \cong \widehat{EF}$

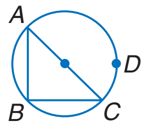
**Prove:** $\angle FAE \cong \angle CBD$




**36.** paragraph proof: Theorem 10.7

**Given:** $\widehat{ABC}$ is a semicircle.

**Prove:** $\angle ABC$ is a right angle.




**EXTRA PRACTICE**
See pages 820, 837.

**Math Online**
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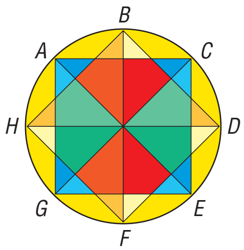
**37.** Write a paragraph proof for Theorem 10.8, which states: If a quadrilateral is inscribed in a circle, then opposite angles are supplementary.

**STAINED GLASS** In the stained glass window design, all of the small arcs around the circle are congruent. Suppose the center of the circle is point $O$.




**38.** What is the measure of each of the small arcs?

**39.** What kind of figure is $\triangle AOC$? Explain.

**40.** What kind of figure is quadrilateral $BDFH$? Explain.

**41.** What kind of figure is quadrilateral $ACEG$? Explain.

**H.O.T. Problems**

**42. REASONING** Compare and contrast an inscribed angle and a central angle that intercepts the same arc.

**43. OPEN ENDED** Find a real-world logo with an inscribed polygon.

**44. CHALLENGE** A trapezoid $ABCD$ is inscribed in $\odot O$. Explain how you can verify that $ABCD$ must be an isosceles trapezoid.

**45. *Writing in Math*** Use the information about sockets on page 578 and the definition of an inscribed polygon to explain how a socket is like an inscribed polygon. Expain how you would find the length of a regular hexagon inscribed in a circle with a diameter of $\frac{3}{4}$ inch.

## STANDARDIZED TEST PRACTICE

**46.** A square is inscribed in a circle. What is the ratio of the area of the circle to the area of the square?

A $\frac{1}{4}$

B $\frac{1}{2}$

C $\frac{\pi}{2}$

D $\frac{\pi}{4}$

**47.** **REVIEW** Simplify $4(3x - 2)(2x + 4) + 3x^2 + 5x - 6$.

F $9x^2 + 3x - 14$

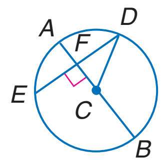
G $9x^2 + 13x - 14$

H $27x^2 + 37x - 38$

J $27x^2 + 27x - 26$

## Spiral Review

**Find each measure.** (Lesson 10-3)




**48.** If $AB = 60$ and $DE = 48$, find $CF$.

**49.** If $AB = 32$ and $FC = 11$, find $FE$.

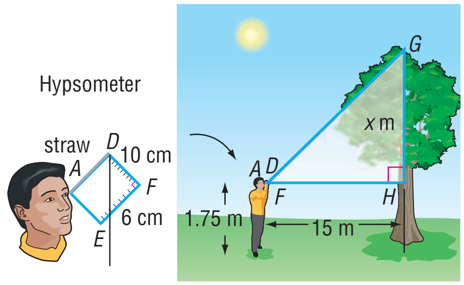
**50.** If $DE = 60$ and $FC = 16$, find $AB$.

**Points $Q$ and $R$ lie on $\odot P$. Find the length of $\widehat{QR}$ for the given radius and angle measure.** (Lesson 10-2)

**51.** $PR = 12$, and $m\angle QPR = 60$

**52.** $m\angle QPR = 90$, $PR = 16$

**53.** **FORESTRY** A hypsometer as shown can be used to estimate the height of a tree. Bartolo looks through the straw to the top of the tree and obtains the readings given. Find the height of the tree. (Lesson 7-3)




**Complete each sentence with *sometimes*, *always*, or *never*.** (Lesson 4-1)

**54.** Equilateral triangles are _?_ isosceles.

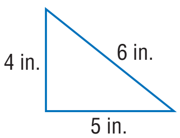
**55.** Acute triangles are _?_ equilateral.

**56.** Obtuse triangles are _?_ scalene.

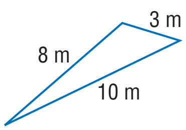
## GET READY for the Next Lesson

**PREREQUISITE SKILL** **Determine whether each figure is a right triangle.** (Lesson 8-2)

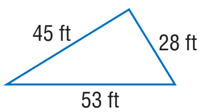
**57.**




**58.**




**59.**



# CHAPTER 10 Mid-Chapter Quiz

**Lessons 10-1 through 10-4**

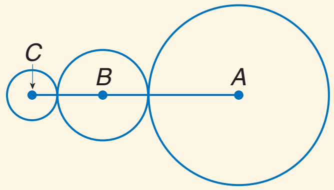
**1. MULTIPLE CHOICE** In the figure, the radius of circle $A$ is twice the radius of circle $B$ and four times the radius of circle $C$. If the sum of the circumferences of the three circles is $42\pi$, find the measure of $\overline{AC}$.

**A** 22

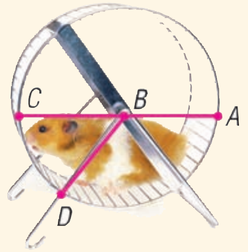
**B** 27

**C** 30

**D** 34




**For Exercises 2–8, refer to the front circular edge of the hamster wheel shown below.** (Lessons 10-1 and 10-2)




**2.** Name the circle.

**3.** Name three radii of the wheel.

**4.** If $BD = 3x$ and $CB = 7x - 3$, find $AC$.

**5.** If $m\angle CBD = 85$, find $m\overset{\frown}{AD}$.

**6.** If $r = 3$ inches, find the circumference of circle $B$ to the nearest tenth of an inch.

**7.** There are 40 equally spaced rungs on the wheel. What is the degree measure of an arc connecting two consecutive rungs?

**8.** What is the length of $\overset{\frown}{CAD}$ to the nearest tenth if $m\angle ABD = 150$ and $r = 3$?

**ENTERTAINMENT For Exercises 9–11, refer to the table, which shows the number of movies students at West Lake High School see in the theater each week.** (Lesson 10-2)

| Movies | |
|---|---|
| No movies | 17% |
| 1 movie | 53% |
| 2 movies | 23% |
| 3 or more movies | 7% |

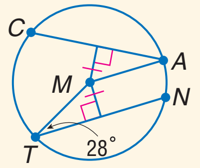
**9.** If you were to construct a circle graph of the data, how many degrees would be allotted to each category?

**10.** Describe the arcs for each category.

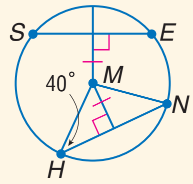
**11.** Construct a circle graph for these data.

**Find each measure.** (Lesson 10-3)

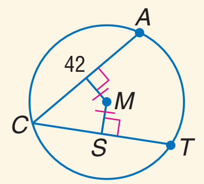
**12.** $m\angle CAM$



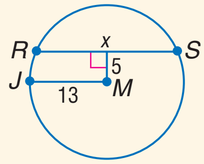

**13.** $m\overset{\frown}{ES}$




**14.** $SC$




**15.** $x$




**16. MULTIPLE CHOICE** The diameter of a circle is 30 inches, and a chord of the circle is 24 inches long. How far is the chord from the center of the circle? (Lesson 10-3)

**F** 5 inches

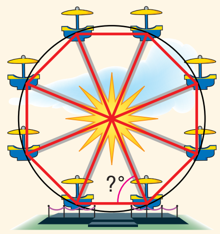
**G** 7 inches

**H** 9 inches

**J** 11 inches

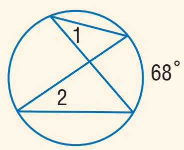
**17.** Quadrilateral $WXYZ$ is inscribed in a circle. If $m\angle X = 50$ and $m\angle Y = 70$, find $m\angle W$ and $m\angle Z$. (Lesson 10-4)

**18. AMUSEMENT RIDES** A Ferris wheel is shown. If the distances between the seat axles are the same, what is the measure of an angle formed by the braces attaching consecutive seats? (Lesson 10-4)




**19. PROBABILITY** In $\odot A$, point $X$ is randomly located so that it does not coincide with points $P$ or $Q$. If $m\overset{\frown}{PQ} = 160$, what is the probability that $m\angle PXQ = 80$? (Lesson 10-4)

**20.** Find the measure of each numbered angle. (Lesson 10-4)



# 10-5 Tangents

## Main Ideas

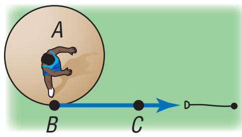
- Use properties of tangents.
- Solve problems involving circumscribed polygons.

## New Vocabulary

tangent
point of tangency

## GET READY for the Lesson

In April 2004, Yipsi Moreno of Cuba set the hammer throw record for North America, Central America, and the Carribean with a throw of 75.18 meters in La Habana, Cuba. The hammer is a metal ball, usually weighing 16 pounds, attached to a steel wire at the end of which is a grip. The ball is spun around by the thrower and then released, with the greatest distance thrown winning the event.



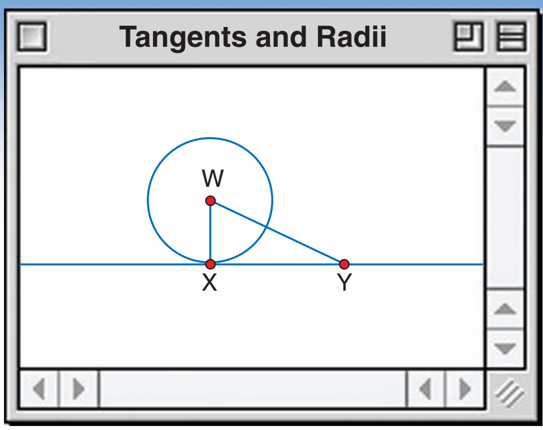

**Tangents** The figure at the right models the hammer throw event. Circle $A$ represents the circular area containing the spinning thrower. Ray $BC$ represents the path the hammer takes when released. $\overrightarrow{BC}$ is **tangent** to $\odot A$, because the line containing $\overrightarrow{BC}$ intersects the circle in exactly one point. This point is called the **point of tangency**.

## GEOMETRY SOFTWARE LAB

### Tangents and Radii

**MODEL**

- Use The Geometer's Sketchpad to draw a circle with center $W$. Then draw a segment tangent to $\odot W$. Label the point of tangency as $X$.
- Choose another point on the tangent and name it $Y$. Draw $\overline{WY}$.




**THINK AND DISCUSS**

1. What is $\overline{WX}$ in relation to the circle?
2. Measure $\overline{WY}$ and $\overline{WX}$. Write a statement to relate $WX$ and $WY$.
3. Move point $Y$. How does the location of $Y$ affect the statement you wrote in Exercise 2?
4. Measure $\angle WXY$. What conclusion can you make?
5. **Make a conjecture** about the shortest distance from the center of the circle to a tangent of the circle.



Andy Lyons/Getty Images

The lab suggests that the shortest distance from a tangent to the center of a circle is the radius drawn to the point of tangency. Since the shortest distance from a point to a line is a perpendicular, the radius and the tangent must be perpendicular.

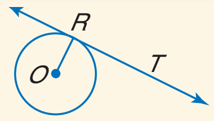
**Study Tip**

**Tangent Lines**
All of the theorems applying to tangent lines also apply to parts of the line that are tangent to the circle.

## THEOREM 10.9

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

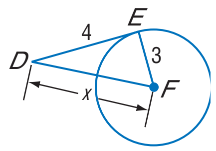
**Example:** If $\overleftrightarrow{RT}$ is a tangent, $\overline{OR} \perp \overleftrightarrow{RT}$.




You will prove Theorem 10.9 in Exercise 24.

## EXAMPLE 1 Find Lengths

**ALGEBRA** $\overline{ED}$ **is tangent to** $\odot F$ **at point** $E$. **Find** $x$.




Because the radius is perpendicular to the tangent at the point of tangency, $\overline{EF} \perp \overline{DE}$. This makes $\angle DEF$ a right angle and $\triangle DEF$ a right triangle. Use the Pythagorean Theorem to find $x$.

$(EF)^2 + (DE)^2 = (DF)^2$ Pythagorean Theorem

$3^2 + 4^2 = x^2$ $EF = 3$, $DE = 4$, $DF = x$

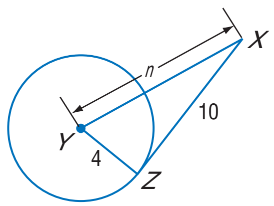
$25 = x^2$ Simplify.

$\pm 5 = x$ Take the square root of each side.

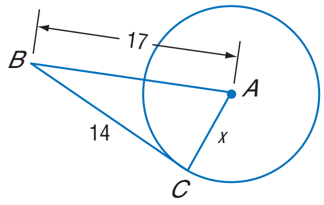
Because $x$ is the length of $\overline{DF}$, ignore the negative result. Thus, $x = 5$.

**CHECK Your Progress**

**1A.** $\overline{XZ}$ is tangent to $\odot Y$ at point $Z$. Find $n$.




**1B.** $\overline{BC}$ is tangent to $\odot A$ at point $C$. Find $x$.

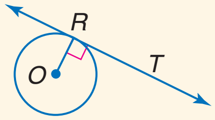



The converse of Theorem 10.9 is also true.

## THEOREM 10.10

If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

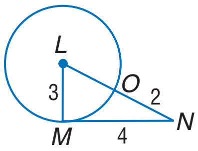
**Example:** If $\overline{OR} \perp \overleftrightarrow{RT}$, $\overleftrightarrow{RT}$ is a tangent.




You will prove this theorem in Exercise 25.

## EXAMPLE 2 Identify Tangents

**a. Determine whether $\overline{MN}$ is tangent to $\odot L$. Justify your reasoning.**




First determine whether $\triangle LMN$ is a right triangle by using the converse of the Pythagorean Theorem.

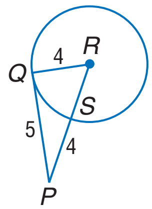
$(LM)^2 + (MN)^2 \stackrel{?}{=} (LN)^2$ Converse of Pythagorean Theorem

$3^2 + 4^2 \stackrel{?}{=} 5^2$ $LM = 3$, $MN = 4$, $LN = 3 + 2$ or $5$

$25 = 25$ ✓ Simplify.

Because $3^2 + 4^2 = 5^2$, the converse of the Pythagorean Theorem allows us to conclude that $\triangle LMN$ is a right triangle and $\angle LMN$ is a right angle. Thus, $\overline{LM} \perp \overline{MN}$, making $\overline{MN}$ a tangent to $\odot L$.

**b. Determine whether $\overline{PQ}$ is tangent to $\odot R$. Justify your reasoning.**




Since $RQ = RS$, $RP = 4 + 4$ or 8 units.

$(RQ)^2 + (PQ)^2 \stackrel{?}{=} (RP)^2$ Converse of Pythagorean Theorem

$4^2 + 5^2 \stackrel{?}{=} 8^2$ $RQ = 4$, $PQ = 5$, $RP = 8$

$41 \neq 64$ Simplify.

Because $RQ^2 + PQ^2 \neq RP^2$, $\triangle RQP$ is not a right triangle. So, $\overline{PQ}$ is not tangent to $\odot R$.

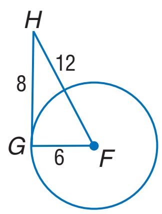
### Study Tip

**Identifying Tangents**

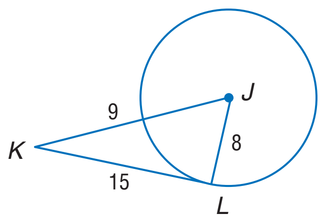
Never assume that a segment is tangent to a circle by appearance unless told otherwise. The figure must either have a right angle symbol or include the measurements that confirm a right angle.

### CHECK Your Progress

**2A.** Determine whether $\overline{GH}$ is tangent to $\odot F$. Justify your reasoning.




**2B.** Determine whether $\overline{KL}$ is tangent to $\odot J$. Justify your reasoning.

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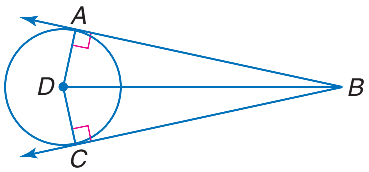
More than one line can be tangent to the same circle. In the figure, $\overline{AB}$ and $\overline{BC}$ are tangent to $\odot D$. So, $(AB)^2 + (AD)^2 = (DB)^2$ *and* $(BC)^2 + (CD)^2 = (DB)^2$.




$(AB)^2 + (AD)^2 = (BC)^2 + (CD)^2$ Substitution

$(AB)^2 + (AD)^2 = (BC)^2 + (AD)^2$ $AD = CD$

$(AB)^2 = (BC)^2$ Subtract $(AD)^2$ from each side.

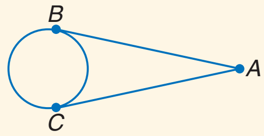
$AB = BC$ Take the square root of each side.

The last statement implies that $\overline{AB} \cong \overline{BC}$. This is a proof of Theorem 10.11.

## THEOREM 10.11

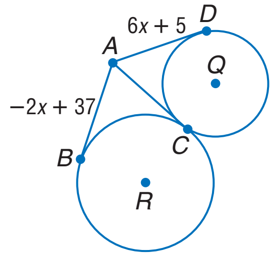
If two segments from the same exterior point are tangent to a circle, then they are congruent.

**Example:** $\overline{AB} \cong \overline{AC}$




## EXAMPLE Congruent Tangents

**3** **ALGEBRA** **Find $x$. Assume that segments that appear tangent to circles are tangent.**




$\overline{AD}$ and $\overline{AC}$ are drawn from the same exterior point and are tangent to $\odot Q$, so $\overline{AD} \cong \overline{AC}$. $\overline{AC}$ and $\overline{AB}$ are drawn from the same exterior point and are tangent to $\odot R$, so $\overline{AC} \cong \overline{AB}$. By the Transitive Property, $\overline{AD} \cong \overline{AB}$.

$AD = AB$ Definition of congruent segments

$6x + 5 = -2x + 37$ Substitution

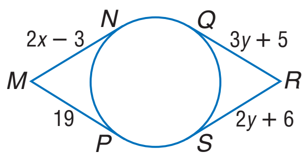
$8x + 5 = 37$ Add $2x$ to each side.

$8x = 32$ Subtract 5 from each side.

$x = 4$ Divide each side by 8.

### CHECK Your Progress

**3.** Find $x$ and $y$.

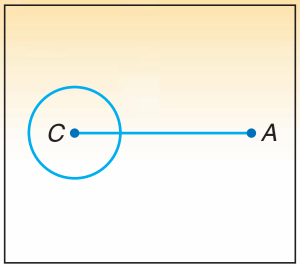



In the construction that follows, you will learn how to construct a line tangent to a circle through a point exterior to the circle.

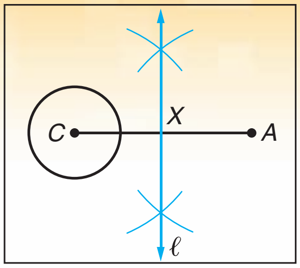
## CONSTRUCTION

### Line Tangent to a Circle Through a Point Exterior to the Circle

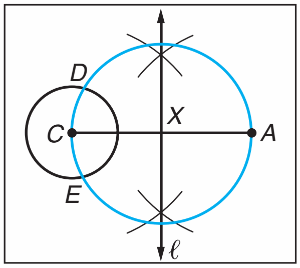
**Step 1** Construct a circle. Label the center *C*. Draw a point outside ⊙*C*. Then draw $\overline{CA}$.



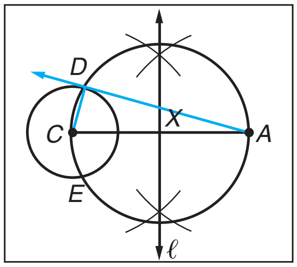

**Step 2** Construct the perpendicular bisector of $\overline{CA}$ and label it line ℓ. Label the intersection of ℓ and $\overline{CA}$ as point *X*.




**Step 3** Construct circle *X* with radius $\overline{XC}$. Label the points where the circles intersect as *D* and *E*.




**Step 4** Draw $\overleftrightarrow{AD}$. △*ADC* is inscribed in a semicircle. So ∠*ADC* is a right angle, and $\overleftrightarrow{AD}$ is a tangent.

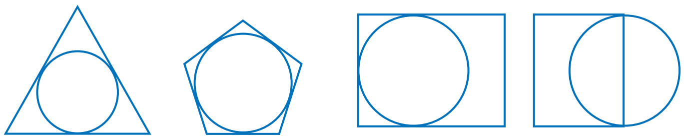



You will construct a line tangent to a circle through a point on the circle in Exercise 23.

**Circumscribed Polygons** In Lesson 10-3, you learned that circles can be circumscribed about a polygon. Likewise, polygons can be circumscribed about a circle, or the circle is inscribed in the polygon. Notice that the vertices of the polygon *do not* lie on the circle, but every side of the polygon is tangent to the circle.

**Study Tip**

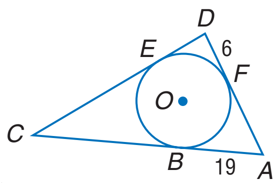
**Common Misconceptions**
Just because the circle is tangent to one or more of the sides of a polygon does not mean that the polygon is circumscribed about the circle, as shown in the second pair of figures.



Polygons are circumscribed. Polygons are *not* circumscribed.

## EXAMPLE 4 Triangles Circumscribed About a Circle

**Triangle *ADC* is circumscribed about ⊙*O*. Find the perimeter of △*ADC* if *EC* = *DE* + *AF*.**




Use Theorem 10.10 to determine the equal measures:
$AB = AF = 19$, $FD = DE = 6$, and $EC = CB$.
We are given that $EC = DE + AF$, so $EC = 6 + 19$ or 25.

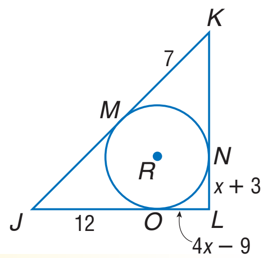
$P = AB + BC + EC + DE + FD + AF$ Definition of perimeter

$= 19 + 25 + 25 + 6 + 6 + 19$ or 100 Substitution

The perimeter of △*ADC* is 100 units.

**CHECK Your Progress**

4. Triangle *JKL* is circumscribed about ⊙*R*. Find *x* and the perimeter of △*JKL*.

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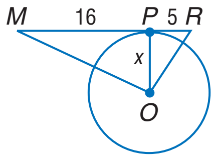
## CHECK Your Understanding

**Examples 1 and 2** (pp. 589–590)

**For Exercises 1 and 2, use the figure at the right.**

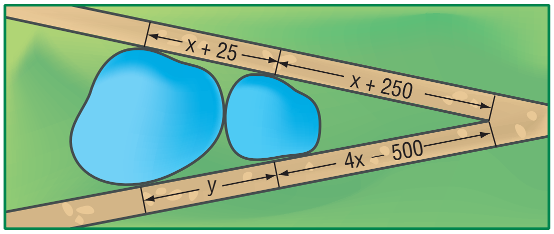



**1.** Tangent $\overline{MP}$ is drawn to ⊙$O$. Find $x$ if $MO = 20$.

**2.** If $RO = 13$, determine whether $\overline{PR}$ is tangent to ⊙$O$.

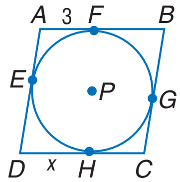
**Example 3** (p. 591)

**3. LANDSCAPE ARCHITECT** A landscape architect is planning to pave two walking paths beside two ponds, as shown. Find the values of $x$ and $y$. What is the total length of the walking paths?




**Example 4** (p. 592)

**4.** Rhombus $ABCD$ is circumscribed about ⊙$P$ and has a perimeter of 32 units. Find $x$.

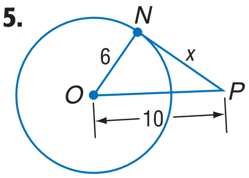



## Exercises

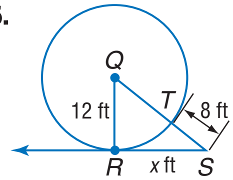
| HOMEWORK HELP | |
|---|---|
| For Exercises | See Examples |
| 5–10 | 1 |
| 11–14 | 2 |
| 15–16 | 3 |
| 17–22 | 4 |

**Find $x$. Assume that segments that appear to be tangent are tangent.**

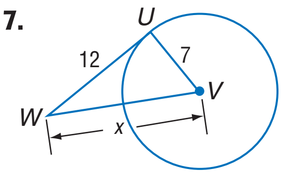
**5.**



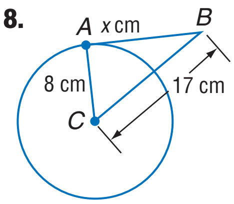

**6.**



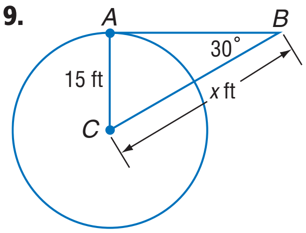

**7.**



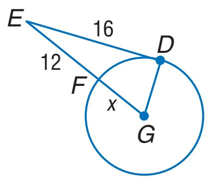

**8.**




**9.**

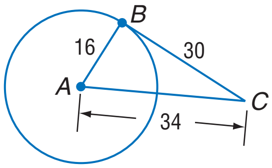



**10.**

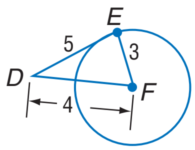



**Determine whether each segment is tangent to the given circle.**

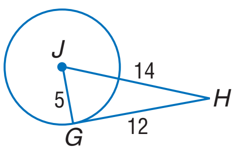
**11.** $\overline{BC}$



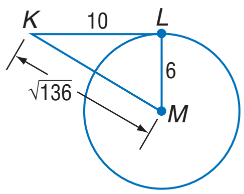

**12.** $\overline{DE}$




**13.** $\overline{GH}$

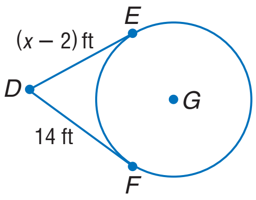



**14.** $\overline{KL}$

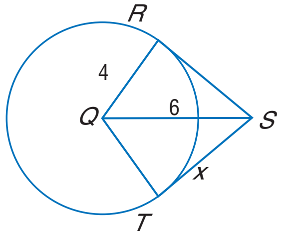


**Find $x$. Assume that segments that appear to be tangent are tangent.**

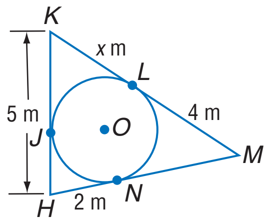
**15.**



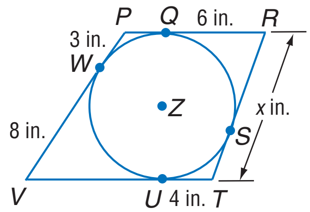

**16.**




**17.**

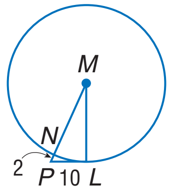



**18.**

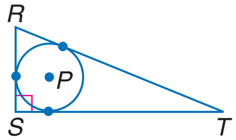



**Find the perimeter of each polygon for the given information.**

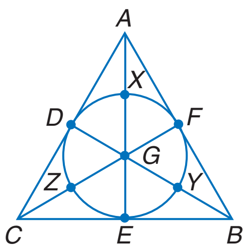
**19.**



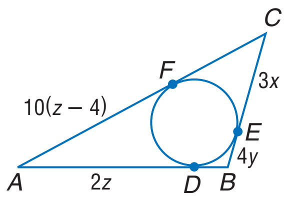

**20.** $ST = 18$, radius of $\odot P = 5$




**21.** $BY = CZ = AX = 2$
radius of $\odot G = 3$




**22.** $CF = 6(3 - x)$, $DB = 12y - 4$



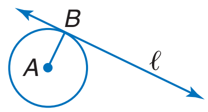

**23. CONSTRUCTION** Construct a line tangent to a circle through a point on the circle following these steps.

- Construct a circle with center $T$ and locate a point $P$ on $\odot T$ and draw $\overrightarrow{TP}$.
- Construct a perpendicular to $\overrightarrow{TP}$ through point $T$.

**Study Tip**

**Look Back**
To review **constructing perpendiculars to a line**, see Lesson 3-6.

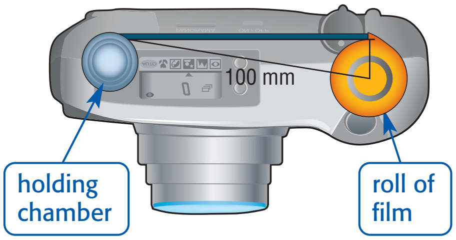
**24. PROOF** Write an indirect proof of Theorem 10.9 by assuming that $\ell$ is not perpendicular to $\overline{AB}$.
**Given:** $\ell$ is tangent to $\odot A$ at $B$, $\overline{AB}$ is a radius of $\odot A$.
**Prove:** Line $\ell$ is perpendicular to $\overline{AB}$.

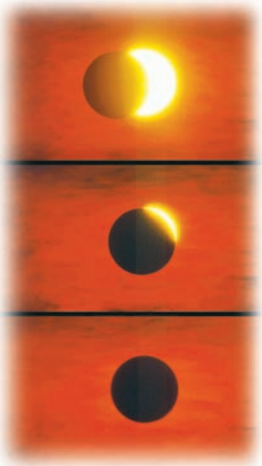



**25. PROOF** Write an indirect proof of Theorem 10.10 by assuming that $\ell$ is not tangent to $\odot A$.
**Given:** $\ell \perp \overline{AB}$, $\overline{AB}$ is a radius of $\odot A$.
**Prove:** Line $\ell$ is tangent to $\odot A$.

**26. PROOF** Write a two-column proof to show that if a quadrilateral is circumscribed about a circle, then the sum of the measures of the two opposite sides is equal to the sum of the measures of the two remaining sides.

**27. PHOTOGRAPHY** The film in a 35-mm camera unrolls from a cylinder, travels across an opening for exposure, and then goes into another circular chamber as each photograph is taken. The roll of film has a diameter of 25 millimeters, and the distance from the center of the roll to the intake of the chamber is 100 millimeters. To the nearest millimeter, how much of the film would be exposed if the camera were opened before the roll had been used up?



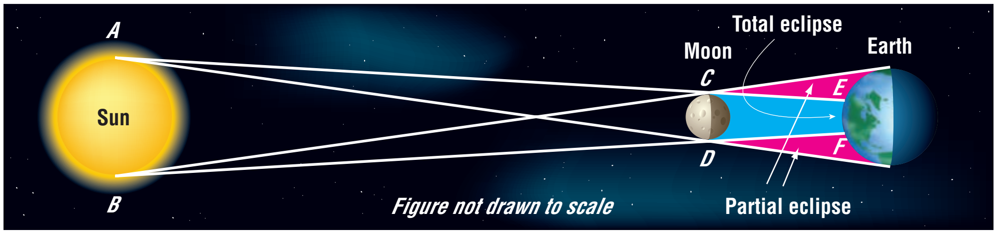



**Real-World Link**

During the 20th century, there were 78 total solar eclipses, but only 15 of these affected parts of the United States. The next total solar eclipse visible in the U.S. will be in 2017.

**Source:** *World Almanac*

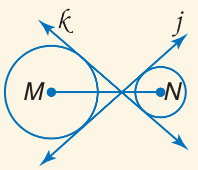
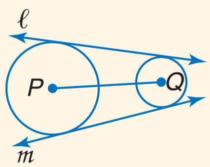
**ASTRONOMY For Exercises 28 and 29, use the following information.**
A solar eclipse occurs when the Moon blocks the Sun's rays from hitting Earth. Some areas of the world will experience a total eclipse, others a partial eclipse, and some no eclipse at all, as shown in the diagram below.




**28.** The blue section denotes a total eclipse on that part of Earth. Which tangents define the blue area?

**29.** The pink section denotes the area that will have a partial eclipse. Which tangents define the northern and southern boundaries of the partial eclipse?

**COMMON TANGENTS A line that is tangent to two circles in the same plane is called a *common tangent*.**

| ***Common internal tangents*** **intersect the segment connecting the centers.** | ***Common external tangents*** **do not intersect the segment connecting the centers.** |
| --- | --- |
|  Lines $k$ and $j$ are common internal tangents. |  Lines $\ell$ and $m$ are common external tangents. |





**Refer to the diagram of the eclipse above.**

**30.** Name two common internal tangents.

**31.** Name two common external tangents.

**H.O.T. Problems**

**32. REASONING** Determine the number of tangents that can be drawn to a circle for each point. Explain your reasoning.

**a.** containing a point outside the circle

**b.** containing a point inside the circle

**c.** containing a point on the circle

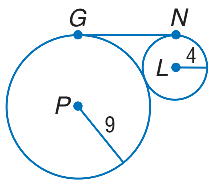
**33. OPEN ENDED** Draw an example of a circumscribed polygon and an example of an inscribed polygon, and give real-life examples of each.

**EXTRA PRACTICE**
See pages 820, 837.

**Math Online**
Self-Check Quiz at geometryonline.com

Ray Massey/Getty Images

**34. CHALLENGE** Find the measure of tangent $\overline{GN}$. Explain.

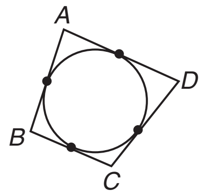



**35. REASONING** Write an argument to support this statement or provide a counterexample. *If two lines are tangent to the same circle, the lines intersect.*

**36. *Writing in Math*** Using the information about tangents and track and field on page 588, explain how the hammer throw models a tangent. Determine the distance the hammer landed from Moreno if the wire and handle are 1.2 meters long and her arm is 0.8 meter long.

## STANDARDIZED TEST PRACTICE

**37.** Quadrilateral *ABCD* is circumscribed about a circle. If $AB = 19$, $BC = 6$, and $CD = 14$, what is the measure of $\overline{AD}$?




**A** 11 **C** 25

**B** 20 **D** 27

**38. REVIEW** A paper company ships reams of paper in a box that weighs 1.3 pounds. Each ream of paper weighs 4.4 pounds, and a box can carry no more than 12 reams of paper. Which inequality best describes the total weight in pounds $w$ to be shipped in terms of the number of reams of paper $r$ in each box?

**F** $w \geq 1.3 + 4.4r,\ r \geq 12$

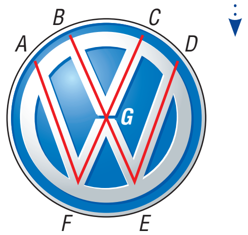
**G** $w = 1.3 + 4.4r,\ r \leq 12$

**H** $w \leq 1.3 + 4.4r,\ r \leq 12$

**J** $w = 1.3 + 4.4r,\ r \geq 12$

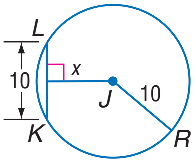
## Spiral Review

**39. ADVERTISING** Circles are often used in logos for commercial products. The logo at the right shows two inscribed angles and two central angles. If $\overparen{AC} \cong \overparen{BD}$, $m\overparen{AF} = 90$, $m\overparen{FE} = 45$, and $m\overparen{ED} = 90$, find $m\angle AFC$ and $m\angle BED$. (Lesson 10-4)

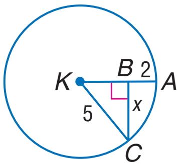



**Find each measure to the nearest tenth.** (Lesson 10-3)

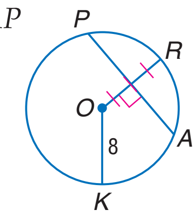
**40.** $x$




**41.** $BC$




**42.** $AP$




## GET READY for the Next Lesson

**PREREQUISITE SKILL** **Solve each equation.** (pages 781 and 782)

**43.** $x + 3 = \frac{1}{2}[(4x + 6) - 10]$

**44.** $2x - 5 = \frac{1}{2}[(3x + 16) - 20]$

**45.** $2x + 4 = \frac{1}{2}[(x + 20) - 10]$

**46.** $x + 3 = \frac{1}{2}[(4x + 10) - 45]$

EXTEND 10-5

## Geometry Lab

# Inscribed and Circumscribed Triangles

In Lesson 5-1, you learned that there are special points of concurrency in a triangle. Two of these will be used in these activities.

- The *incenter* is the point at which the angle bisectors meet. It is equidistant from the sides of the triangle.
- The *circumcenter* is the point at which the perpendicular bisectors of the sides intersect. It is equidistant from the vertices of the triangle.

Concepts in Motion
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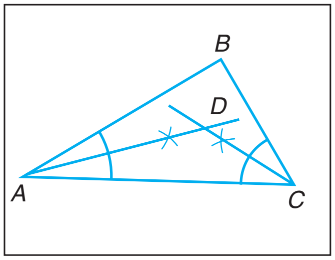
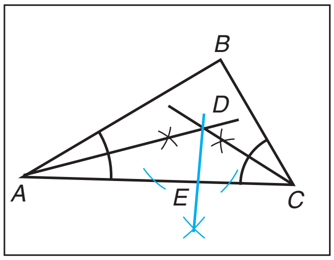
### ACTIVITY 1 Construct a circle inscribed in a triangle.
The triangle is circumscribed about the circle.

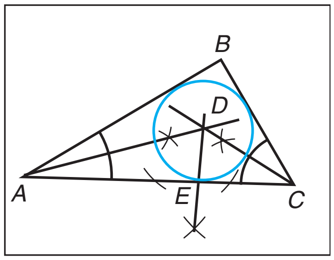
**Step 1** Draw a triangle and label its vertices $A$, $B$, and $C$. Construct two angle bisectors of the triangle to locate the incenter. Label it $D$.




**Step 2** Construct a segment perpendicular to a side of $\triangle ABC$ through the incenter. Label the intersection $E$.

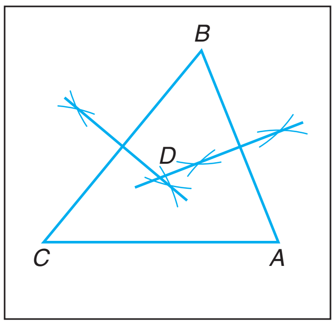



**Step 3** Use the compass to measure $DE$. Then put the point of the compass on $D$, and draw a circle with that radius.

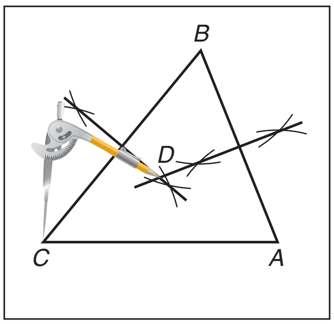



### ACTIVITY 2 Construct a circle through any three noncollinear points.
This construction may be referred to as circumscribing a circle about a triangle.

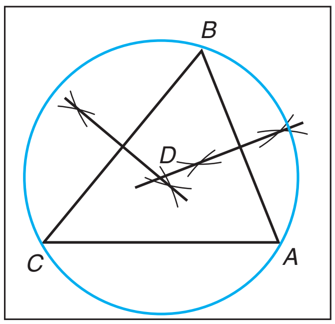
**Step 1** Draw a triangle and label its vertices $A$, $B$, and $C$. Construct perpendicular bisectors of two sides of the triangle to locate the circumcenter. Label it $D$.




**Step 2** Use the compass to measure the distance from the circumcenter $D$ to any of the three vertices.



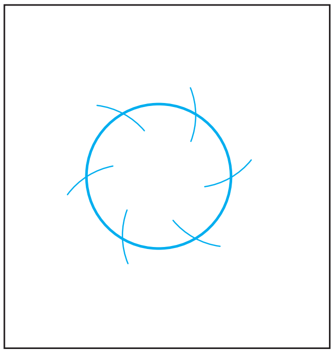

**Step 3** Using that setting, place the compass point at $D$, and draw a circle about the triangle.



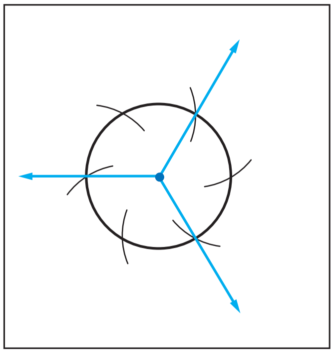
For the next activity, refer to the construction of an inscribed regular hexagon on page 576.

**ACTIVITY 3** **Construct an equilateral triangle circumscribed about a circle.**

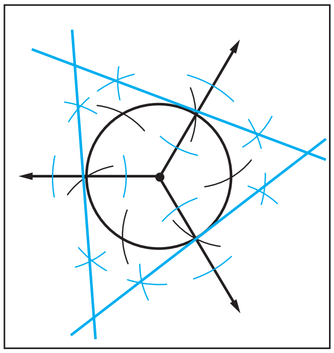
**Step 1** Construct a circle and divide it into six congruent arcs.



**Step 2** Place a point at every other arc. Draw rays from the center through these points.



**Step 3** Construct a line perpendicular to each of the rays through the points.



## Analyze the Results

1. Draw an obtuse triangle and inscribe a circle in it.
2. Draw a right triangle and circumscribe a circle about it.
3. Draw a circle of any size and circumscribe an equilateral triangle about it.

**Refer to Activity 1.**

4. Why do you only have to construct the perpendicular to one side of the triangle?
5. How can you use the Incenter Theorem to explain why this construction is valid?

**Refer to Activity 2.**

6. Why do you only have to measure the distance from the circumcenter to any one vertex?
7. How can you use the Circumcenter Theorem to explain why this construction is valid?

**Refer to Activity 3.**

8. What is the measure of each of the six congruent arcs?
9. Write a convincing argument as to why the lines constructed in Step 3 form an equilateral triangle.
10. Why do you think the terms *incenter* and *circumcenter* are good choices for the points they define?

# 10-6 Secants, Tangents, and Angle Measures

## Main Ideas

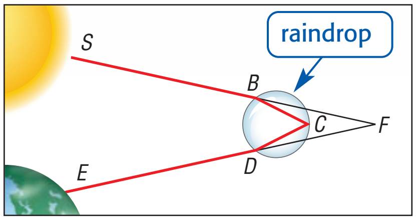
- Find measures of angles formed by lines intersecting on or inside a circle.
- Find measures of angles formed by lines intersecting outside the circle.

## New Vocabulary

secant

## GET READY for the Lesson

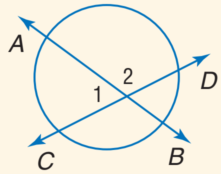
Droplets of water in the air refract or bend sunlight as it passes through them, creating a rainbow. The various angles of refraction result in an arch of colors. In the figure, the sunlight from point $S$ enters the raindrop at $B$ and is bent. The light proceeds to the back of the raindrop, and is reflected at $C$ to leave the raindrop at point $D$ heading to Earth. Angle $F$ represents the measure of how the resulting ray of light deviates from its original path.




**Intersections on or Inside a Circle** A line that intersects a circle in exactly two points is called a **secant**. In the figure above, $\overline{SF}$ and $\overline{EF}$ are secants of the circle. When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.

### THEOREM 10.12

If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.




**Examples:** $m\angle 1 = \frac{1}{2}(m\widehat{AC} + m\widehat{BD})$

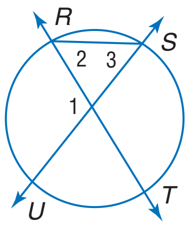
$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

### PROOF Theorem 10.12

**Given:** secants $\overleftrightarrow{RT}$ and $\overleftrightarrow{SU}$

**Prove:** $m\angle 1 = \frac{1}{2}(m\widehat{ST} + m\widehat{RU})$

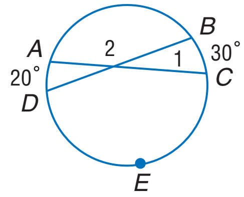
**Proof:**




| Statements | Reasons |
|---|---|
| **1.** $m\angle 1 = m\angle 2 + m\angle 3$ | **1.** Exterior Angle Theorem |
| **2.** $m\angle 2 = \frac{1}{2}m\widehat{ST}$, $m\angle 3 = \frac{1}{2}m\widehat{RU}$ | **2.** The measure of the inscribed $\angle$ = half the measure of the intercepted arc. |
| **3.** $m\angle 1 = \frac{1}{2}m\widehat{ST} + \frac{1}{2}m\widehat{RU}$ | **3.** Substitution |
| **4.** $m\angle 1 = \frac{1}{2}(m\widehat{ST} + m\widehat{RU})$ | **4.** Distributive Property |

## EXAMPLE Secant-Secant Angle

**1** **Find $m\angle 2$ if $m\widehat{BC} = 30$ and $m\widehat{AD} = 20$.**




**Method 1 Find $m\angle 1$.**

$m\angle 1 = \frac{1}{2}(m\widehat{BC} + m\widehat{AD})$ Theorem 10.12

$= \frac{1}{2}(30 + 20)$ or 25 Substitution

$m\angle 2 = 180 - m\angle 1$

$= 180 - 25$ or 155

**Method 2 Find $m\widehat{AB}$ and $m\widehat{DEC}$ first.**

$m\angle 2 = \frac{1}{2}(m\widehat{AB} + m\widehat{DEC})$ Theorem 10.12

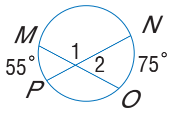
$= \frac{1}{2}[360 - (m\widehat{BC} + m\widehat{AD})]$ $m\widehat{AB} + m\widehat{DEC} = 360 - (m\widehat{BC} + m\widehat{AD})$

$= \frac{1}{2}[360 - (30 + 20)]$ Substitution

$= \frac{1}{2}(310)$ or 155 Simplify.

**CHECK Your Progress**

**1.** Find $m\angle 1$.

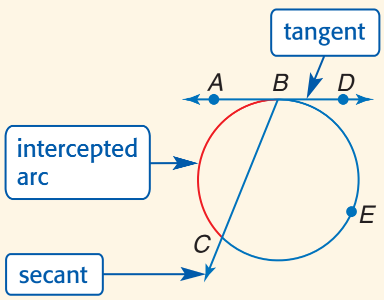



## THEOREM 10.13

If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

**Examples:** $m\angle ABC = \frac{1}{2}m\widehat{BC}$

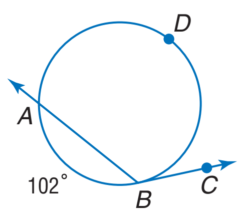
$m\angle DBC = \frac{1}{2}m\widehat{BEC}$




You will prove Theorem 10.13 in Exercise 41.

## EXAMPLE Secant-Tangent Angle

**2** **Find $m\angle ABC$ if $m\widehat{AB} = 102$.**



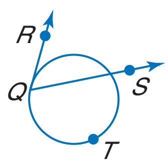

$m\widehat{ADB} = 360 - m\widehat{AB}$

$= 360 - 102$ or 258

$m\angle ABC = \frac{1}{2}m\widehat{ADC}$

$= \frac{1}{2}(258)$ or 129

**CHECK Your Progress**




**2.** Find $m\angle RQS$ if $m\widehat{QTS} = 238$.

**Intersections Outside a Circle** Secants and tangents can also meet outside a circle. The measure of the angle formed also involves half of the measures of the arcs they intercept.

**Study Tip**

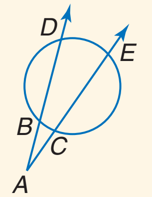
**Absolute Value**

The measure of each $\angle A$ can also be expressed as one-half the absolute value of the difference of the arc measures. In this way, the order of the arc measures does not affect the outcome of the calculation.

## THEOREM 10.14

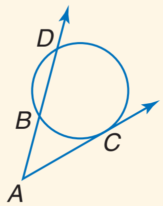
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

**Two Secants**



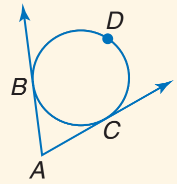

$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$

**Secant-Tangent**




$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$

**Two Tangents**

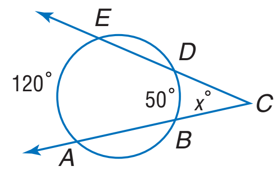



$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

You will prove Theorem 10.14 in Exercise 40.

## EXAMPLE Secant-Secant Angle

**3** Find $x$.



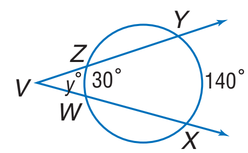

$m\angle C = \frac{1}{2}(m\widehat{EA} - m\widehat{DB})$

$x = \frac{1}{2}(120 - 50)$ Substitution

$x = \frac{1}{2}(70)$ or 35 Simplify.

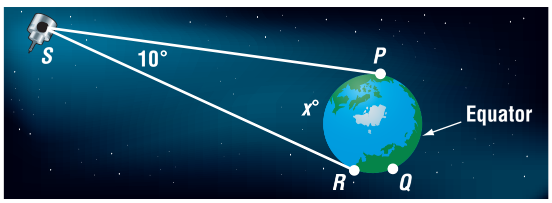
**CHECK Your Progress**

**3.** Find $y$.




## Real-World EXAMPLE

**4** **SATELLITES** **Suppose a satellite $S$ orbits above Earth rotating so that it appears to hover directly over the equator. Use the figure to determine the arc measure on the equator visible to this satellite.**




$\widehat{PR}$ represents the arc along the equator visible to the satellite $S$. If $x = m\widehat{PR}$, then $m\widehat{PQR} = 360 - x$. Use the measure of the given angle to find $m\widehat{PR}$.

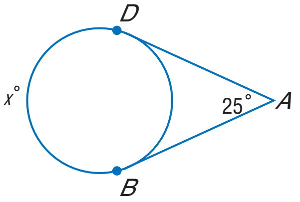
$$m\angle S = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$$

| | |
|---|---|
| $10 = \frac{1}{2}[(360 - x) - x]$ | Substitution |
| $20 = 360 - 2x$ | Multiply each side by 2 and simplify. |
| $-340 = -2x$ | Subtract 360 from each side. |
| $170 = x$ | Divide each side by $-2$. |

The measure of the arc on Earth visible to the satellite is 170.

**CHECK Your Progress**

**4.** Find $x$.

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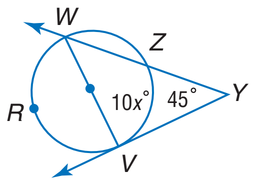
## EXAMPLE 5 Secant-Tangent Angle

**Find $x$.**



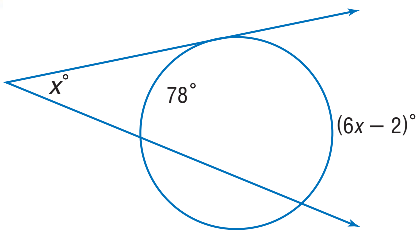

$\widehat{WRV}$ is a semicircle because $\overline{WV}$ is a diameter.
So, $m\widehat{WRV} = 180$.

$$m\angle Y = \frac{1}{2}(m\widehat{WV} - m\widehat{ZV})$$

| | |
|---|---|
| $45 = \frac{1}{2}(180 - 10x)$ | Substitution |
| $90 = 180 - 10x$ | Multiply each side by 2. |
| $-90 = -10x$ | Subtract 180 from each side. |
| $9 = x$ | Divide each side by $-10$. |

**CHECK Your Progress**

**5.** Find $x$.

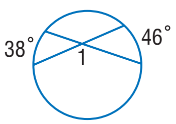



## CHECK Your Understanding

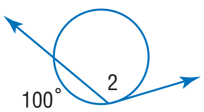
Examples 1, 2 (p. 600)

**Find each measure.**

**1.** $m\angle 1$

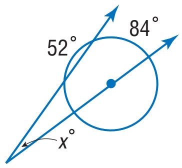


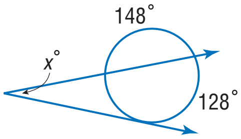

**2.** $m\angle 2$

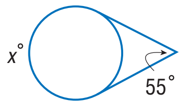


**Examples 3 to 5**
(pp. 601, 602)

**Find $x$.**

**3.**


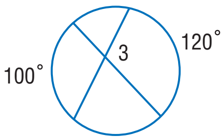

**4.**


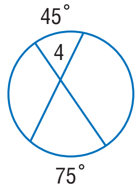

**5.**


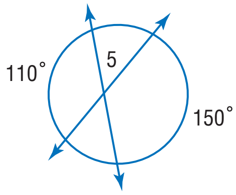

## Exercises

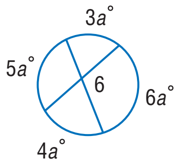
| HOMEWORK HELP | |
|---|---|
| For Exercises | See Examples |
| 6–9, 14 | 1 |
| 10–13 | 2 |
| 15–18 | 3 |
| 19–20 | 4 |
| 21–24 | 5 |

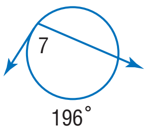
**Find each measure.**

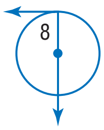
**6.** $m\angle 3$


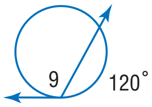

**7.** $m\angle 4$


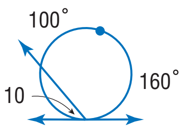

**8.** $m\angle 5$


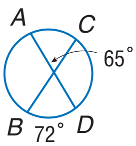

**9.** $m\angle 6$



**10.** $m\angle 7$


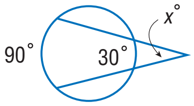

**11.** $m\angle 8$


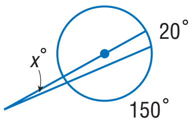

**12.** $m\angle 9$


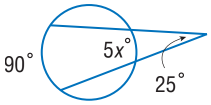

**13.** $m\angle 10$


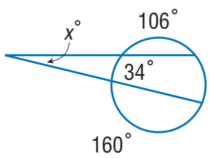

**14.** $m\overset{\frown}{AC}$


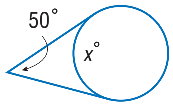

**Find $x$. Assume that any segment that appears to be tangent is tangent.**

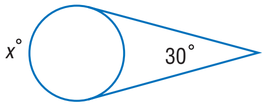
**15.**


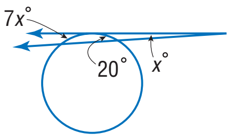

**16.**


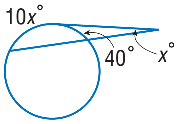

**17.**


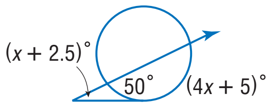

**18.**


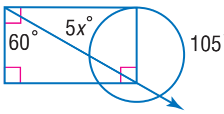

**19.**


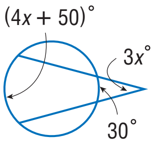

**20.**


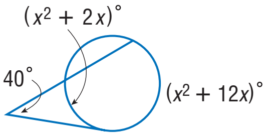

**21.**



**22.**



**23.**


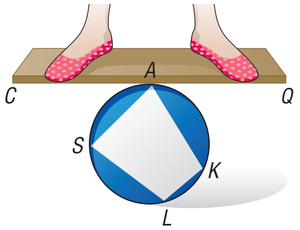

**24.**



**25.**



**26.**


**CIRCUS For Exercises 27–30, refer to the figure and the information below.**

One of the acrobatic acts in the circus requires the artist to balance on a board that is placed on a round drum, as shown at the right. Find each measure if $\overline{SA} \parallel \overline{LK}$, $m\angle SLK = 78$, and $m\widehat{SA} = 46$.



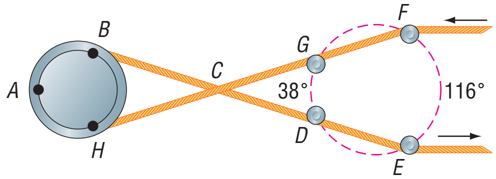

**27.** $m\angle CAS$

**28.** $m\angle QAK$

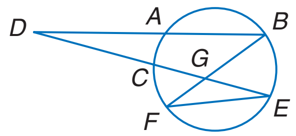
**29.** $m\widehat{KL}$

**30.** $m\widehat{SL}$

**31. WEAVING** Once yarn is woven from wool fibers, it is often dyed and then threaded along a path of pulleys to dry. One set of pulleys is shown below. Note that the yarn appears to intersect itself at $C$, but in reality it does not. Use the information from the diagram to find $m\widehat{BH}$.




**Find each measure if $m\widehat{FE} = 118$, $m\widehat{AB} = 108$, $m\angle EGB = 52$, and $m\angle EFB = 30$.**




**32.** $m\widehat{AC}$

**33.** $m\widehat{CF}$

**34.** $m\angle EDB$

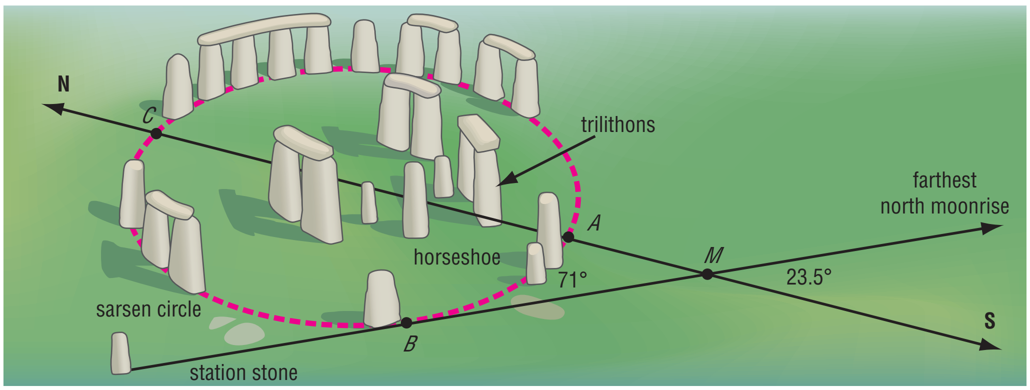
**Real-World Link**

Stonehenge is located in southern England near Salisbury. In its final form, Stonehenge included 30 upright stones about 18 feet tall by 7 feet thick.

**Source:** *World Book Encyclopedia*

**LANDMARKS For Exercises 35–37, use the following information.**

Stonehenge is a British landmark made of huge stones arranged in a circular pattern that reflects the movements of Earth and the moon. The diagram shows that the angle formed by the north/south axis and the line aligned from the station stone to the northmost moonrise position measures 23.5°.




**35.** Find $m\widehat{BC}$.

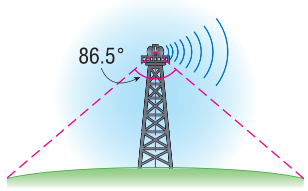
**36.** Is $\widehat{ABC}$ a semicircle? Explain.

**37.** If the circle measures about 100 feet across, approximately how far would you walk around the circle from point $B$ to point $C$?



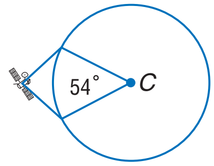
file photo

**38. TELECOMMUNICATIONS** The signal from a telecommunication tower follows a ray that has its endpoint on the tower and is tangent to Earth. Suppose a tower is located at sea level, as shown in the figure. Determine the measure of the arc intercepted by the two tangents.




*Note:* Art not drawn to scale

**39. SATELLITES** A satellite is orbiting so that it maintains a constant altitude above the equator. The camera on the satellite can detect an arc of 6000 kilometers on Earth's surface. This arc measures 54°. What is the measure of the angle of view of the camera located on the satellite?

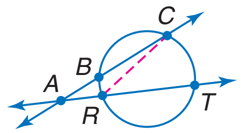



**40. PROOF** Write a two-column proof of Theorem 10.14. Consider each case.

**a. Case 1: two secants**

**Given:** $\overleftrightarrow{AC}$ and $\overleftrightarrow{AT}$ are secants to the circle.

**Prove:** $m\angle CAT = \frac{1}{2}(m\widehat{CT} - m\widehat{BR})$

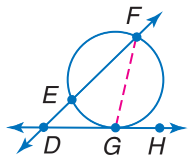



**b. Case 2: secant and a tangent**

**Given:** $\overleftrightarrow{DG}$ is a tangent to the circle.

$\overleftrightarrow{DF}$ is a secant to the circle.

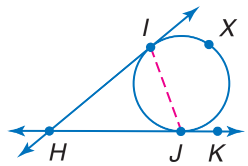
**Prove:** $m\angle FDG = \frac{1}{2}(m\widehat{FG} - m\widehat{GE})$




**c. Case 3: two tangents**

**Given:** $\overleftrightarrow{HI}$ and $\overleftrightarrow{HJ}$ are tangents to the circle.

**Prove:** $m\angle IHJ = \frac{1}{2}(m\widehat{IXJ} - m\widehat{IJ})$



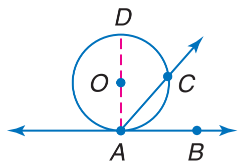

**41. PROOF** Write a paragraph proof of Theorem 10.13.

**a. Given:** $\overleftrightarrow{AB}$ is a tangent of $\odot O$.

$\overrightarrow{AC}$ is a secant of $\odot O$.

$\angle CAB$ is acute.

**Prove:** $m\angle CAB = \frac{1}{2}m\widehat{CA}$




**b.** Prove Theorem 10.13 if the angle in part **a** is obtuse.

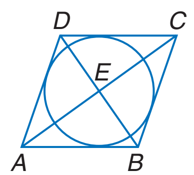
**EXTRA PRACTICE**
See pages 821, 837.

**Math Online**
Self-Check Quiz at
geometryonline.com

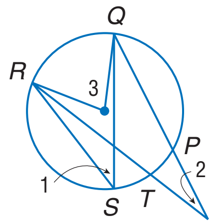
**H.O.T. Problems**

**42. OPEN ENDED** Draw a circle and one of its diameters. Call the diameter $\overline{AC}$. Draw a line tangent to the circle at $A$. What type of angle is formed by the tangent and the diameter? Explain.

**43. CHALLENGE** Circle $E$ is inscribed in rhombus $ABCD$. The diagonals of the rhombus are 10 centimeters and 24 centimeters long. To the nearest tenth centimeter, how long is the radius of circle $E$? (*Hint:* Draw an altitude from $E$.)



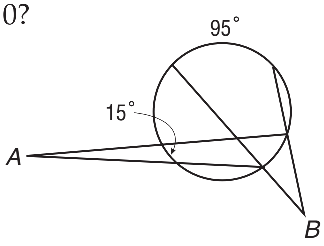
**44. CHALLENGE** In the figure, $\angle 3$ is a central angle. List the numbered angles in order from greatest measure to least measure. Explain your reasoning.




**45. *Writing in Math*** Refer to the information on page 599 to explain how you would calculate the angle representing how the light deviates from its original path. Include in your description the types of segments represented in the figure on page 599.

## STANDARDIZED TEST PRACTICE

**46.** What is the measure of $\angle B$ if $m\angle A = 10$?




**A** 30

**B** 35

**C** 47.5

**D** 90

**47. REVIEW** Larry's Fish Food comes in tubes that have a radius of 2 centimeters and a height of 7 centimeters. Odell bought a full tube, but he thinks he's used about $\frac{1}{4}$ of it. About how much is left?

**F** 88 $cm^3$

**G** 66 $cm^3$

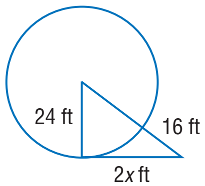
**H** 53 $cm^3$

**J** 41 $cm^3$

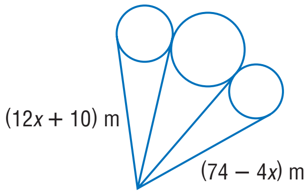
## Spiral Review

**Find $x$. Assume that segments that appear to be tangent are tangent.** (Lesson 10-5)

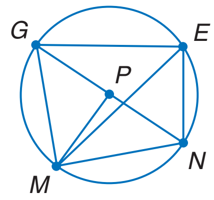
**48.**




**48.**




**In $\odot P$, $m\widehat{EN} = 66$ and $m\angle GPM = 89$. Find each measure.** (Lesson 10-4)




**50.** $m\angle EGN$

**51.** $m\angle GME$

**52.** $m\angle GNM$

**RAMPS For Exercises 53 and 54, use the following information.**
The Americans with Disabilities Act requires that wheelchair ramps have at least a 12-inch run for each rise of 1 inch. (Lesson 3-3)

**53.** Determine the slope represented by this requirement.

**54.** The maximum length the law allows for a ramp is 30 feet. How many inches tall is the highest point of this ramp?

## GET READY for the Next Lesson

**PREREQUISITE SKILL Use the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$, to solve each equation to the nearest tenth.**

**55.** $x^2 + 6x - 40 = 0$

**56.** $2x^2 + 7x - 30 = 0$

**57.** $3x^2 - 24x + 45 = 0$

# 10-7 Special Segments in a Circle

## Main Ideas

- Find measures of segments that intersect in the interior of a circle.
- Find measures of segments that intersect in the exterior of a circle.

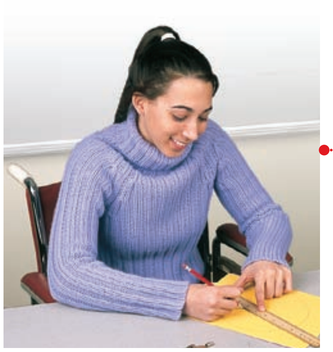
## GET READY for the Lesson

The U.S. Marshals Service is the nation's oldest federal law enforcement agency, serving the country since 1789. Appointed by the President, there are 94 U.S. Marshals, one for each federal court district in the country. The "Eagle Top" badge, introduced in 1941, was the first uniform U.S. Marshals badge.


**Segments Intersecting Inside a Circle** In Lesson 10-2, you learned how to find lengths of parts of a chord that is intersected by the perpendicular diameter. But how do you find lengths for other intersecting chords?

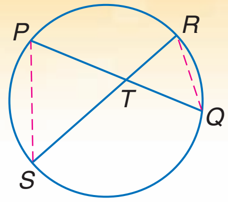
## GEOMETRY LAB

### Intersecting Chords



**MAKE A MODEL**

- Draw a circle and two intersecting chords.
- Name the chords $\overline{PQ}$ and $\overline{RS}$ intersecting at $T$.
- Draw $\overline{PS}$ and $\overline{RQ}$.




**ANALYZE**

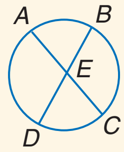
1. Name pairs of congruent angles. Explain your reasoning.
2. How are $\triangle PTS$ and $\triangle RTQ$ related? Why? Cut out both triangles, move them, and verify your conjecture.
3. **Make a conjecture** about the relationship of $\overline{PT}$, $\overline{TQ}$, $\overline{RT}$, and $\overline{ST}$.
4. Measure each angle and verify your conjecture.

The results of the lab suggest a proof for Theorem 10.15.

**THEOREM 10.15**

If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.

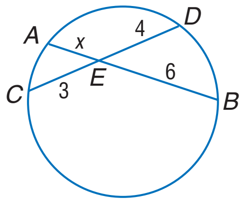
**Example:** $AE \cdot EC = BE \cdot ED$




You will prove Theorem 10.15 in Exercise 16.

## EXAMPLE Intersection of Two Chords

**1** **Find $x$.**




$AE \cdot EB = CE \cdot ED$

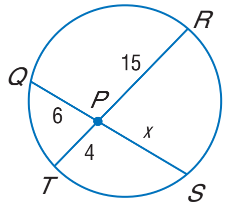
$x \cdot 6 = 3 \cdot 4$ Substitution

$6x = 12$ Multiply.

$x = 2$ Divide each side by 6.

**CHECK Your Progress**

**1.** Find $x$.




Intersecting chords can also be used to measure arcs.

### Real-World Link

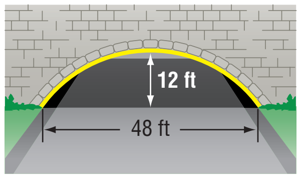
The Astrodome in Houston was the first ballpark to be built with a roof over the playing field. It originally had real grass and clear panels to allow sunlight in. This setup made it difficult to see the ball in the air, so they painted the ceiling and replaced the grass with carpet, which came to be known as "astro-turf."

**Source:** ballparks.com

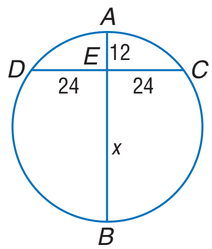
## Real-World EXAMPLE

Personal Tutor

**2** **TUNNELS Tunnels are constructed to allow roadways to pass through mountains. What is the radius of the circle containing the arc if the opening is not a semicircle?**




Draw a model using a circle. Let $x$ represent the unknown measure of the segment of diameter $\overline{AB}$. Use the products of the lengths of the intersecting chords to find the length of the diameter.




$AE \cdot EB = DE \cdot EC$ Segment products

$12x = 24 \cdot 24$ Substitution

$x = 48$ Divide each side by 12.

$AB = AE + EB$ Segment Addition Postulate

$AB = 12 + 48$ or $60$ Substitution and addition

Since the diameter is 60, $r = 30$.

**CHECK Your Progress**

**2. ASTRODOME** The highest point, or apex, of the Astrodome is 208 feet high, and the diameter of the circle containing the arc is 710 feet. How long is the stadium from one side to the other?

Online Personal Tutor at geometryonline.com

**Segments Intersecting Outside a Circle** Nonparallel chords of a circle that do not intersect inside the circle can be extended to form secants that intersect in the exterior of a circle. The special relationship among secant segments excludes the chord.

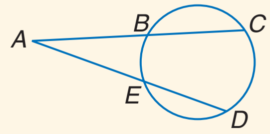
**Study Tip**

**Helping You Remember**

To remember this concept, the wording of Theorem 10.16 can be simplified by saying that each side of the equation is the product of the exterior part and the whole segment.

## THEOREM 10.16

If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

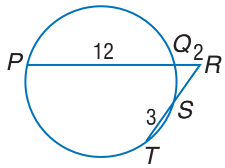



**Example:** $AB \cdot AC = AE \cdot AD$

You will prove Theorem 10.16 in Exercise 25.

## EXAMPLE 3 Intersection of Two Secants

**Find $RS$ if $PQ = 12$, $QR = 2$, and $TS = 3$.**

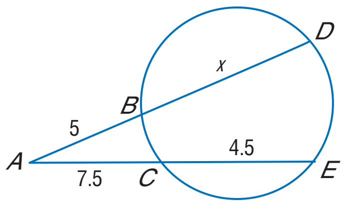



Let $RS = x$.

| | |
|---|---|
| $QR \cdot PR = RS \cdot RT$ | Secant Segment Products |
| $2 \cdot (12 + 2) = x \cdot (x + 3)$ | Substitution |
| $28 = x^2 + 3x$ | Distributive Property |
| $0 = x^2 + 3x - 28$ | Subtract 28 from each side. |
| $0 = (x + 7)(x - 4)$ | Factor. |
| $x + 7 = 0 \quad x - 4 = 0$ | |
| $x = -7 \quad x = 4$ | Disregard negative value. |

**CHECK Your Progress**

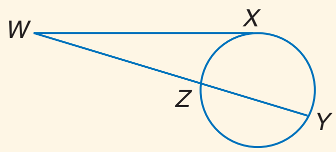
**3.** Find $x$.




The same secant segment product can be used with a secant segment and a tangent. In this case, the tangent is both the exterior part and the whole segment. This is stated in Theorem 10.17.

## THEOREM 10.17

If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.

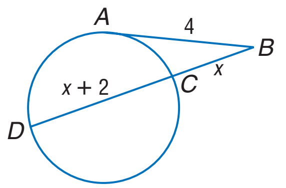



**Example:** $WX \cdot WX = WZ \cdot WY$

You will prove Theorem 10.17 in Exercise 26.

## EXAMPLE 4 Intersection of a Secant and a Tangent

**Find $x$. Assume that segments that appear to be tangent are tangent.**




$$(AB)^2 = BC \cdot BD$$

$$4^2 = x\,(x + x + 2)$$

$$16 = x\,(2x + 2)$$

$$16 = 2x^2 + 2x$$

$$0 = 2x^2 + 2x - 16$$

$$0 = x^2 + x - 8$$

This expression is not factorable. Use the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

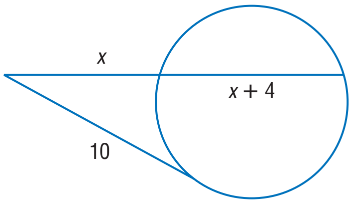
$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(-8)}}{2(1)}$$ $a = 1, b = 1, c = -8$

$$= \frac{-1 + \sqrt{33}}{2} \text{ or } x = \frac{-1 - \sqrt{33}}{2}$$ Disregard the negative solution.

$$\approx 2.37$$ Use a calculator.

**CHECK Your Progress**

**4.** Find $x$.

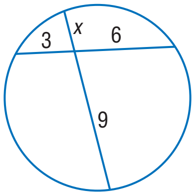



## CHECK Your Understanding

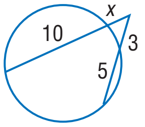
**Examples 1, 3, 4 (pp. 608, 609, 610)**

**Find $x$ to the nearest tenth. Assume that segments that appear to be tangent are tangent.**

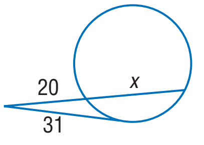
**1.**




**2.**

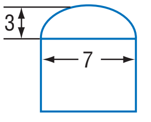



**3.**




**Example 2 (p. 608)**

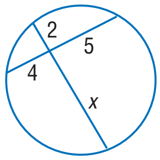
**4. HISTORY** The Roman Coliseum has many "entrances" in the shape of a door with an arched top. The ratio of the arch width to the arch height is 7:3. Find the ratio of the arch width to the radius of the circle that contains the arch.

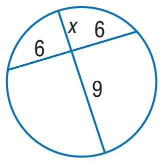


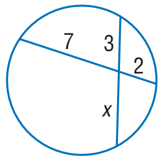
# Exercises

| HOMEWORK HELP | |
|---|---|
| For Exercises | See Examples |
| 5–7 | 1 |
| 8–9 | 2 |
| 10–12 | 3 |
| 13–15 | 4 |

**Find $x$ to the nearest tenth. Assume that segments that appear to be tangent are tangent.**

**5.**



**6.**



**7.**



**8. KNOBS** If you remove a knob from a kitchen appliance, you will notice that the hole is not completely round. Suppose the flat edge is 4 millimeters long and the distance from the curved edge to the flat edge is about 4.25 millimeters. Find the radius of the circle containing the hole.

**Real-World Career**

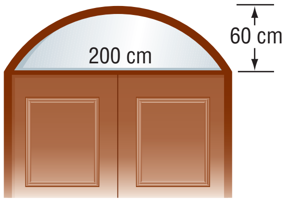
**Construction Worker**

Construction workers must know how to measure and fit shapes together to make a sound building that will last for years to come. These workers also must master using machines to cut wood and metal to certain specifications that are based on geometry.

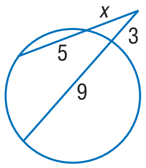
**Math Online**

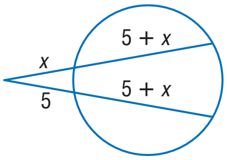
For more information, go to geometryonline.com.

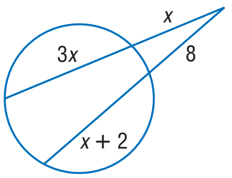
**9. ARCHITECTURE** An arch over a courtroom door is 60 centimeters high and 200 centimeters wide. Find the radius of the circle containing the arc of the arch.

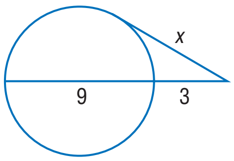


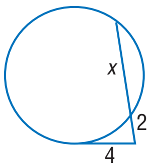

**Find $x$ to the nearest tenth. Assume that segments that appear to be tangent are tangent.**

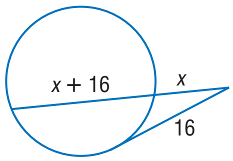
**10.**



**11.**



**12.**



**13.**


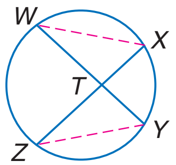

**14.**



**15.**



**16. PROOF** Copy and complete the proof of Theorem 10.15.

**Given:** $\overline{WY}$ and $\overline{ZX}$ intersect at $T$.

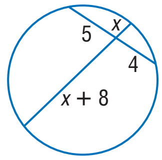
**Prove:** $WT \cdot TY = ZT \cdot TX$



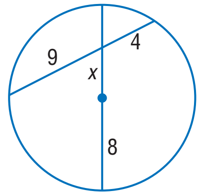

| Statements | Reasons |
|---|---|
| **a.** $\overline{WY}$ and $\overline{ZX}$ intersect at $T$ | **a.** Given |
| **b.** $\angle W \cong \angle Z$, $\angle X \cong \angle Y$ | **b.** ___?___ |
| **c.** ___?___ | **c.** AA Similarity |
| **d.** $\frac{WT}{ZT} = \frac{TX}{TY}$ | **d.** ___?___ |
| **e.** ___?___ | **e.** Cross products |

**Find each variable to the nearest tenth.**

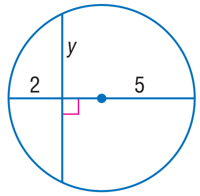
**17.**



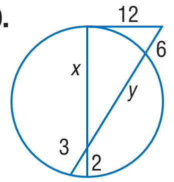

**18.**



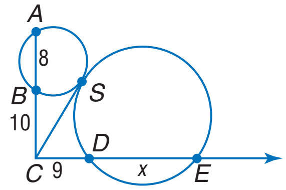

**19.**



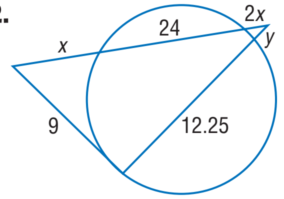

**20.**



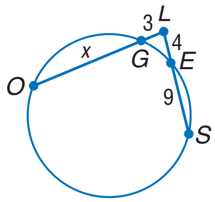

**21.**



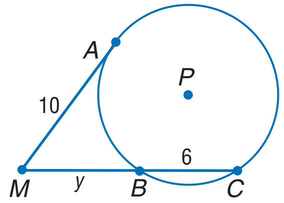

**22.**




**23.**



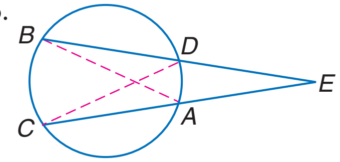

**24.**




**25. PROOF** Write a paragraph proof of Theorem 10.16.

**Given:** secants $\overline{EC}$ and $\overline{EB}$

**Prove:** $EA \cdot EC = ED \cdot EB$

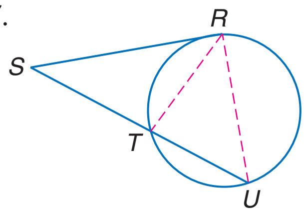



**EXTRA PRACTICE**
See pages 821, 837.
**Math Online**
Self-Check Quiz at
geometryonline.com

**26. PROOF** Write a two-column proof of Theorem 10.17.

**Given:** tangent $\overline{SR}$, secant $\overline{SU}$

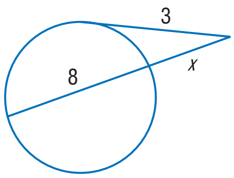
**Prove:** $(SR)^2 = ST \cdot SU$




**H.O.T. Problems**

**27. REASONING** Explain how the products for secant segments are similar to the products for a tangent and a secant segment.

**28. FIND THE ERROR** Becky and Latisha are writing products to find $x$. Who is correct? Explain your reasoning.




Becky

$3^2 = x \cdot 8$

$9 = 8x$

$\frac{9}{8} = x$

Latisha

$3^2 = x(x + 8)$

$9 = x^2 + 8x$

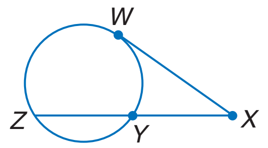
$0 = x^2 + 8x - 9$

$0 = (x + 9)(x - 1)$

$x = 1$

**29. OPEN ENDED** Draw a circle with two secant segments and one tangent segment that intersect at the same point. Give a real-life object that could be modeled by this drawing.

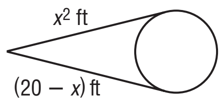
**30. CHALLENGE** In the figure, $Y$ is the midpoint of $\overline{XZ}$. Find $WX$. Explain how you know this.




**31. *Writing in Math*** Use the figure on page 607 to explain how the lengths of intersecting chords are related. Describe the segments that are formed by the intersecting segments, $\overline{AD}$ and $\overline{EF}$, and the relationship among these segments.

## STANDARDIZED TEST PRACTICE

**32.** Find two possible values for $x$ from the information in the figure.




**A** $-4, -5$

**B** $-4, 5$

**C** $4, 5$

**D** $4, -5$

**33. REVIEW** In the system of equations $4x + 3y = 6$ and $-5x + 2y = 13$, which expression can be substituted for $x$ in the equation $4x + 3y = 6$?

**F** $\frac{3}{2} - \frac{3}{4}y$

**G** $6 - 3y$

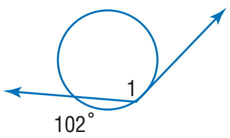
**H** $-\frac{13}{5} + \frac{2}{5}y$

**J** $13 - 2y$

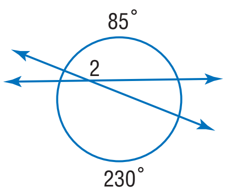
## Spiral Review

**Find the measure of each numbered angle. Assume that segments that appear tangent are tangent.** (Lesson 10-6)

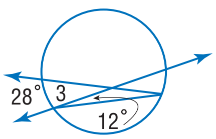
**34.**




**35.**

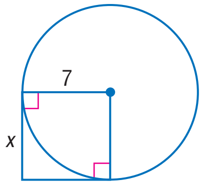



**36.**

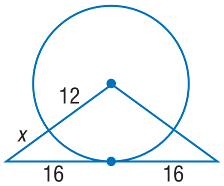



**Find $x$. Assume that segments that appear to be tangent are tangent.** (Lesson 10-5)

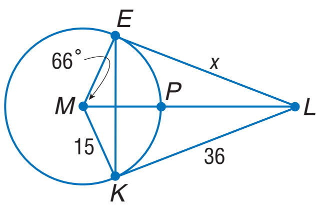
**37.**




**38.**




**39.**




**40. INDIRECT MEASUREMENT** Joseph is measuring the width of a stream to build a bridge over it. He picks out a rock across the stream as landmark $A$ and places a stone on his side as point $B$. Then he measures 5 feet at a right angle from $\overline{AB}$ and marks this $C$. From $C$, he sights a line to point $A$ on the other side of the stream and measures the angle to be about 67°. How far is it across the stream rounded to the nearest whole foot? (Lesson 8-5)

## GET READY for the Next Lesson

**PREREQUISITE SKILL Find the distance between each pair of points.** (Lesson 1-3)

**41.** $C(-2, 7), D(10, 12)$

**42.** $E(1, 7), F(3, 4)$

**43.** $G(9, -4), H(15, -2)$

# 10-8 Equations of Circles

## Main Ideas

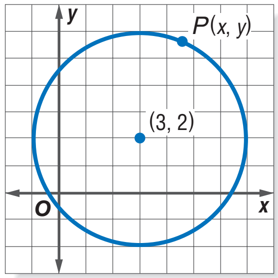
- Write the equation of a circle.
- Graph a circle on the coordinate plane.

## GET READY for the Lesson

When a rock enters the water, ripples move out from the center forming concentric circles. If the rock is assigned coordinates, each ripple can be modeled by an equation of a circle.

**Equation of a Circle** The fact that a circle is the *locus* of points in a plane equidistant from a given point creates an equation for any circle.

Suppose the center is at (3, 2) and the radius is 4. The radius is the distance from the center. Let $P(x, y)$ be the endpoint of any radius.




$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

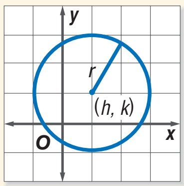
$4 = \sqrt{(x - 3)^2 + (y - 2)^2}$ $d = 4$, $(x_1, y_1) = (3, 2)$

$16 = (x - 3)^2 + (y - 2)^2$ Square each side.

Applying this same procedure to an unknown center $(h, k)$ and radius $r$ yields a general equation for any circle.

### KEY CONCEPT — Standard Equation of a Circle

An equation for a circle with center at $(h, k)$ and radius of $r$ units is $(x - h)^2 + (y - k)^2 = r^2$.




### Study Tip

**Equations of Circles**

Note that the equation of a circle is kept in the form shown above. The terms being squared are not expanded.

## EXAMPLE Equation of a Circle

**1** **Write an equation for the circle with center at (–2, 4), $d = 4$.**

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$[x - (-2)]^2 + [y - 4]^2 = 2^2$ $(h, k) = (-2, 4)$, If $d = 4$, $r = 2$.

$(x + 2)^2 + (y - 4)^2 = 4$ Simplify.

### CHECK Your Progress

**Write an equation for each circle described below.**

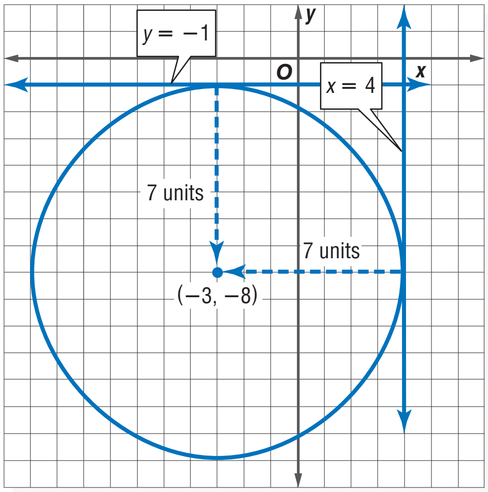
**1A.** center at (3, –2), $d = 10$ **1B.** center at origin, $r = 6$

Other information about a circle can be used to find the equation of the circle.

## EXAMPLE Use Characteristics of Circles

**2** **A circle with a diameter of 14 has its center in the third quadrant. The lines $y = -1$ and $x = 4$ are tangent to the circle. Write an equation of the circle.**

Sketch a drawing of the two tangent lines.




Since $d = 14$, $r = 7$. The line $x = 4$ is perpendicular to a radius. Since $x = 4$ is a vertical line, the radius lies on a horizontal line. Count 7 units to the left from $x = 4$. Find the value of $h$.

$$h = 4 - 7 \text{ or } -3$$

Likewise, the radius perpendicular to the line $y = -1$ lies on a vertical line. The value of $k$ is 7 units down from $-1$.

$$k = -1 - 7 \text{ or } -8$$

The center is at $(-3, -8)$, and the radius is 7. An equation for the circle is $(x + 3)^2 + (y + 8)^2 = 49$.

**CHECK Your Progress**

**2.** A circle with center at (5, 4) has a radius with endpoint at (−3, 4). Write an equation of the circle.

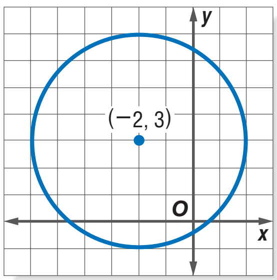
**Online** Personal Tutor at geometryonline.com

**Graph Circles** You can analyze the equation of a circle to find information that will help you graph the circle on a coordinate plane.

## EXAMPLE Graph a Circle

**3** **Graph $(x + 2)^2 + (y - 3)^2 = 16$.**

Compare each expression in the equation to the standard form.




| | |
|---|---|
| $(x - h)^2 = (x + 2)^2$ | $(y - k)^2 = (y - 3)^2$ |
| $x - h = x + 2$ | $y - k = y - 3$ |
| $-h = 2$ | $-k = -3$ |
| $h = -2$ | $k = 3$ |

$r^2 = 16$, so $r = 4$.

The center is at $(-2, 3)$, and the radius is 4.

Graph the center. Use a compass set at a width of 4 grid squares to draw the circle.

**Study Tip**

**Graphing Calculator**

To use the center and radius to graph a circle, select a suitable window that contains the center of the circle. For a TI-83/84 Plus, press ZOOM 5.

Then use **9: Circle (** on the **Draw** menu. Put in the coordinates of the center and then the radius so that the screen shows "Circle (−2, 3, 4)." Then press ENTER.

**CHECK Your Progress**

**3A.** $(x - 4)^2 + (y + 1)^2 = 9$ **3B.** $x^2 + y^2 = 25$

**Math Online** Extra Examples at geometryonline.com

If you know three points on the circle, you can find the center and radius of the circle and write its equation.

**Study Tip**

**Locus**
The center of the circle is the locus of points equidistant from the three given points. This is a **compound locus** because the point satisfies more than one condition.

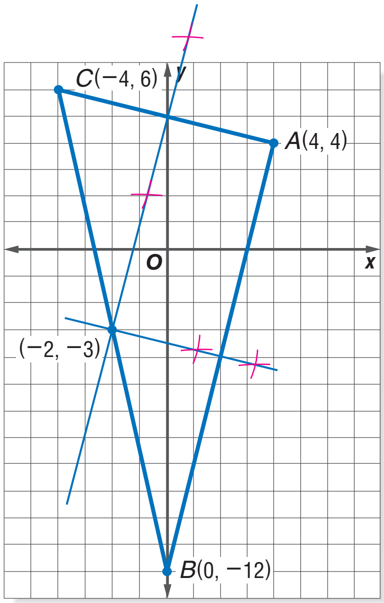
## Real-World EXAMPLE

**4 CELL PHONES Cell phones work by the transfer of phone signals from one tower to another via satellite. Cell phone companies try to locate towers so that they service multiple communities. Suppose three large metropolitan areas are modeled by the points $A(4, 4)$, $B(0, -12)$, and $C(-4, 6)$, and each unit equals 100 miles. Determine the location of a tower equidistant from all three cities, and write an equation for the circle.**

**Explore** You are given three points that lie on a circle.

**Plan** Graph $\triangle ABC$. Construct the perpendicular bisectors of two sides to locate the center, which is the location of the tower. Find the length of a radius. Use the center and radius to write an equation.

**Solve** Graph $\triangle ABC$ and construct the perpendicular bisectors of two sides. The center appears to be at $(-2, -3)$. This is the location of the tower.




Find $r$ by using the Distance Formula with the center and any of the three points.

$$r = \sqrt{[-2 - 4]^2 + [-3 - 4]^2}$$
$$= \sqrt{85}$$

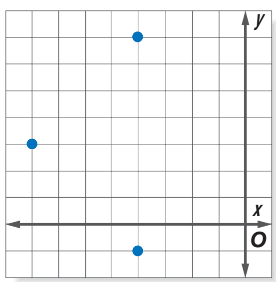
Write an equation.

$$[x - (-2)]^2 + [y - (-3)]^2 = (\sqrt{85})^2$$
$$(x + 2)^2 + (y + 3)^2 = 85$$

**Check** You can verify the location of the center by finding the equations of the two bisectors and solving a system of equations. You can verify the radius by finding the distance between the center and another of the three points on the circle.

**CHECK Your Progress**

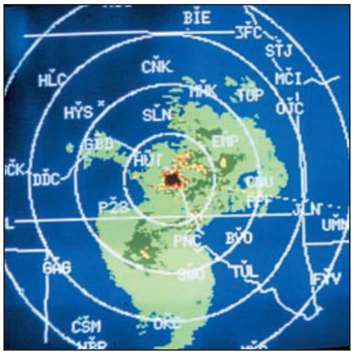
**4.** Three tornado sirens are placed strategically on a circle around a town so that they can be heard by all of the people. Write the equation of the circle on which they are placed.



## CHECK Your Understanding

**Example 1** (p. 614)

**1. WEATHER** Meteorologists track severe storms using Doppler radar. A polar grid is used to measure distances as the storms progress. If the center of the radar screen is the origin and each ring is 10 miles farther from the center, what is the equation of the fourth ring?



**Example 2** (p. 615)

**Write an equation for each circle described below.**

**2.** center at $(-3, 5)$, $r = 10$
**3.** center at origin, $r = \sqrt{7}$
**4.** diameter with endpoints at $(2, 7)$ and $(-6, 15)$
**5.** diameter with endpoints at $(-7, -2)$ and $(-15, 6)$

**Example 3** (p. 615)

**Graph each equation.**

**6.** $(x + 5)^2 + (y - 2)^2 = 9$
**7.** $(x - 3)^2 + y^2 = 16$

**Example 4** (p. 616)

**8.** Write an equation of a circle that contains $M(-2, -2)$, $N(2, -2)$, and $Q(2, 2)$. Then graph the circle.

## Exercises

**HOMEWORK HELP**

| For Exercises | See Examples |
|---|---|
| 9–16 | 1 |
| 17–21 | 2 |
| 22–27 | 3 |
| 28–29 | 4 |

**Write an equation for each circle described below.**

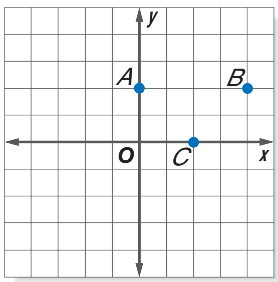
**9.** center at origin, $r = 3$
**10.** center at $(-2, -8)$, $r = 5$
**11.** center at $(1, -4)$, $r = \sqrt{17}$
**12.** center at $(0, 0)$, $d = 12$
**13.** center at $(5, 10)$, $r = 7$
**14.** center at $(0, 5)$, $d = 20$
**15.** center at $(-8, 8)$, $d = 16$
**16.** center at $(-3, -10)$, $d = 24$
**17.** a circle with center at $(-3, 6)$ and a radius with endpoint at $(0, 6)$.
**18.** a circle whose diameter has endpoints at $(2, 2)$ and $(-2, 2)$
**19.** a circle with center at $(-2, 1)$ and a radius with endpoint at $(1, 0)$
**20.** a circle with $d = 12$ and a center translated 18 units left and 7 units down from the origin
**21.** a circle with its center in quadrant I, radius of 5 units, and tangents $x = 2$ and $y = 3$

**Graph each equation.**

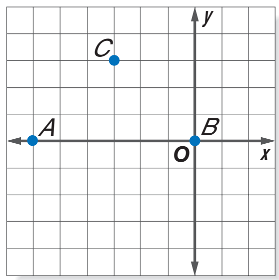
**22.** $x^2 + y^2 = 25$
**23.** $x^2 + y^2 = 36$
**24.** $x^2 + y^2 - 1 = 0$
**25.** $x^2 + y^2 - 49 = 0$
**26.** $(x - 2)^2 + (y - 1)^2 = 4$
**27.** $(x + 1)^2 + (y + 2)^2 = 9$

**Write an equation of the circle containing each set of points. Copy and complete the graph of the circle.**

**28.**




**29.**



**30.** Find the radius of a circle that has equation $(x - 2)^2 + (y - 3)^2 = r^2$ and contains (2, 5).

**31.** Find the radius of a circle that has equation $(x - 5)^2 + (y - 3)^2 = r^2$ and contains (5, 1).

**32.** **COORDINATE GEOMETRY** Refer to the Check part of Example 4. Verify the coordinates of the center by solving a system of equations that represent the perpendicular bisectors.

**MODEL ROCKETS For Exercises 33–35, use the following information.**

Different sized engines will launch model rockets to different altitudes. The higher the rocket goes, the larger the circle of possible landing sites becomes. Under normal wind conditions, the landing radius is three times the altitude of the rocket.

**33.** Write the equation of the landing circle for a rocket that travels 300 feet in the air.

**34.** What type of circles are modeled by the landing areas for engines that take the rocket to different altitudes?

**35.** What would the radius of the landing circle be for a rocket that travels 1000 feet in the air?

**36.** The equation of a circle is $(x - 6)^2 + (y + 2)^2 = 36$. Determine whether the line $y = 2x - 2$ is a secant, a tangent, or neither of the circle. Explain.

**37.** The equation of a circle is $x^2 - 4x + y^2 + 8y = 16$. Find the center and radius of the circle.

**38.** **WEATHER** The geographic center of Tennessee is near Murfreesboro. The closest Doppler weather radar is in Nashville. If Murfreesboro is designated as the origin, then Nashville has coordinates (−58, 55), where each unit is one mile. If the radar has a radius of 80 miles, write an equation for the circle that represents the radar coverage from Nashville.

**39.** **SPACE TRAVEL** Apollo 8 was the first manned spacecraft to orbit the Moon at an average altitude of 185 kilometers above the Moon's surface. Write an equation to model a single circular orbit of the command module if the radius of the Moon is 1740 kilometers. Let the center of the Moon be at the origin.

**Real-World Link**

The Apollo program was designed to successfully land a man on the moon. The first landing was July 20, 1969. There were a total of six landings on the moon during 1969–1972.

**Source:** infoplease.com

**EXTRA PRACTICE**

See pages 821, 837.

**Math Online**

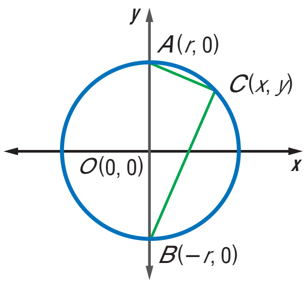
Self-Check Quiz at geometryonline.com

**40.** **RESEARCH** Use the Internet or other materials to find the closest Doppler radar to your home. Write an equation of the circle for the radar coverage if your home is the center.

**H.O.T. Problems**

**41.** **OPEN ENDED** Draw an obtuse triangle on a coordinate plane and construct the circle that circumscribes it.

**42.** **CHALLENGE** Write a coordinate proof to show that if an inscribed angle intercepts the diameter of a circle, as shown the angle is a right angle.




**43.** **REASONING** Explain how the definition of a circle leads to its equation.

**44.** ***Writing in Math*** Refer to the information on page 614 to describe the kinds of equations used to describe the ripples of a splash. Include the general form of the equation of a circle in your answer. Then produce the equations of five ripples if each ripple is 3 inches farther from the center.

## STANDARDIZED TEST PRACTICE

**45.** Which of the following is an equation of a circle with center at $(-2, 7)$ and a diameter of 18?

**A** $x^2 + y^2 - 4x + 14y + 53 = 324$

**B** $x^2 + y^2 + 4x - 14y + 53 = 81$

**C** $x^2 + y^2 - 4x + 14y + 53 = 18$

**D** $x^2 + y^2 + 4x - 14y + 53 = 3$

**46.** **REVIEW** A rectangle has an area of 180 square feet and a perimeter of 54 feet. What are the dimensions of the rectangle?

**F** 13 ft and 13 ft

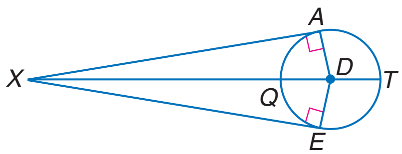
**G** 13 ft and 14 ft

**H** 15 ft and 12 ft

**J** 16 ft and 9 ft

## Spiral Review

**Find each measure if $EX = 24$ and $DE = 7$.** (Lesson 10-7)




**47.** $AX$

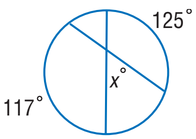
**48.** $DX$

**49.** $QX$

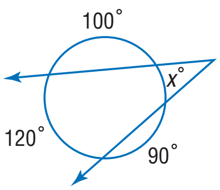
**50.** $TX$

**Find $x$.** (Lesson 10-6)

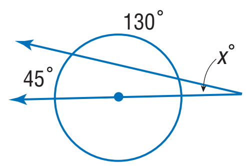
**51.**




**52.**




**53.**

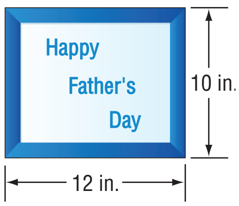



**Use the following information for Exercises 54 and 55.**
Triangle $ABC$ has vertices $A(-3, 2)$, $B(4, -1)$, and $C(0, -4)$.

**54.** What are the coordinates of the image after moving $\triangle ABC$ 3 units left and 4 units up? (Lesson 9-2)

**55.** What are the coordinates of the image of $\triangle ABC$ after a reflection in the $y$-axis? (Lesson 9-1)

**56.** **CRAFTS** For a Father's Day present, a kindergarten class is making foam plaques. The edge of each plaque is covered with felt ribbon all the way around with 1 inch overlap. There are 25 children in the class. How much ribbon does the teacher need for all 25 children to complete this craft? (Lesson 1-6)



# CHAPTER 10 Study Guide and Review

STUDY TO GO

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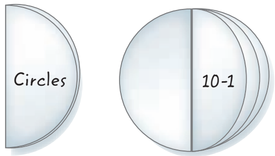
## FOLDABLES™ Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.




## Key Concepts

### Circles and Circumference (Lesson 10-1)

- Circumference: $C = \pi d$ or $C = 2\pi r$

### Angles, Arcs, Chords, and Inscribed Angles

(Lessons 10-2 to 10-4)

- The sum of the measures of the central angles of a circle is 360. The measure of each arc is related to the measure of its central angle.
- The length of an arc is proportional to the length of the circumference.
- Diameters perpendicular to chords bisect chords and intercepted arcs.
- The measure of the inscribed angle is half the measure of its intercepted arc.

### Tangents, Secants, and Angle Measures

(Lessons 10-5 and 10-6)

- A line that is tangent to a circle intersects the circle in exactly one point and is perpendicular to a radius.
- Two segments tangent to a circle from the same exterior point are congruent.
- The measure of an angle formed by two secant lines is half the positive difference of its intercepted arcs.
- The measure of an angle formed by a secant and tangent line is half its intercepted arc.

### Special Segments and Equation of a Circle

(Lessons 10-7 and 10-8)

- The lengths of intersecting chords in a circle can be found by using the products of the measures of the segments.
- The equation of a circle with center $(h, k)$ is $(x - h)^2 + (y - k)^2 = r^2$.

## Key Vocabulary

| | |
|---|---|
| arc (p. 564) | intercepted (p. 578) |
| center (p. 554) | major arc (p. 564) |
| central angle (p. 563) | minor arc (p. 564) |
| chord (p. 554) | pi ($\pi$) (p. 556) |
| circle (p. 554) | point of tangency (p. 588) |
| circumference (p. 556) | radius (p. 554) |
| circumscribed (p. 571) | secant (p. 599) |
| diameter (p. 554) | semicircle (p. 564) |
| inscribed (p. 571) | tangent (p. 588) |

## Vocabulary Check

**Choose the term that best matches each phrase. Choose from the list above.**

**1.** a line that intersects a circle in exactly one point

**2.** a polygon with all of its vertices on the circle

**3.** an angle with a vertex that is at the center of the circle

**4.** a segment that has its endpoints on the circle

**5.** a line that intersects a circle in exactly two points

**6.** the distance around a circle

**7.** a chord that passes through the center of a circle

**8.** an irrational number that is the ratio of $\frac{C}{d}$

**9.** an arc that measures greater than 180

**10.** a point where a circle meets a tangent

**11.** locus of all points in a plane equidistant from a given point

**12.** a central angle separates the circle into two of these



Math Online Vocabulary Review at geometryonline.com

# Lesson-by-Lesson Review

## 10–1 Circles and Circumference (pp. 554–561)

**The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary.**

**13.** $d = 15$ in., $r = \underline{?}$, $C = \underline{?}$

**14.** $C = 68$ yd, $r = \underline{?}$, $d = \underline{?}$

**15.** $r = 11$ mm, $d = \underline{?}$, $C = \underline{?}$

**16. BICYCLES** If the circumference of the bicycle tire is 81.7 inches, how long is one spoke?


**Example 1** **Find $r$ to the nearest hundredth if $C = 76.2$ feet.**

$C = 2\pi r$ Circumference formula

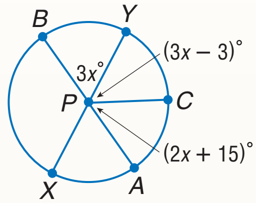
$76.2 = 2\pi r$ Substitution

$\frac{76.2}{2\pi} = r$ Divide each side by $2\pi$.

$12.13 \approx r$ Use a calculator.

## 10–2 Measuring Angles and Arcs (pp. 563–569)

**Find each measure.**



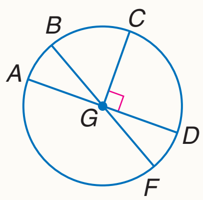

**17.** $m\widehat{YC}$

**18.** $m\widehat{BC}$

**19.** $m\widehat{BX}$

**20.** $m\widehat{BCA}$

**In $\odot G$, $m\angle AGB = 30$ and $\overline{CG} \perp \overline{GD}$. Find each measure.**



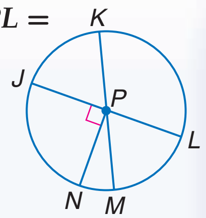

**21.** $m\widehat{AB}$ **22.** $m\widehat{BC}$

**23.** $m\widehat{FD}$ **24.** $m\widehat{CDF}$

**25.** $m\widehat{BCD}$ **26.** $m\widehat{FAB}$

**27. CLOCKS** If a clock has a diameter of 6 inches, what is the distance along the edge of the clock from the minute hand to the hour hand at 5:00?

**Example 2** **In $\odot P$, $m\angle MPL = 65$ and $\overline{NP} \perp \overline{PL}$.**




**a. Find $m\widehat{NM}$.**

$\widehat{NM}$ is a minor arc, so $m\widehat{NM} = m\angle NPM$. $\angle JPN$ is a right angle and $m\angle MPL = 65$, so $m\angle NPM = 25$.

$m\widehat{NM} = 25$

**b. Find $m\widehat{NJK}$.**

$\widehat{NJK}$ is composed of adjacent arcs $\widehat{NJ}$ and $\widehat{JK}$. $\angle MPL \cong \angle JPK$, so $m\angle JPK = 65$.

$m\widehat{NJ} = m\angle NPJ$ or 90 $\angle NPJ$ is a right angle.

$m\widehat{NJK} = m\widehat{NJ} + m\widehat{JK}$ Arc Addition Postulate

$m\widehat{NJK} = 90 + 65$ or 155 Substitution

## 10–3 Arcs and Chords (pp. 570–577)

**In $\odot R$, $SU = 20$, $YW = 20$, and $m\widehat{YX} = 45$. Find each measure.**

**28.** $SV$

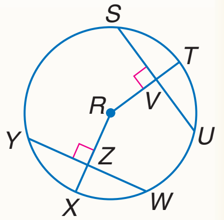
**29.** $WZ$

**30.** $UV$

**31.** $m\widehat{YW}$

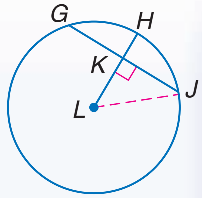
**32.** $m\widehat{ST}$

**33.** $m\widehat{SU}$




**34. ART** Leonardo DaVinci saw the ideal proportions for man as being measured in relation to two geometric shapes: the circle and the square. The square inscribed in a circle and the circle inscribed in a square are useful to artists, architects, engineers, and designers. Find the measure of each arc of the circle circumscribed about the square.

**Example 4** **Circle $L$ has a radius of 32. Find $LK$ if $GJ = 40$.**




Draw radius $\overline{LJ}$. $LJ = 32$ and $\triangle LKJ$ is a right triangle.

Since $\overline{LH} \perp \overline{GJ}$, $\overline{LH}$ bisects $\overline{GJ}$.

$KJ = \frac{1}{2}(GJ)$ — Definition of segment bisector

$= \frac{1}{2}(40)$ or 20 — $GJ = 40$

Use the Pythagorean Theorem to find $LK$.

$(LK)^2 + (KJ)^2 = (LJ)^2$ — Pythagorean Theorem

$(LK)^2 + 20^2 = 32^2$ — $KJ = 20$, $LJ = 32$

$(LK)^2 + 400 = 1024$ — Simplify.

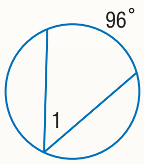
$(LK)^2 = 624$ — Subtract.

$LK = \sqrt{624}$ or about 24.98

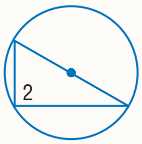
## 10–4 Inscribed Angles (pp. 578–586)

**Find the measure of each numbered angle.**

**35.**




**36.**

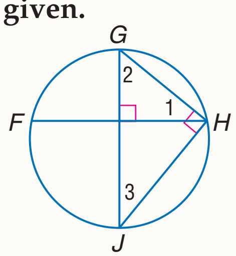



**Find the measure of each numbered angle for each situation given.**

**37.** $m\widehat{GH} = 78$

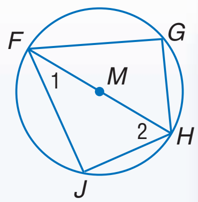
**38.** $m\angle 2 = 2x$, $m\angle 3 = x$

**39.** $m\widehat{JH} = 114$




**40. ICE SKATING** Sara skates on a circular ice rink and inscribes quadrilateral $ABCD$ in the circle. If $m\angle A = 120$ and $m\angle B = 66$, find $m\angle C$ and $m\angle D$.

**Example 5** **Triangle $FGH$ and $FHJ$ are inscribed in $\odot M$ with $FG \cong FJ$. Find $x$ if $m\angle 1 = 6x - 5$, and $m\angle 2 = 7x + 4$.**




$FJH$ is a right angle because $\widehat{FJH}$ is a semicircle.

$m\angle 1 + m\angle 2 + m\angle FJH = 180$ — Angle Sum Th.

$(6x - 5) + (7x + 4) + 90 = 180$ — Substitution

$13x + 89 = 180$ — Simplify.

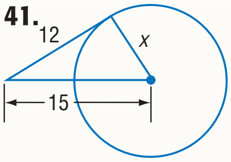
$x = 7$ — Solve for $x$.

**Mixed Problem Solving**
For mixed problem-solving practice, see page 835.

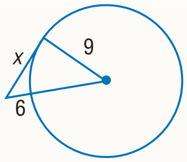
## 10–5 Tangents (pp. 588–596)

Find $x$. Assume that segments that appear to be tangent are tangent.

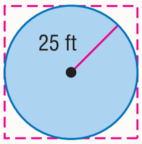
**41.**



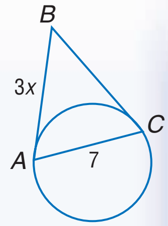

**42.**




**43. SPRINKLER** A sprinkler waters a circular section of lawn that is surrounded by a fenced-in square field. If the spray extends to a distance of 25 feet, what is the total length of the fence around the field?




**Example 6** Given that the perimeter of $\triangle ABC = 25$, find $x$. Assume that segments that appear to be tangent to circles are tangent.




In the figure, $\overline{AB}$ and $\overline{BC}$ are drawn from the same exterior point and are tangent to $\odot Q$. So, $\overline{AB} \cong \overline{BC}$.

The perimeter of the triangle, $AB + BC + AC$, is 25.

$AB + BC + AC = 25$ Definition of perimeter

$3x + 3x + 7 = 25$ $AB = BC = 3x$, $AC = 7$

$6x + 7 = 25$ Simplify.

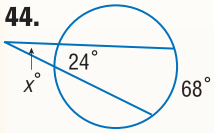
$6x = 18$ Subtract 7 from each side.

$x = 3$ Divide each side by 6.

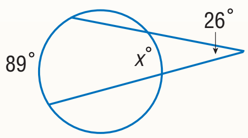
## 10–6 Secants, Tangents, and Angle Measures (pp. 599–606)

Find $x$.

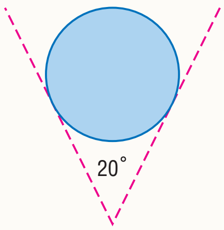
**44.**



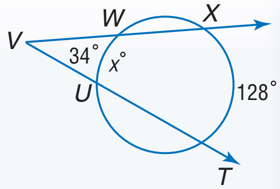

**45.**




**46. JEWELRY** Mary has a circular pendant hanging from a chain around her neck. The chain is tangent to the pendant and then forms an angle of 20° below the pendant. Find the measure of the arc at the bottom of the pendant.




**Example 7** Find $x$.




$m\angle V = \frac{1}{2}(m\widehat{XT} - m\widehat{WU})$

$34 = \frac{1}{2}(128 - x)$ Substitution

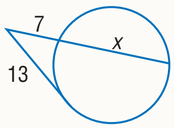
$-30 = -\frac{1}{2}x$ Simplify.

$x = 60$ Multiply each side by −2.

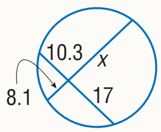
## 10–7 Special Segments in a Circle (pp. 607–613)

**Find $x$ to the nearest tenth. Assume that segments that appear to be tangent are tangent.**

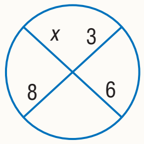
**47.**



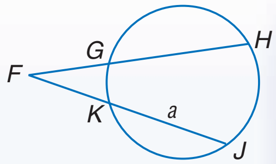

**48.**




**49. LAMPSHADE** The top of a lampshade is a circle with two intersecting chords. Use the figure to find $x$.




**Example 8** **Find $a$, if $FG = 18$, $GH = 42$, and $FK = 15$.**




Let $KJ = a$.

| | |
|---|---|
| $FK \cdot FJ = FG \cdot FH$ | Secant Segment Products |
| $15(a + 15) = 18(18 + 42)$ | Substitution |
| $15a + 225 = 1080$ | Distributive Property |
| $15a = 855$ | Subtract. |
| $a = 57$ | Divide each side by 15. |

## 10–8 Equations of Circles (pp. 614–619)

**Write an equation for each circle.**

**50.** center at $(0, 0)$, $r = \sqrt{5}$

**51.** center at $(-4, 8)$, $d = 6$

**52.** center at $(-1, 4)$ and is tangent to $x = 1$

**Graph each equation.**

**53.** $x^2 + y^2 = 2.25$

**54.** $(x - 4)^2 + (y + 1)^2 = 9$

**For Exercises 55 and 56, use the following information.**

A circle graphed on a coordinate plane contains $A(0, 6)$, $B(6, 0)$, $C(6, 6)$.

**55.** Write an equation of the circle.

**56.** Graph the circle.

**57. PIZZA** A pizza parlor is located at the coordinates (7, 3) on a coordinate grid. The pizza parlor's delivery service extends for 15 miles. Write the equation of the circle which represents the outer edge of the pizza delivery service area.

**Example 9** **Write an equation of a circle with center $(-1, 4)$ and radius 3.**

| | |
|---|---|
| $(x - h)^2 + (y - k)^2 = r^2$ | Equation of a circle |
| $[x - (-1)]^2 + (y - 4)^2 = 3^2$ | $h = -1, k = 4, r = 3$ |
| $(x + 1)^2 + (y - 4)^2 = 9$ | Simplify. |

**Example 10**
**Graph $(x - 2)^2 + (y + 3)^2 = 6.25$.**

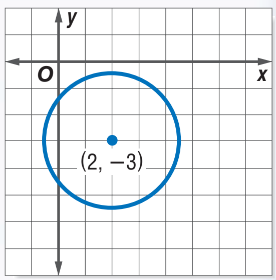
Identify the values of $h$, $k$, and $r$ by writing the equation in standard form.

$$(x - 2)^2 + (y + 3)^2 = 6.25$$

$$(x - 2)^2 + [y - (-3)]^2 = 2.5^2$$

$h = 2$, $k = -3$, and $r = 2.5$

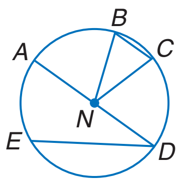
Graph the center $(2, -3)$ and use a compass to construct a circle with radius 2.5 units.



# Chapter 10 Practice Test

1. Determine the radius of a circle with circumference $25\pi$ units. Round to the nearest tenth.

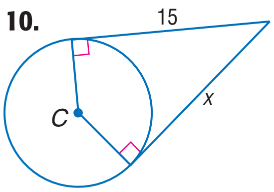
**For Questions 2–9, refer to $\odot N$.**



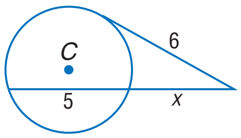

2. Name the radii of $\odot N$.
3. If $AD = 24$, find $CN$.
4. Is $ED > AD$? Explain.
5. If $AN$ is 5 meters long, find the exact circumference of $\odot N$.
6. If $m\angle BNC = 20$, find $m\widehat{BC}$.
7. If $\overline{BE} \cong \overline{ED}$ and $m\widehat{ED} = 120$, find $m\widehat{BE}$.
8. If $m\widehat{BC} = 30$ and $\widehat{AB} \cong \widehat{CD}$, find $m\widehat{AB}$.
9. If $\widehat{AE} = 75$, find $m\angle ADE$.

**Find $x$. Assume that segments that appear to be tangent are tangent.**

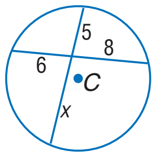
**10.**



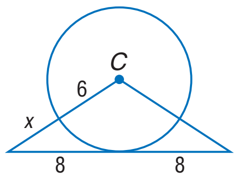

**11.**



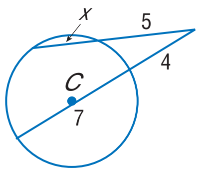

**12.**



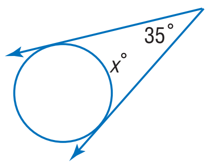

**13.**



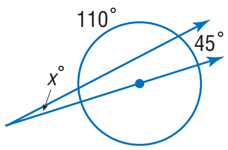

**14.**



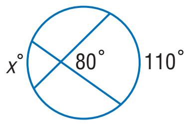

**15.**




**16.**




**17.**




18. **AMUSEMENT RIDES** Suppose a Ferris wheel is 50 feet wide. Approximately how far does a rider travel in one rotation of the wheel?

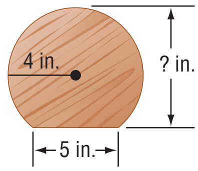
19. Write an equation of a circle with center at $(-2, 5)$ and a diameter of 50.

20. **EARTHQUAKES** When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Suppose a seismograph station determines that the epicenter of an earthquake is located 63 miles from the station. If the station is located at the origin, write an equation for the circle that represents a possible epicenter of the earthquake.

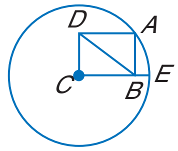
21. Graph $(x - 1)^2 + (y + 2)^2 = 4$.

22. **PROOF** Write a two-column proof.
**Given:** $\odot X$ with diameters $\overline{RS}$ and $\overline{TV}$
**Prove:** $\widehat{RT} \cong \widehat{VS}$

23. **CRAFTS** Takita is making bookends out of circular wood pieces, as shown at the right. What is the height of the cut piece of wood?




24. **MULTIPLE CHOICE** Circle $C$ has radius $r$ and $ABCD$ is a rectangle. Find $DB$.




**A** $r$

**B** $r\frac{\sqrt{2}}{2}$

**C** $r\sqrt{3}$

**D** $r\frac{\sqrt{3}}{2}$

25. **CONSTRUCTION** An arch over a doorway is 2 feet high and 7 feet wide. Find the radius of the circle containing the arch.

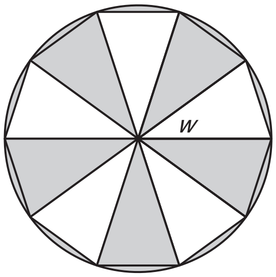
Math Online Chapter Test at geometryonline.com

# Standardized Test Practice

**Cumulative, Chapters 1–10**

**Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.**

**1.** A regular decagon is drawn in a circle as a design on the cover of a school yearbook. Opposite vertices are connected by a line segment.




What is the measure of $\angle w$?

**A** 45° **C** 60°

**B** 50° **D** 90°

**2.** The vertices of $\triangle EFG$ are $E(3, 1)$, $F(4, 5)$, $G(1, -2)$. If $\triangle EFG$ is translated 2 units down and 3 units to the right to create $\triangle MNP$, what are the coordinates of the vertices of $\triangle MNP$?

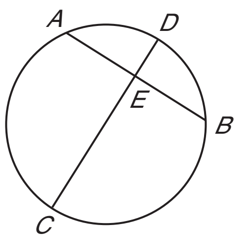
**F** $M(1, 4)$, $N(3, 8)$, $P(-1, 1)$

**G** $M(5, 4)$, $N(6, 8)$, $P(3, 1)$

**H** $M(6, -1)$, $N(7, 3)$, $P(4, -4)$

**J** $M(6, 3)$, $N(7, 7)$, $P(4, 0)$

**3. GRIDDABLE** In the circle below, $\overline{AB}$ and $\overline{CD}$ are chords intersecting at $E$.




If $AE = 8$, $DE = 4$, and $EB = 9$, what is the length of $\overline{EC}$?

**4. ALGEBRA** Which inequality is equivalent to $7x < 9x - 3(2x + 5)$?

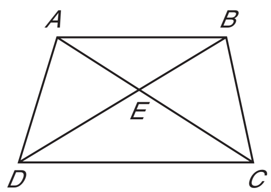
**A** $4x < 15$

**B** $4x > -15$

**C** $4x < -15$

**D** $4x > 15$

**5.** Trapezoid $ABCD$ is shown below.




Which pair of triangles can be established as similar to prove that $\frac{AE}{AB} = \frac{EC}{DC}$?

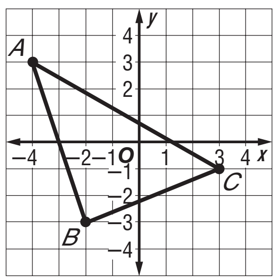
**F** $\triangle ADB$ and $\triangle BCA$

**G** $\triangle AEB$ and $\triangle CED$

**H** $\triangle ADC$ and $\triangle BCD$

**J** $\triangle AED$ and $\triangle BEC$

**6.** If $\triangle ABC$ is reflected across the $y$-axis, what are the coordinates of $C'$?




**A** (3, 1)

**B** (−1, 3)

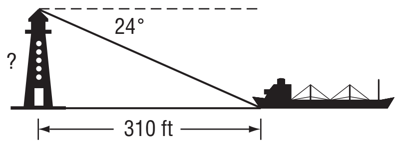
**C** (−3, −1)

**D** (−1, −3)

## Preparing for Standardized Tests

For test-taking strategies and more practice, see pages 841–856.

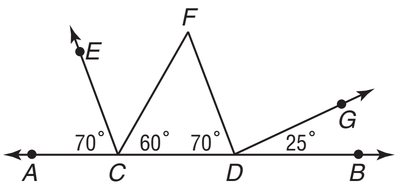
**7. GRIDDABLE** A passing ship is 310 feet from the base of a lighthouse. The angle of depression from the top of the lighthouse to the ship is 24°.




| |
|---|
| sin 24° ≈ 0.41 |
| cos 24° ≈ 0.91 |
| tan 24° ≈ 0.45 |

What is the height of the lighthouse in feet to the nearest tenth?

**8.** Which of the following statements is true?



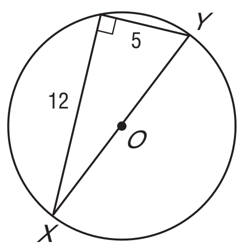

**F** $\overline{CE} \cong \overline{DF}$

**G** $\overline{CF} \parallel \overline{DG}$

**H** $\overline{CF} \cong \overline{DF}$

**J** $\overline{CE} \parallel \overline{DF}$

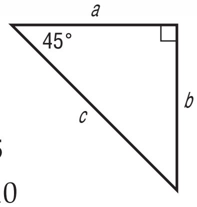
**9.** If $\overline{XY}$ is a diameter of circle $O$, what is the circumference of circle $O$?




**A** $5\pi$ **C** $7.5\pi$

**B** $7\pi$ **D** $13\pi$

**10.** If $a = 5\sqrt{2}$ in the right triangle below, what is the value of $c$?



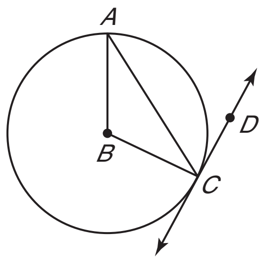

**F** $5\sqrt{6}$ **H** 5

**G** $10\sqrt{2}$ **J** 10

**TEST-TAKING TIP**

**Question 10** Question 10 is a multi-step problem. First, identify the figure shown as a 45°-45°-90° right triangle. Next, use the relationship between the sides in this kind of triangle to determine that $c = \sqrt{2}a$. Finally, recall from algebra that $\sqrt{m} \cdot \sqrt{n} = \sqrt{m \cdot n}$.

**11. GRIDDABLE** $\overleftrightarrow{CD}$ is tangent at point $C$ to a circle, with $B$ at its center. $\overline{BC}$ is a radius. If $m\angle ACB = 35°$, what is $m\angle ACD$?




**12.** A triangle is dilated so that the ratio between the areas of the triangle and its image is 9 to 8. What is the ratio between the perimeters of the two triangles?

**A** 3 to 22 **C** 18 to 16

**B** 4.5 to 4 **D** 81 to 64

### Pre-Ap

**Record your answers on a sheet of paper. Show your work.**

**13.** The segment with endpoints $A(1, -2)$ and $B(1, 6)$ is the diameter of a circle.

**a.** Graph the points and draw the circle.

**b.** What is the circumference of the circle?

**c.** What is the equation of the circle?

| NEED EXTRA HELP? | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| If You Missed Question... | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| Go to Lesson or Page... | 10-4 | 9-2 | 10-7 | 783 | 7-3 | 8-5 | 9-1 | 3-5 | 10-1 | 8-3 | 10-5 | 9-5 | 10-8 |