CHAPTER 11

Areas of Polygons and Circles

BIG Ideas

- Find areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, and circles.
- Find areas of composite figures.
- Find geometric probability and areas of sectors and segments of circles.

Key Vocabulary

apothem (p. 649)

composite figure (p. 658)

geometric probability (p. 665)

sector (p. 666)

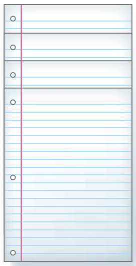
Real-World Link

Hang Gliding The shape of the nylon parachute of the hang glider comprises two triangular wings.

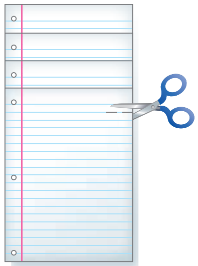
FOLDABLES™ Study Organizer

Areas of Polygons and Circles Make this Foldable to help you organize your notes. Begin with five sheets of notebook paper.

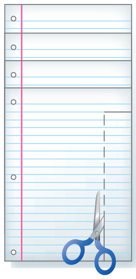
1. **Stack** 4 of the 5 sheets of notebook paper as illustrated.



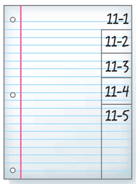
2. **Cut** in about 1 inch along the heading line on the top sheet of paper.



3. **Cut** the margins off along the right edge.



4. **Stack** in order of cuts, placing the uncut fifth sheet at the back. Label the tabs as shown. Staple edge to form a book.



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GET READY for Chapter 11

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at geometryonline.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

The area and width of a rectangle are given. Find the length of the rectangle. (Lesson 1-6)

1. $A = 150, w = 15$ **2.** $A = 38, w = 19$

3. $A = 21.16, w = 4.6$ **4.** $A = 2000, w = 32$

5. $A = 450, w = 25$ **6.** $A = 256, w = 20$

7. GARDENS The area of a rectangular garden is 115 square feet. If the width is 11 feet, what is the length? Round to the nearest tenth. (Lesson 1-6)

Evaluate each expression for $a = 6$, $b = 8$, $c = 10$, and $d = 11$. (Prerequisite Skills)

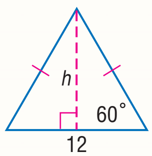
8. $\frac{1}{2}a(b + c)$ **9.** $\frac{1}{2}ab$

10. $\frac{1}{2}(2b + c)$ **11.** $\frac{1}{2}d(a + c)$

12. $\frac{1}{2}(b + c)$ **13.** $\frac{1}{2}cd$

Find h in each triangle. (Lesson 8-3)

14.



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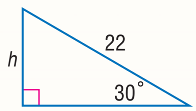
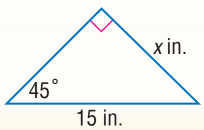


16. WINDOWS Miss Valdez has a triangular window pane above the door of her house, as shown. Find the length of each of the legs of the triangle. (Lesson 8-3)



QUICK Review

EXAMPLE 1

The area of a rectangle is 81 square units and the width is 3 units. Find the length.

$A = \ell w$	Definition of Area
$81 = \ell(3)$	Substitution
$27 = \ell$	Divide each side by 3.

A rectangle with an area of 81 square units and a width of 3 units has a length of 27 units.

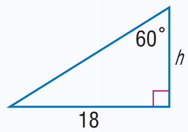
EXAMPLE 2

Evaluate $\frac{1}{2}(2x + y)$ for $x = 5$ and $y = 18$.

$$\begin{aligned} \tfrac{1}{2}(2x + y) &= \tfrac{1}{2}(2(5) + 18) && \text{Substitution} \\ &= \tfrac{1}{2}(10 + 18) && \text{Multiply.} \\ &= \tfrac{1}{2}(28) \text{ or } 14 && \text{Simplify.} \end{aligned}$$

EXAMPLE 3

Find h in the triangle.



By the Angle Sum Theorem, the third angle is 30°. The sides of a 30°-60°-90° triangle are in the ratio $x{:}x\sqrt{3}{:}2x$.

$$x\sqrt{3} = 18$$

$$x = \frac{18}{\sqrt{3}} \text{ or } 6\sqrt{3}$$

Since h is opposite the 30° angle, $h = 6\sqrt{3}$.

11-1 Areas of Parallelograms

Main Ideas

- Find perimeters and areas of parallelograms.
- Determine whether points on a coordinate plane define a parallelogram.

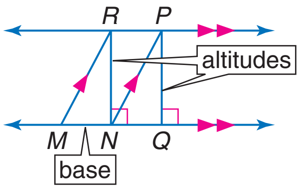
New Vocabulary

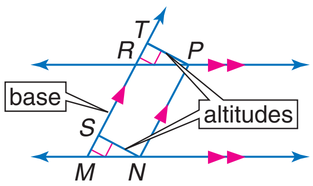
height of a parallelogram

GET READY for the Lesson

Land is usually measured in acres. Acre is a historic Saxon term that means "field." An acre was a unit of measure that represented a field that could be plowed in one day. Unlike other units of measure for area, an acre is not a square, but a rectangle 22 yards by 220 yards.

Areas of Parallelograms Recall that a *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a base. For each base, any segment that is perpendicular to the base is an altitude. The length of an altitude is called the **height of the parallelogram**.



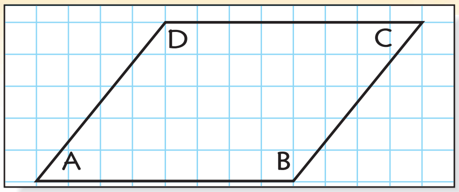


GEOMETRY LAB

Area of a Parallelogram

MODEL

- Draw parallelogram *ABCD* on grid paper. Label the vertices on the interior of the angles with letters *A*, *B*, *C*, and *D*.
- Fold ▱*ABCD* so that *A* lies on *B* and *C* lies on *D*, forming a rectangle.



ANALYZE

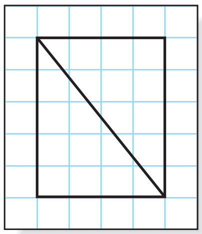
1. What is the area of the rectangle?

2. How many rectangles form the parallelogram?

3. What is the area of the parallelogram?

4. How do the base and altitude of the parallelogram relate to the length and width of the rectangle?

5. **MAKE A CONJECTURE** Use what you observed to write a formula for the area of a parallelogram.



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Study Tip

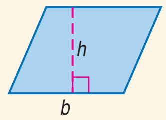
Units

Length is measured in linear units, and area is measured in square units.

The Geometry Lab leads to the formula for the area of a parallelogram.

KEY CONCEPT — *Area of a Parallelogram*

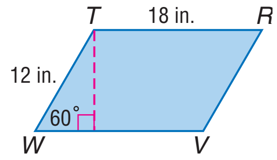
Words If a parallelogram has an area of A square units, a base of b units, and a height of h units, then area equals the product of the base and the height.



Symbols $A = bh$

EXAMPLE 1 Perimeter and Area of a Parallelogram

Find the perimeter and area of ▱*TRVW*.



Bases and Sides: Each pair of opposite sides of a parallelogram has the same measure. Each base is 18 inches long, and each side is 12 inches long.

Perimeter: The perimeter of a polygon is the sum of the measures of its sides. So, the perimeter of ▱*TRVW* is 2(18) + 2(12) or 60 inches.

Study Tip

Look Back

To review **special right triangles**, see Lesson 8-3.

Height: Use a 30°-60°-90° triangle to find the height. Recall that if the measure of the leg opposite the 30° angle is x, then the length of the hypotenuse is $2x$, and the length of the leg opposite the 60° angle is $x\sqrt{3}$.

$12 = 2x$ Substitute 12 for the hypotenuse.

$6 = x$ Divide each side by 2.

So, the height of the parallelogram is $x\sqrt{3}$ or $6\sqrt{3}$ inches.

Area:

$A = bh$ Area of a parallelogram

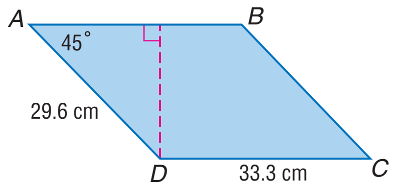
$= 18(6\sqrt{3})$ $b = 18$, $h = 6\sqrt{3}$

$= 108\sqrt{3}$ or about 187.1

The perimeter of ▱*TRVW* is 60 inches, and the area is about 187.1 square inches.

CHECK Your Progress

1. Find the perimeter and area of ▱*ABCD*.



Real-World EXAMPLE

2 **INTERIOR DESIGN** **The Navarros are painting a room in their house. The rectangular room is 14 feet long and 12 feet wide. The walls are 10 feet high. Find the area of the walls to be painted.**

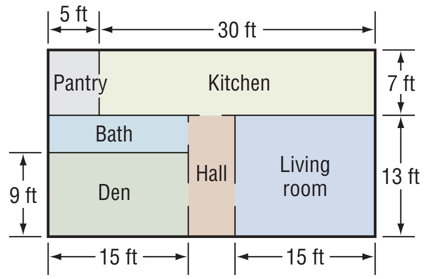
The dimensions of the floor and the height of the walls are given. The area of the walls to be painted is the sum of the area of each wall in the room.

Area of each long wall		Area of each short wall	
$A = bh$	Area of a rectangle	$A = bh$	Area of a rectangle
$= (14)(10)$	$b = 14$ ft, $h = 10$ ft	$= (12)(10)$	$b = 12$ ft, $h = 10$ ft
$= 140$	Multiply.	$= 120$	Multiply.

Since the room is rectangular, the total area is $2(140) + 2(120)$ or 520 square feet.

CHECK Your Progress

2. **INTERIOR DESIGN** The Waroners are planning to carpet part of their house. The carpet they plan to buy is sold by the square yard. Find the amount of carpeting needed to cover the living room, den, and hall if all are rectangular rooms.



Parallelograms on the Coordinate Plane Recall the properties of quadrilaterals that you studied in Chapter 6. Using these properties as well as the formula for slope and the Distance Formula, you can find the perimeters and areas of quadrilaterals on the coordinate plane.

Study Tip

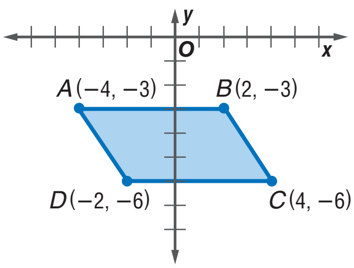
Look Back
To review properties of **parallelograms, rectangles,** and **squares,** see Lessons 6-3, 6-4, and 6-5.

EXAMPLE Perimeter and Area on the Coordinate Plane

3 **COORDINATE GEOMETRY** **The vertices of a quadrilateral are $A(-4, -3)$, $B(2, -3)$, $C(4, -6)$, and $D(-2, -6)$.**

a. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

First graph each point and draw the quadrilateral. Then determine the slope of each side.



$$\text{slope of } \overline{AB} = \frac{-3-(-3)}{-4-2}$$
$$= \frac{0}{-6} \text{ or } 0$$

$$\text{slope of } \overline{CD} = \frac{-6-(-6)}{4-(-2)}$$
$$= \frac{0}{6} \text{ or } 0$$

$$\text{slope of } \overline{BC} = \frac{-3-(-6)}{2-4} = \frac{3}{-2} \qquad \text{slope of } \overline{AD} = \frac{-3-(-6)}{-4-(-2)} = \frac{3}{-2}$$

Opposite sides have the same slope, so they are parallel. *ABCD* is a parallelogram. The slopes of the consecutive sides are *not* negative reciprocals of each other, so the sides are not perpendicular. Thus, the parallelogram is neither a square nor a rectangle.

b. Find the perimeter of quadrilateral *ABCD*.

Since $\overline{AB}$ and $\overline{CD}$ are parallel to the x-axis, you can subtract the x-coordinates of the endpoints to find their measures.

$$AB = 2-(-4) = |6| \text{ or } 6 \qquad CD = 4-(-2) = |6| \text{ or } 6$$

Use the Distance Formula to find *BC* and *AD*.

$$BC = \sqrt{(4-2)^2 + (-6-(-3))^2} = \sqrt{2^2 + (-3)^2} = \sqrt{13}$$

$$AD = \sqrt{(-2-(-4))^2 + (-6-(-3))^2} = \sqrt{2^2 + (-3)^2} = \sqrt{13}$$

Now add to find the perimeter.

perimeter of $ABCD$	$= AB + BC + CD + AD$	Definition of perimeter
	$= 6 + \sqrt{13} + 6 + \sqrt{13}$	Substitution
	$= 12 + 2\sqrt{13}$	Add like terms.

The perimeter of quadrilateral *ABCD* is $12 + 2\sqrt{13}$ or about 19.21 units.

c. Find the area of quadrilateral *ABCD*.

Base: From Part b, $CD = 6$.

Height: Since $\overline{AB}$ and $\overline{CD}$ are horizontal segments, the distance between them, or the height, can be measured on any vertical segment. Reading from the graph, the height is 3.

$A = bh$	Area formula
$= 6(3)$	$b = 6, h = 3$
$= 18$	Simplify.

The area of $\square ABCD$ is 18 square units.

Study Tip

Alternative Method

It was already proved that *ABCD* is a parallelogram. So since opposite sides of a parallelogram are congruent, it could be assumed that $AB = CD$ and $BC = AD$.

CHECK Your Progress

COORDINATE GEOMETRY The vertices of a quadrilateral are $J(0, -3)$, $K(-3, 1)$, $L(-15, -8)$, and $M(-12, -12)$.

3A. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

3B. Find the perimeter of quadrilateral *JKLM*.

3C. Find the area of quadrilateral *JKLM*.

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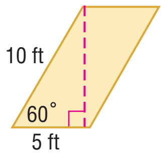
CHECK Your Understanding

Example 1 (p. 631)

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

1.



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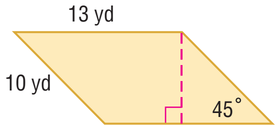

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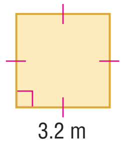
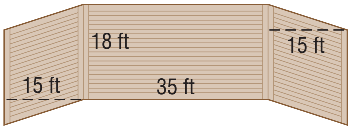


Example 2 (p. 632)

4. **HOME IMPROVEMENT** Mr. Esperanza is planning to stain his deck. To know how much stain to buy, he needs to find the area of the deck. What is the area?



Example 3 (p. 632)

Given the coordinates of the vertices of quadrilateral *TVXY*, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the perimeter and area of *TVXY*.

5. $T(0, 0)$, $V(2, 6)$, $X(6, 6)$, $Y(4, 0)$

6. $T(10, 16)$, $V(2, 18)$, $X(-3, -2)$, $Y(5, -4)$

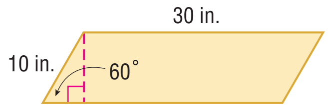
Exercises

HOMEWORK HELP

For Exercises	See Examples
7–12	1
13–17	2
18–21	3

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

7.



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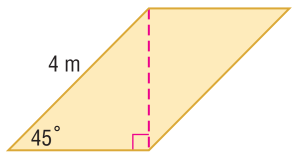

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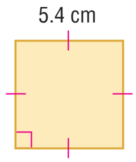

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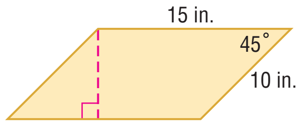

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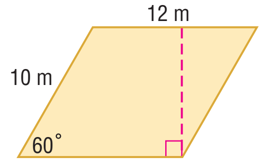

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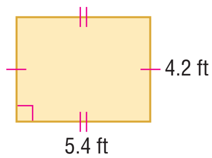
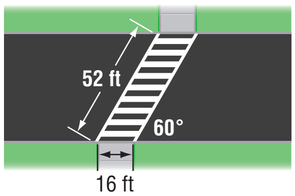
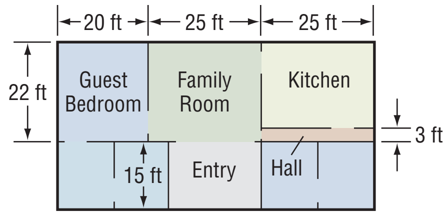


13. **ROADS** A crosswalk with two stripes, each 52 feet long, is at a 60° angle to the curb. The width of the crosswalk at the curb is 16 feet. Find the perpendicular distance between the stripes of the crosswalk.

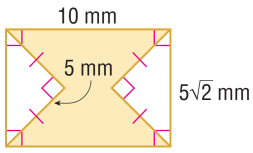


14. **INTERIOR DESIGN** The Bessos are planning to have new carpet installed in their guest bedroom, family room, and hallway. Find the number of square yards of carpet they should order if all rooms are rectangular.



Find the area of each shaded region. Round to the nearest tenth if necessary.

15.



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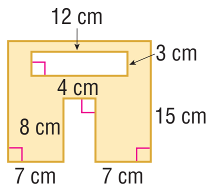

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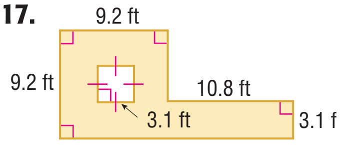


COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the perimeter and area of the quadrilateral.**

18. $A(0, 0), B(4, 0), C(5, 5), D(1, 5)$

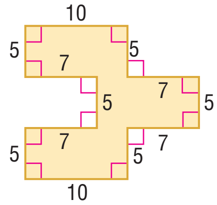
19. $E(-5, -3), F(3, -3), G(5, 4), H(-3, 4)$

20. $R(-2, 4), S(8, 4), T(8, -3), U(-2, -3)$

21. $V(1, 10), W(4, 8), X(2, 5), Y(-1, 7)$

Find the area of each figure.

22.



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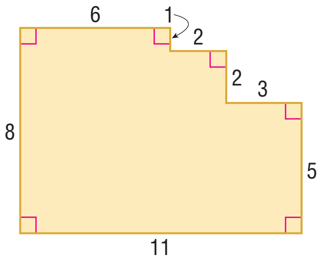

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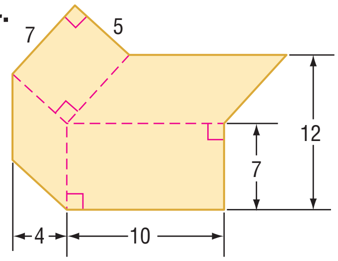
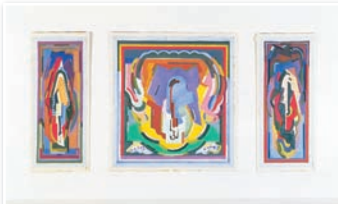


ART **For Exercises 25 and 26, use the following information.**

A triptych painting is a series of three pieces with a similar theme displayed together. Suppose the center panel is a 12-inch square and the panels on either side are 12 inches by 5 inches. The panels are 2 inches apart with a 3 inch wide border around the edges.



25. Determine whether the triptych will fit a 45-inch by 20-inch frame. Explain.

26. Find the area of the artwork, including the border.

CHANGING DIMENSIONS **For Exercises 27–29, use the following information.**

A parallelogram has a base of 8 meters, sides of 11 meters, and a height of 10 meters.

27. Find the perimeter and area of the parallelogram.

28. Suppose the base of the parallelogram was divided in half. Find the new perimeter and area. Compare to the perimeter and area of the original parallelogram.

29. Suppose the original dimensions of the parallelogram were divided in half. Find the perimeter and the area. Compare the perimeter and area of the parallelogram with the original.

EXTRA PRACTICE
See pages 822, 838.

Math Online
Self-Check Quiz at
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H.O.T. Problems

30. OPEN ENDED Make and label a scale drawing of your bedroom. Then find its area in square yards.

31. REASONING Given a parallelogram of base b and height h, determine an expression for the area of a parallelogram with each dimension halved. Determine the formulas for the area and perimeter. Compare to the original formulas. **Make a conjecture** about the area and perimeter of a parallelogram in which each dimension was divided in half.

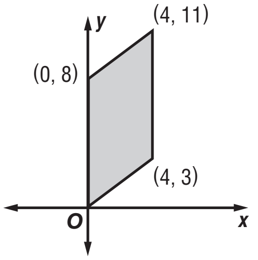
32. **REASONING** Determine the formulas for the area and perimeter of a parallelogram in which one dimension was divided in half. Compare to the original formula. **Make a conjecture** about the area of a parallelogram in which one dimension was divided in half.

33. **CHALLENGE** A piece of twine 48 inches long is cut into two lengths. Each length is then used to form a square. The sum of the areas of the two squares is 74 square inches. Find the length of each side of the smaller square and the larger square.

34. ***Writing in Math*** Refer to the information on acres on page 630. Explain how area is related to acres.

STANDARDIZED TEST PRACTICE

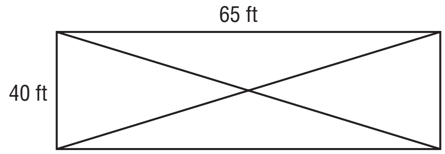
35. What is the area, in square units, of the parallelogram shown?



A 12 **C** 24

B 20 **D** 40

36. **REVIEW** Tia is going to spray paint a rectangle and its two diagonals in a field for a game. If each can of spray paint covers about 100 feet, how many cans of spray paint should Tia buy?



F 3 **H** 5

G 4 **J** 6

Spiral Review

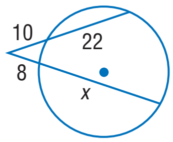
Determine the coordinates of the center and the measure of the radius for each circle with the given equation. (Lesson 10-8)

37. $(x - 5)^2 + (y - 2)^2 = 49$

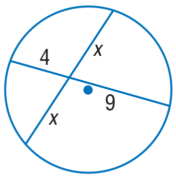
38. $(x + 3)^2 + (y + 9)^2 - 81 = 0$

Find *x*. Assume that segments that appear to be tangent are tangent. (Lesson 10-7)

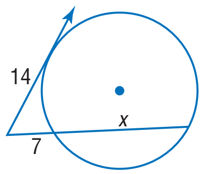
39.



40.



41.



42. **BIKES** Tariq is making a ramp for bike jumps. The ramp support forms a right angle. The base is 12 feet long, and the height is 5 feet. What length of plywood does Tariq need for the ramp? (Lesson 8-2)

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $w = 8$, $x = 4$, $y = 2$, and $z = 5$.** (Page 780)

43. $\frac{1}{2}(7y)$

44. $\frac{1}{2}wx$

45. $\frac{1}{2}z(x + y)$

46. $\frac{1}{2}x(y + w)$

EXPLORE 11-2

Graphing Calculator Lab

Investigating Area

ACTIVITY 1

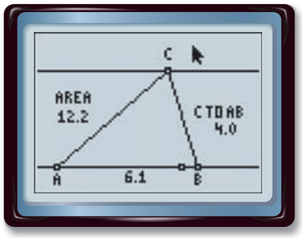
Step 1 Construct a line using the line tool on the F2 menu. Label two points on the line as A and B.

Step 2 Use the Parallel tool on the F3 menu to construct a line parallel to the first line. Pressing ENTER will draw the line and a point on the line. Label the point C.

Step 3 Construct triangle ABC using the Triangle tool on the F2 menu.

Step 4 Access the Area tool under Measure on the F5 menu. Display the area of $\triangle ABC$. Then display the measure of $\overline{AB}$ and the distance from C to $\overleftrightarrow{AB}$.

Step 5 Click on point C and drag it along the line to change the shape of $\triangle ABC$.



1A. What do you observe about the base and height of $\triangle ABC$?

1B. What do you observe about the area of $\triangle ABC$?

1C. Use what you know about the formula for the area of a rectangle to write a conjecture about the formula for the area of a triangle.

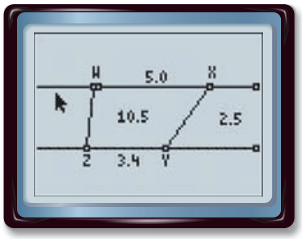
ACTIVITY 2

Step 1 Construct a line using the line tool on the F2 menu. Label two points on the line as W and X. Then use the Parallel tool on the F3 menu to construct a line parallel. Label points on this line as Y and Z.

Step 2 Access the Quadrilateral tool on the F2 menu. Construct quadrilateral $WXYZ$.

Step 3 Use the Area tool under Measure on the F5 menu to display the area of $WXYZ$. Then display the measures of $\overline{WX}$ and $\overline{YZ}$, and find the distance from $\overleftrightarrow{WX}$ to $\overleftrightarrow{YZ}$.

Step 4 Click on point W and drag it along the line.



2A. What kind of quadrilateral is $WXYZ$? Explain.

2B. Use what you know about the formula for the area of a rectangle to write a conjecture about the formula for the area of this type of quadrilateral. Verify your conjecture.

ANALYZE THE RESULTS

The area of a rhombus is dependent upon the measures of the diagonals. Use Cabri Jr. to draw a rhombus and make a conjecture about the formula for the area of a rhombus.

11-2 Areas of Triangles, Trapezoids, and Rhombi

Main Ideas

- Find areas of triangles.
- Find areas of trapezoids and rhombi.

GET READY for the Lesson

Umbrellas can protect you from rain, wind, and sun. The umbrella shown at the right is made of triangular panels. To cover the umbrella frame with canvas panels, you need to know the area of each panel.

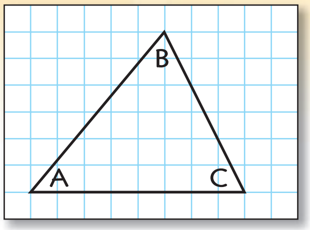
Areas of Triangles You have learned how to find the areas of squares, rectangles, and parallelograms. The formula for the area of a triangle is related to these formulas.

GEOMETRY LAB

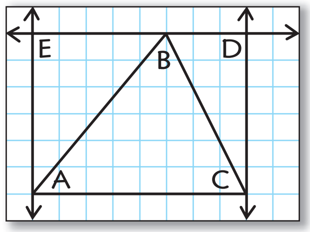
Area of a Triangle

MODEL

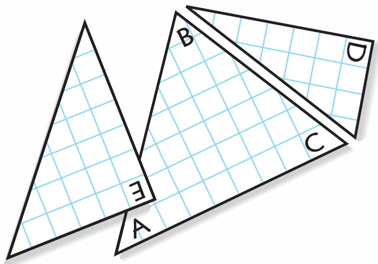
- Draw a triangle on grid paper so that one edge is along a horizontal line. Label the vertices on the interior of the angles of the triangle as A, B, and C.



- Draw a line perpendicular to $\overline{AC}$ through A.
- Draw a line perpendicular to $\overline{AC}$ through C.
- Draw a line parallel to $\overline{AC}$ through B.
- Label the points of intersection of the lines drawn as D and E as shown.



- Find the area of rectangle $ACDE$ in square units.
- Cut out rectangle $ACDE$. Then cut out $\triangle ABC$. Place the two smaller pieces over $\triangle ABC$ to completely cover the triangle.



ANALYZE THE RESULTS

1. What do you observe about the two smaller triangles and $\triangle ABC$?
2. What fraction of rectangle $ACDE$ is $\triangle ABC$?
3. Derive a formula that could be used to find the area of $\triangle ABC$.

The Geometry Lab suggests the formula for finding the area of a triangle.

Study Tip

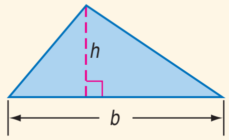
Look Back

To review the **height and altitude of a triangle**, see Lesson 5-1.

KEY CONCEPT — *Area of a Triangle*

Words If a triangle has an area of A square units, a base of b units, and a corresponding height of h units, then the area equals one half the product of the base and the height.

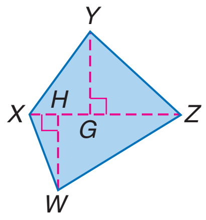
Symbols $A = \frac{1}{2}bh$



EXAMPLE Areas of Triangles

1 **Find the area of quadrilateral $XYZW$ if $XZ = 39$, $HW = 20$, and $YG = 21$.**

The area of the quadrilateral is equal to the sum of the areas of $\triangle XWZ$ and $\triangle XYZ$.

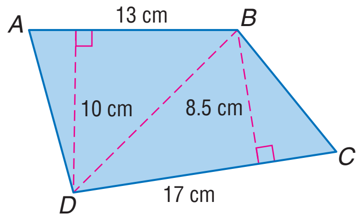


$$\begin{aligned} \text{area of } XYZW &= \text{area of } \triangle XYZ + \text{area of } \triangle XWZ \\ &= \tfrac{1}{2}bh_1 + \tfrac{1}{2}bh_2 \\ &= \tfrac{1}{2}(39)(21) + \tfrac{1}{2}(39)(20) && \text{Substitution} \\ &= 409.5 + 390 && \text{Simplify.} \\ &= 799.5 \end{aligned}$$

The area of quadrilateral $XYZW$ is 799.5 square units.

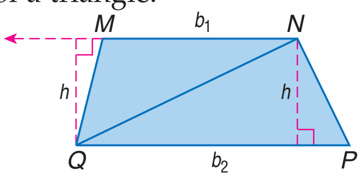
CHECK Your Progress

1. Find the area of quadrilateral $ABCD$.



Areas of Trapezoids and Rhombi The formulas for the areas of trapezoids and rhombi are related to the formula for the area of a triangle.

The diagonal $\overline{QN}$ separates trapezoid $MNPQ$ into two triangles.



$$\underset{\text{trapezoid } MNPQ}{\text{area of}} = \underset{\triangle MNQ}{\text{area of}} + \underset{\triangle NPQ}{\text{area of}}$$

$$\begin{aligned} A &= \tfrac{1}{2}(b_1)h + \tfrac{1}{2}(b_2)h && \text{Let } MN = b_1, \text{ and } QP = b_2. \\ &= \tfrac{1}{2}(b_1 + b_2)h && \text{Factor.} \\ &= \tfrac{1}{2}h(b_1 + b_2) && \text{Commutative Property} \end{aligned}$$

This is the formula for the area of any trapezoid.

Study Tip

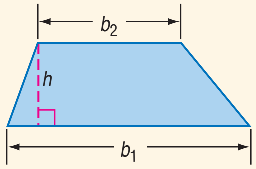
Alternate Method

Notice that the formula for the area of a trapezoid can also be expressed as a product of the height and the mean of the lengths of the bases.

KEY CONCEPT *Area of a Trapezoid*

Words If a trapezoid has an area of A square units, bases of b_1 units and b_2 units, and a height of h units, then the area equals the product of one half the height and the sum of the lengths of each base.

Symbols $A = \frac{1}{2}h(b_1 + b_2)$



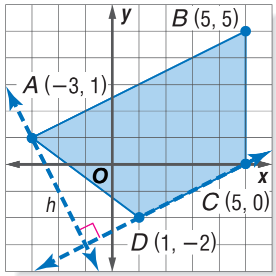
EXAMPLE Area of a Trapezoid on a Coordinate Plane

2 **COORDINATE GEOMETRY** **Find the area of trapezoid $ABCD$ with vertices $A(-3, 1)$, $B(5, 5)$, $C(5, 0)$, and $D(1, -2)$.**

Height: To find the height, extend the line that passes through D and C.

The slope of this line is $\frac{1}{2}$.

Next, graph the line perpendicular to the bases that passes through A. From the graph, you can determine that the coordinates of the point of intersection are $(-1, -3)$.



The height of the trapezoid is the distance between $(-3, 1)$ and $(-1, -3)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$h = \sqrt{(-1 - (-3))^2 + (-3 - 1)^2}$ $(x_1, y_1) = (-3, 1)$, $(x_2, y_2) = (-1, -3)$

$= \sqrt{2^2 + (-4)^2}$ Subtract.

$= \sqrt{4 + 16}$ or $\sqrt{20}$ Simplify.

Bases: Use the Distance Formula to determine the length of each base.

$\overline{AB}$: $A(-3, 1)$ and $B(5, 5)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-3 - 5)^2 + (1 - 5)^2}$

$= \sqrt{(-8)^2 + (-4)^2}$

$= \sqrt{64 + 16}$ or $\sqrt{80}$

$\overline{DC}$: $D(1, -2)$ and $C(5, 0)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1 - 5)^2 + (-2 - 0)^2}$

$= \sqrt{(-4)^2 + (-2)^2}$

$= \sqrt{16 + 4}$ or $\sqrt{20}$

Area: $A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}\left(\sqrt{20}\right)\left(\sqrt{80} + \sqrt{20}\right)$ $h = \sqrt{20}$, $b_1 = \sqrt{80}$, $b_2 = \sqrt{20}$

$= \frac{1}{2}\sqrt{1600} + \frac{1}{2}\sqrt{400}$ Distributive Property

$= \frac{1}{2}(40) + \frac{1}{2}(20)$ or 30 The area of the trapezoid is 30 square units.

CHECK Your Progress

2. **COORDINATE GEOMETRY** Find the area of trapezoid $ABCD$ with vertices $A(-10, 5)$, $B(13, 5)$, $C(7, -3)$, $D(-8, -3)$.

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The formula for the area of a triangle can also be used to derive the formula for the area of a rhombus.

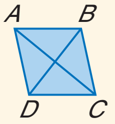
Study Tip

Area of a Rhombus

Because a rhombus is also a parallelogram, you can also use the formula $A = bh$ to determine the area.

KEY CONCEPT — *Area of a Rhombus*

Words If a rhombus has an area of A square units and diagonals of d_1 and d_2 units, then area equals one half the product of the length of each diagonal.



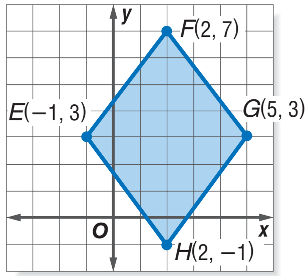
Symbols $A = \frac{1}{2}d_1d_2$

Example $A = \frac{1}{2}(AC)(BD)$

You will derive this formula in Exercise 41.

EXAMPLE Area of a Rhombus on the Coordinate Plane

3 **COORDINATE GEOMETRY** **Find the area of rhombus *EFGH*.**



Explore To find the area of the rhombus, we need to know the lengths of each diagonal.

Plan Use coordinate geometry to find the length of each diagonal. Use the formula to find the area of rhombus *EFGH*.

Solve Let $\overline{EG}$ be d_1 and $\overline{FH}$ be d_2.

Subtract the x-coordinates of E and G to find that d_1 is 6. Subtract the y-coordinates of F and H to find that d_2 is 8.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$= \frac{1}{2}(6)(8)$ or 24 $d_1 = 6, d_2 = 8$

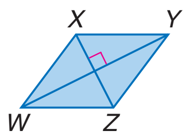
Check The area of rhombus *EFGH* is 24 square units.

CHECK Your Progress

3. **COORDINATE GEOMETRY** Find the area of rhombus *JKLM* with vertices $J(0, 2)$, $K(2, 6)$, $L(4, 2)$, $M(2, -2)$.

EXAMPLE Find Missing Measures

4 **ALGEBRA** **Rhombus *WXYZ* has an area of 100 square meters. Find *WY* if *XZ* = 10 meters.**



Use the formula for the area of a rhombus and solve for d_2.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$100 = \frac{1}{2}(10)(d_2)$ $A = 100, d_1 = 10$

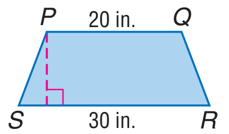
$100 = 5d_2$ Simplify.

$20 = d_2$ Divide.

WY is 20 meters.

CHECK Your Progress

4. Trapezoid *PQRS* has an area of 250 square inches. Find the height of *PQRS*.



Since the dimensions of congruent figures are equal, the areas of congruent figures are also equal.

POSTULATE 11.1

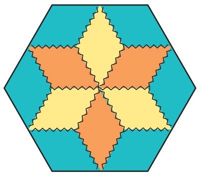
Congruent figures have equal areas.

Study Tip

Look Back
To review **the properties of rhombi and trapezoids**, see Lessons 6-5 and 6-6.

EXAMPLE 5 Area of Congruent Figures

QUILTING This quilt block is composed of twelve congruent rhombi arranged in a regular hexagon. The height of the hexagon is 8 inches. If the total area of the rhombi is 48 square inches, find the lengths of each diagonal and the area of one rhombus.



Step 1 Use the area formula to find the length of the other diagonal.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$4 = \frac{1}{2}(4)d_2$ $A = 4$, $d_1 = 4$

$2 = d_2$ Solve for d_2.

Step 2 Find the length of one diagonal. The height of the hexagon is equal to the sum of the long diagonals of two rhombi. Since the rhombi are congruent, the long diagonals must be congruent. So, the long diagonal is equal to 8 ÷ 2, or 4 inches.

Step 3 Find the area of one rhombus. From Postulate 11.1, the area of each rhombus is the same. So, the area of each rhombus is 48 ÷ 12 or 4 square inches.

Each rhombus in the pattern has an area of 4 square inches and diagonals 3 inches and 2 inches long.

CHECK Your Progress

5A. **RECREATION** Rodrigo wants to cover a kite frame with decorative paper. If the length of one diagonal is 20 inches and the other diagonal measures 25 inches, find the area of the surface of the kite.

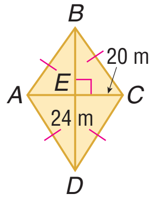
5B. **GARDENS** Clara has enough topsoil to cover 200 square feet. Her garden is shaped like a rhombus with one diagonal that is 25 feet. If she uses all of the topsoil on the garden, what is the length of the other diagonal?

CHECK Your Understanding

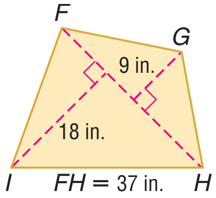
Examples 1–3 (pp. 639–641)

Find the area of each quadrilateral. Round to the nearest tenth.

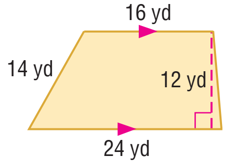
1.



2.



3.



COORDINATE GEOMETRY **Find the area of each figure given the coordinates of the vertices.**

4. $\triangle ABC$ with $A(2, -3)$, $B(-5, -3)$, and $C(-1, 3)$

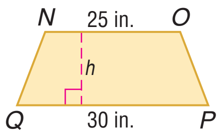
5. trapezoid $FGHJ$ with $F(-1, 8)$, $G(5, 8)$, $H(3, 4)$, and $J(1, 4)$

6. rhombus $LMPQ$ with $L(-4, 3)$, $M(-2, 4)$, $P(0, 3)$, and $Q(-2, 2)$

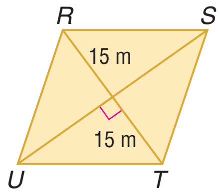
Example 4 (p. 641)

ALGEBRA **Find the missing measure for each quadrilateral.**

7. Trapezoid $NOPQ$ has an area of 302.5 square inches. Find the height of $NOPQ$.

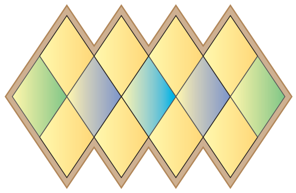


8. Rhombus $RSTU$ has an area of 675 square meters. Find SU.



Example 5 (p. 642)

9. **INTERIOR DESIGN** Jacques is designing a window hanging composed of 13 congruent rhombi. The total width of the window hanging is 15 inches, and the total area is 82 square inches. Find the length of each diagonal and the area of one rhombus.

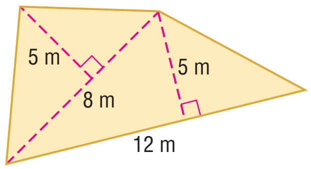


Exercises

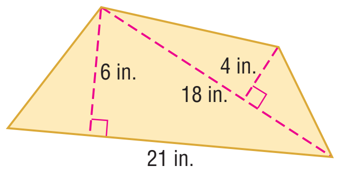
HOMEWORK HELP	
For Exercises	See Examples
10, 11	1
12, 13, 16–19	2
14, 15 20–23	3
24–29	4
30–33	5

Find the area of each figure. Round to the nearest tenth if necessary.

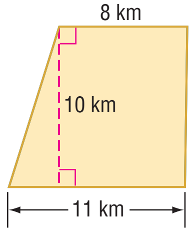
10.



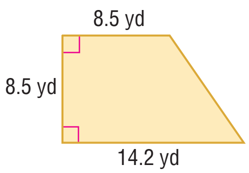
11.



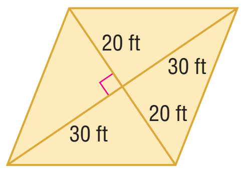
12.



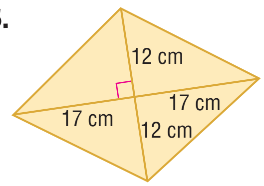
13.



14.



15.



COORDINATE GEOMETRY **Find the area of trapezoid *PQRT* given the coordinates of the vertices.**

16. $P(0, 3), Q(3, 7), R(5, 7), T(6, 3)$

17. $P(-4, -5), Q(-2, -5), R(4, 6), T(-4, 6)$

18. $P(0, 3), Q(3, 1), R(2, -7), T(-7, -1)$

19. $P(-5, 2), Q(10, 7), R(6, -1), T(0, -3)$

COORDINATE GEOMETRY **Find the area of rhombus *JKLM* given the coordinates of the vertices.**

20. $J(2, 1), K(7, 4), L(12, 1), M(7, -2)$

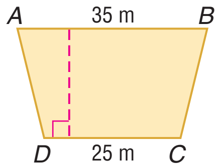
21. $J(-1, 2), K(1, 7), L(3, 2), M(1, -3)$

22. $J(-1, -4), K(2, 2), L(5, -4), M(2, -10)$

23. $J(2, 4), K(6, 6), L(10, 4), M(6, 2)$

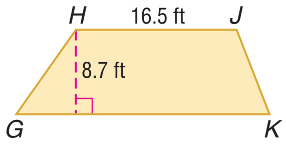
ALGEBRA **Find the missing measure for each figure.**

24. Find the height of trapezoid *ABCD*.



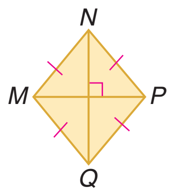
$A = 750\ \text{m}^2$

25. If *HJ* is 16.5 feet, find *GK*.



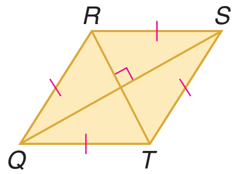
$A = 188.35\ \text{ft}^2$

26. If *MP* is 25 inches, find *NQ*.



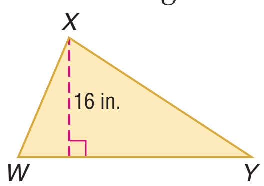
$A = 375\ \text{in}^2$

27. If *RT* is 12.2 meters, find *QS*.



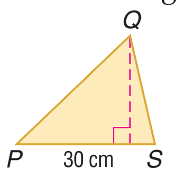
$A = 137.9\ \text{m}^2$

28. Find the length of the base.



$A = 248\ \text{in}^2$

29. Find the height.



$A = 300\ \text{cm}^2$

Real-World Career

Real Estate Agent

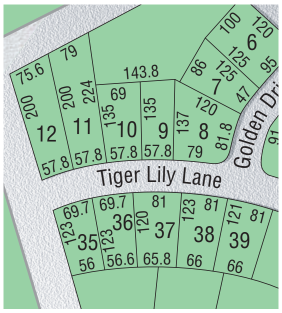
Real estate agents must have a broad knowledge of the neighborhoods in their community. About two-thirds of real estate agents are self-employed. Success is dependent on selling properties.

Source: www.bls.gov

Math Online For more information, go to geometryonline.com.

REAL ESTATE **For Exercises 30 and 31, use the following information.**

The map shows the layout and dimensions of several lot parcels in Aztec Falls. Suppose Lots 35 and 12 are trapezoids.

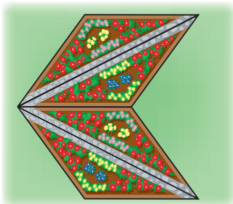


30. If the height of Lot 35 is 122.81 feet, find the area of this lot.

31. If the height of Lot 12 is 199.8 feet, find the area of this lot.

GARDENS **For Exercises 32 and 33, use the following information.**

Keisha designed a garden that is shaped like two congruent rhombi. She wants the long diagonals lined with a stone walkway. The total area of the garden is 150 square feet, and the shorter diagonals are each 12 feet long.



32. Find the length of each stone walkway.

33. Find the length of each side of the garden.

Find the area of each figure.

34. rhombus with a perimeter of 20 meters and a diagonal of 8 meters

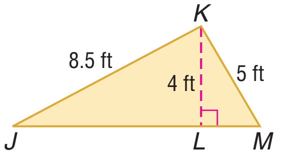
35. rhombus with a perimeter of 52 inches and a diagonal of 24 inches

36. isosceles trapezoid with a perimeter of 52 yards; the measure of one base is 10 yards greater than the other base, the measure of each leg is 3 yards less than twice the length of the shorter base

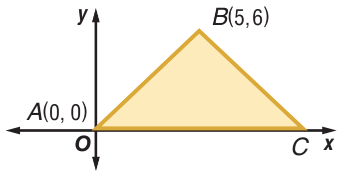
37. equilateral triangle with a perimeter of 15 inches

38. scalene triangle with sides that measure 34.0 meters, 81.6 meters, and 88.4 meters

39. Find the area of $\triangle JKM$.



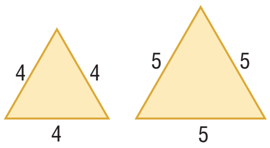
40. In the figure, if point B lies on the perpendicular bisector of $\overline{AC}$, what is the area of $\triangle ABC$?



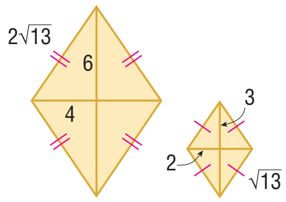
41. Derive the formula for the area of a rhombus using the formula for the area of a triangle.

CHANGING DIMENSIONS **Each pair of figures is similar. Find the area and perimeter of each figure. Describe how changing the dimensions affects the perimeter and area.**

42.

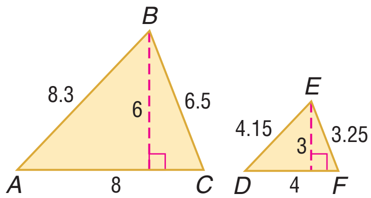


43.



SIMILAR FIGURES **For Exercises 44–49, use the following information.**

Triangle ABC is similar to triangle DEF.



44. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.

45. Find the perimeter of each triangle. Compare the ratio of the perimeters of the triangles to the scale factor.

Study Tip

Look Back

To review **scale factor**, see Lesson 7–2.

46. Find the area of each triangle. Compare the ratio of the areas of the triangles to the scale factor.

47. Compare the ratio of the areas of the triangles to the ratio of the perimeters of the triangles.

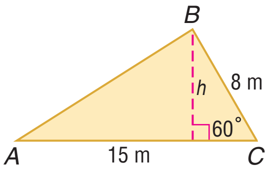
48. **Make a conjecture** about the ratios of the areas of similar triangles as compared to the scale factor.

49. **CHANGING DIMENSIONS** Suppose in $\triangle DEF$ the altitude stays the same, but the base is changed to twice its current measure. The new leg measures are 6 and 4.2 units. How do the perimeter and area of new $\triangle DEF$ compare to those of $\triangle DEF$?

EXTRA PRACTICE
See pages 822, 838.

Math Online
Self-Check Quiz at
geometryonline.com

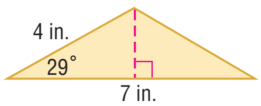
TRIGONOMETRY AND AREA For Exercises 50–53, use the triangle at the right.

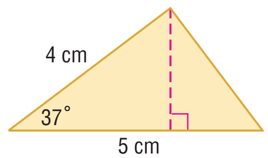


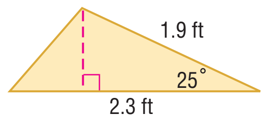
The area of any triangle can be found given the measures of two sides of the triangle and the measure of the included angle. Suppose we are given $AC = 15$, $BC = 8$, and $m\angle C = 60$. To find the height of the triangle, use the sine ratio, $\sin A = \frac{h}{BC}$. Then use the value of h in the formula for the area of a triangle. So, the area is $\frac{1}{2}(15)(8 \sin 60°)$ or 52.0 square units.

50. Derive a formula to find the area of any triangle, given the measures of two sides of the triangle and their included angle.

Find the area of each triangle.

51.


52.


53.


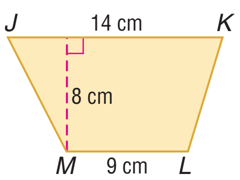
H.O.T. Problems

54. REASONING Determine whether the statement *Two triangles that have the same area also have the same perimeter* is *true* or *false*. Give an example or counterexample.

55. REASONING Determine whether it is *always*, *sometimes*, or *never* true that rhombi with the same area have the same diagonal lengths. Explain your reasoning.

56. OPEN ENDED Draw an isosceles trapezoid that contains at least one isosceles triangle. Then find the area of the trapezoid.

57. FIND THE ERROR Robert and Kiku are finding the area of trapezoid *JKLM*. Who is correct? Explain your reasoning.



Robert

$A = \frac{1}{2}(8)(14 + 9)$

$= \frac{1}{2}(8)(14) + 9$

$= 56 + 9$

$= 65 \text{ cm}^2$

Kiku

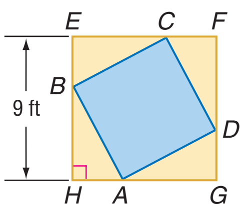
$A = \frac{1}{2}(8)(14 + 9)$

$= \frac{1}{2}(8)(23)$

$= 4(23)$

$= 92 \text{ cm}^2$

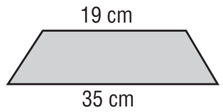
58. CHALLENGE In the figure, the vertices of quadrilateral *ABCD* intersect the square *EFGH* and divide its sides into segments with measures that have a ratio of 1:2. Find the area of *ABCD*. Describe the relationship between the areas of *ABCD* and *EFGH*.



59. *Writing in Math* Describe how to find the area of a triangle. Explain how the area of a triangle can help you find the areas of rhombi and trapezoids.

STANDARDIZED TEST PRACTICE

60. The lengths of the bases of an isosceles trapezoid are shown below.



If the perimeter is 74 centimeters, what is its area?

A 162 cm^2

B 270 cm^2

C 332.5 cm^2

D 342.25 cm^2

61. **REVIEW** What is the effect on the graph of the equation $y = \frac{1}{2}x$ when the equation is changed to $y = -2x$?

F The graph is moved 1 unit down.

G The graph is moved 1 unit up.

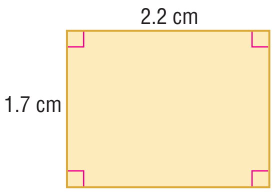
H The graph is rotated 45° about the origin.

J The graph is rotated 90° about the origin.

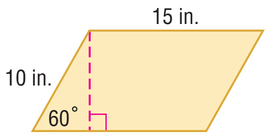
Spiral Review

Find the area of each figure. Round to the nearest tenth. (Lesson 11-1)

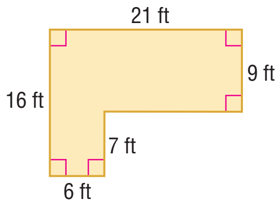
62.



63.



64.



Write an equation of circle R based on the given information. (Lesson 10-8)

65. center: $R(1, 2)$
radius: 7

66. center: $R\left(-4, \frac{1}{2}\right)$
radius: $\frac{11}{2}$

67. center: $R(-1.3, 5.6)$
radius: 3.5

68. **CRAFTS** Andria created a pattern to sew flowers onto a quilt by first drawing a regular pentagon that was 3.5 inches long on each side. Then she added a semicircle onto each side of the pentagon to create the appearance of five petals. How many inches of gold trim does she need to edge 10 flowers? (Lesson 10-1)

Given the magnitude and direction of a vector, find the component form with values rounded to the nearest tenth. (Lesson 9-6)

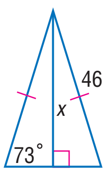
69. magnitude of 136 at a direction of 25 degrees with the positive x-axis

70. magnitude of 280 at a direction of 52 degrees with the positive x-axis

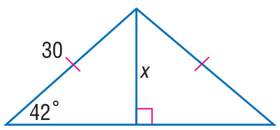
GET READY for the Next Lesson

PREREQUISITE SKILL **Find x. Round to the nearest tenth.** (Lesson 8-4)

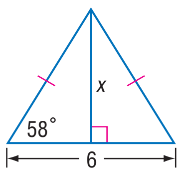
71.



72.



73.



EXPLORE 11-3

Geometry Lab

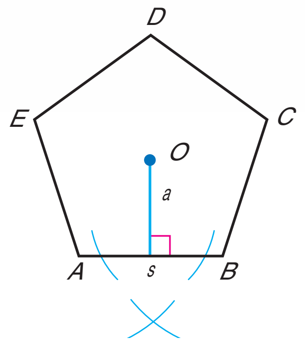
Investigating Areas of Regular Polygons

The point in the interior of a regular polygon that is equidistant from all of the vertices is the *center* of the polygon. A segment from the center to a side of the polygon that is perpendicular to the side is an **apothem**.

ACTIVITY

Step 1 Copy regular pentagon *ABCDE* and its center *O*.

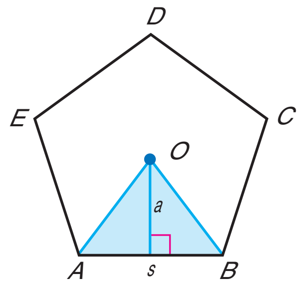
Step 2 Draw the apothem from *O* to side $\overline{AB}$ by constructing the perpendicular bisector of $\overline{AB}$. Label the apothem measure as *a*. Label the measure of $\overline{AB}$ as *s*.



Step 3 Use a straightedge to draw $\overline{OA}$ and $\overline{OB}$.

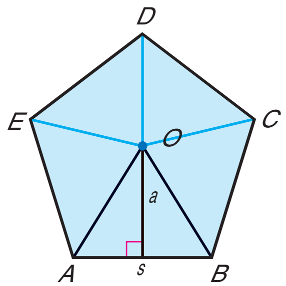
Step 4 What measure in $\triangle AOB$ represents the base of the triangle? What measure represents the height?

Step 5 Find the area of $\triangle AOB$ in terms of *s* and *a*.



Step 6 Draw $\overline{OC}$, $\overline{OD}$, and $\overline{OE}$. What is true of the five small triangles formed?

Step 7 How do the areas of the five triangles compare?



ANALYZE THE RESULTS

1. The area of a pentagon *ABCDE* can be found by adding the areas of the given triangles that make up the pentagonal region.

$A = \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa$

$A = \frac{1}{2}(sa + sa + sa + sa + sa)$ or $\frac{1}{2}(5sa)$

What does 5*s* represent?

2. Write a formula for the area of a pentagon in terms of perimeter *P*.

11-3 Areas of Regular Polygons and Circles

Main Ideas

- Find areas of regular polygons.
- Find areas of circles.

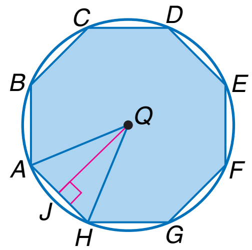
New Vocabulary

apothem

GET READY for the Lesson

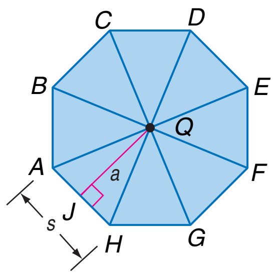
Connecticut's Mystic Seaport is a private maritime museum and home to one of the largest collections of boats in the world. The Village Green is the location of a 50-foot long model of the Mystic River area and a gazebo. The base of the gazebo is an octagon. Suppose the caretakers want to replace the gazebo flooring. How can they determine the area of the floor?

Areas of Regular Polygons In regular octagon $ABCDEFGH$ inscribed in circle Q, $\overline{QA}$ and $\overline{QH}$ are radii from the center of the circle Q to two vertices of the octagon. $\overline{QJ}$ is drawn from the center of the regular polygon perpendicular to a side of the polygon. This segment is called an **apothem**.



Triangle QAH is an isosceles triangle, since the radii are congruent. If all of the radii were drawn, they would separate the octagon into 8 nonoverlapping congruent isosceles triangles.

The area of the octagon can be determined by adding the areas of the triangles. Since $\overline{QJ}$ is perpendicular to $\overline{AH}$, it is an altitude of $\triangle QAH$. Let a represent the length of $\overline{QJ}$ and let s represent the length of a side of the octagon.



$$\text{Area of } \triangle QAH = \frac{1}{2}bh$$
$$= \frac{1}{2}sa$$

The area of one triangle is $\frac{1}{2}sa$ square units. So the area of the octagon is $8\left(\frac{1}{2}sa\right)$ square units. Notice that the perimeter P of the octagon is $8s$ units. We can substitute P for $8s$ in the area formula.

$$\text{Area of octagon} = 8\left(\frac{1}{2}sa\right)$$
$$= 8s\left(\frac{1}{2}a\right) \quad \text{Commutative and Associative Properties}$$
$$= P\left(\frac{1}{2}a\right) \quad \text{Substitution}$$
$$= \frac{1}{2}Pa \quad \text{Commutative Property}$$

This formula can be used for the area of any regular polygon.

KEY CONCEPT — Area of a Regular Polygon

Words If a regular polygon has an area of A square units, a perimeter of P units, and an apothem of a units, then the area is one-half the product of the perimeter and the apothem.

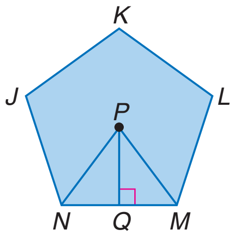
Symbols $A = \frac{1}{2}Pa$

EXAMPLE — Area of a Regular Polygon

1 **Find the area of a regular pentagon with a perimeter of 40 centimeters.**

Draw a diagram of the pentagon. To find the area, you must first find the apothem.

Apothem: The central angles of a regular pentagon are all congruent. Therefore, the measure of each angle is $\frac{360}{5}$ or 72.



$\overline{PQ}$ is an apothem of pentagon $JKLMN$. It bisects $\angle NPM$ and is a perpendicular bisector of $\overline{NM}$. So, $m\angle MPQ = \frac{1}{2}(72)$ or 36.

Since the perimeter is 40 centimeters, each side is 8 centimeters and $QM = 4$ centimeters.

Write a trigonometric ratio to find the length of $\overline{PQ}$.

$\tan \angle MPQ = \frac{QM}{PQ}$ — $\tan \theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$

$\tan 36° = \frac{4}{PQ}$ — $m\angle MPQ = 36$, $QM = 4$

$(PQ) \tan 36° = 4$ — Multiply each side by PQ.

$PQ = \frac{4}{\tan 36°}$ — Divide each side by tan 36°.

$PQ \approx 5.5$ — Use a calculator.

Area: $A = \frac{1}{2}Pa$ — Area of a regular polygon

$\approx \frac{1}{2}(40)(5.5)$ — $P = 40$, $a \approx 5.5$

≈ 110 — Simplify.

So, the area of the pentagon is about 110 square centimeters.

Study Tip

Problem Solving

There is another method for finding the apothem of a regular polygon. You can use the Interior Angle Sum Theorem to find $m\angle PMQ$ and then write a trigonometric ratio to find PQ.

CHECK Your Progress

1A. Find the area of a regular octagon with a perimeter of 124 inches.

1B. Find the area of a square with apothem length of 2.5 meters.

1C. Find the area of a regular hexagon with apothem length of 18 inches.

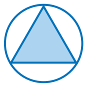
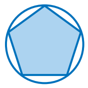
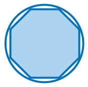
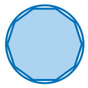
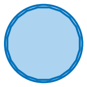
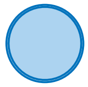
Areas of Circles You can use a calculator to help derive the formula for the area of a circle from the areas of regular polygons.

GEOMETRY LAB

Area of a Circle

Suppose each regular polygon is inscribed in a circle of radius r.

1. Copy and complete the following table. Round to the nearest hundredth.

Inscribed Polygon						
Number of Sides	3	5	8	10	20	50
Measure of a Side	$1.73r$	$1.18r$	$0.77r$	$0.62r$	$0.31r$	$0.126r$
Measure of Apothem	$0.5r$	$0.81r$	$0.92r$	$0.95r$	$0.99r$	$0.998r$
Area						

ANALYZE THE RESULTS

2. What happens to the appearance of the polygon as the number of sides increases?

3. What happens to the measures of the apothems and the areas as the number of sides increases?

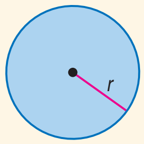
4. **Make a conjecture** about the formula for the area of a circle.

You can see from the Geometry Lab that the more sides a regular polygon has, the more closely it resembles a circle.

KEY CONCEPT — *Area of a Circle*

Words If a circle has an area of A square units and a radius of r units, then the area is the product of π and the square of the radius.

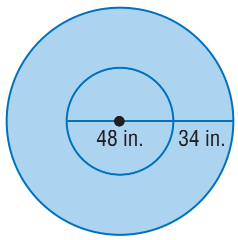
Symbols $A = \pi r^2$



Real-World EXAMPLE

2 **SEWING** **A caterer has a 48-inch diameter table that is 34 inches tall. She wants a tablecloth that will touch the floor. Find the area of the tablecloth in square yards.**

The diameter of the table is 48 inches, and the tablecloth must extend 34 inches in each direction. So the diameter of the tablecloth is 34 + 48 + 34 or 116 inches. Divide by 2 to find that the radius is 58 inches.



$A = \pi r^2$ Area of a circle

$= \pi(58)^2$ Substitution

$\approx 10{,}568.3$ Use a calculator.

The area of the tablecloth is 10,568.3 square inches. To convert to square yards, divide by 1296. The area of the tablecloth is 8.2 square yards to the nearest tenth.

Study Tip

Square Yards

A square yard measures 36 inches by 36 inches or 1296 square inches.

CHECK Your Progress

2. Susana is planning to paint the circular frame for a mirror. She needs to know the area of the frame in order to purchase enough paint. If the diameter of the frame is 18 inches and the diameter of the mirror is 10 inches, what is the area of the frame?

Review Vocabulary

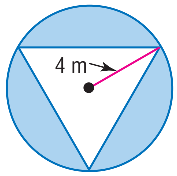
inscribed polygon a polygon in which each vertex lies on a circle

circumscribed polygon a polygon that contains a circle (Lesson 10-3)

You can use the properties of circles and regular polygons to find the areas of inscribed and circumscribed polygons.

EXAMPLE Area of an Inscribed Polygon

3 **Find the area of the shaded region. Assume that the triangle is equilateral.**



The area of the shaded region is the difference between the area of the circle and the area of the triangle.

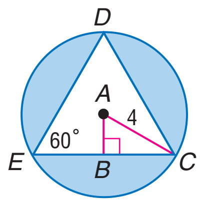
Step 1 Find the area of the circle.

$A = \pi r^2$ Area of a circle

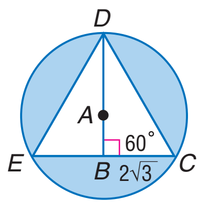
$= \pi(4)^2$ Substitution

≈ 50.3 Use a calculator.

Step 2 Find the area of the triangle. Use properties of 30°-60°-90° triangles. First, find the length of the base. The hypotenuse of $\triangle ABC$ is 4, so BC is $2\sqrt{3}$. Since $EC = 2(BC)$, $EC = 4\sqrt{3}$.



Next, find the height of the triangle, DB. Since $m\angle DCB$ is 60, $DB = 2\sqrt{3}\left(\sqrt{3}\right)$ or 6.



$A = \frac{1}{2}bh$ Area of a triangle

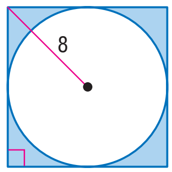
$= \frac{1}{2}\left(4\sqrt{3}\right)(6)$ $b = 4\sqrt{3}$, $h = 6$

≈ 20.8 Use a calculator.

Step 3 The area of the shaded region is $50.3 - 20.8$ or 29.5 square meters to the nearest tenth.

CHECK Your Progress

3A. Find the area of the shaded region. Assume that the quadrilateral is a square. Round to the nearest tenth.



3B. An equilateral triangle is circumscribed around a circle with a radius of 5 units. Find the area of the region between the triangle and the circle.

Online Personal Tutor at geometryonline.com

CHECK Your Understanding

Example 1 (p. 650)

Find the area of each polygon. Round to the nearest tenth.

1. a regular hexagon with a perimeter of 42 yards

2. a regular nonagon with a perimeter of 108 meters

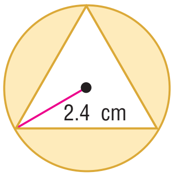
Example 2 (p. 651)

3. FURNITURE DESIGN Tyra wants to cover the cushions of her papasan chair with a different fabric. If there are seven circular cushions that are the same size with a diameter of 12 inches, around a center cushion with a diameter of 20 inches, find the area of fabric in square yards that she will need to cover both sides of the cushions. Allow an extra 3 inches of fabric around each cushion.

Example 3 (p. 652)

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

4.



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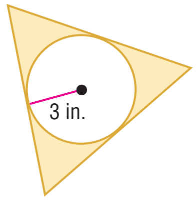


Exercises

HOMEWORK HELP	
For Exercises	See Examples
6–11	1
12–20	3
21–24	2

Find the area of each polygon. Round to the nearest tenth.

6. a regular octagon with a perimeter of 72 inches

7. a square with a perimeter of $84\sqrt{2}$ meters

8. a square with apothem length of 12 centimeters

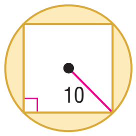
9. a regular hexagon with apothem length of 24 inches

10. a regular triangle with side length of 15.5 inches

11. a regular octagon with side length of 10 kilometers

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

12.



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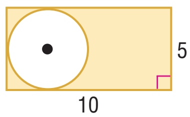

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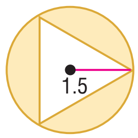
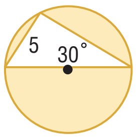


15.

16.



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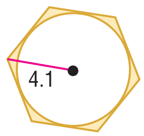

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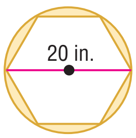

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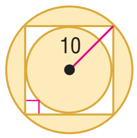

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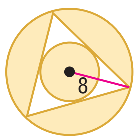


21. **PIZZA** A pizza shop sells 8-inch pizzas for \$5 and 16-inch pizzas for \$10. Which would give you more pizza, two 8-inch pizzas or one 16-inch pizza? Explain.

22. **ALGEBRA** A circle is inscribed in a square, which is circumscribed by another circle. If the diagonal of the square is $2x$, find the ratio of the area of the large circle to the area of the small circle.

23. **ALGEBRA** A circle with radius $3x$ is circumscribed about a square. Find the area of the square.

24. **CAKE** A bakery sells single-layer mini-cakes that are 3 inches in diameter for \$4 each. They also have a cake with a 9-inch diameter for \$15. If both cakes are the same thickness, which option gives you more cake for the money, nine mini-cakes or one 9-inch cake? Explain.

COORDINATE GEOMETRY The coordinates of the vertices of a regular polygon are given. Find the area of each polygon to the nearest tenth.

25. $T(0, 0), U(-7, -7), V(0, -14), W(7, -7)$

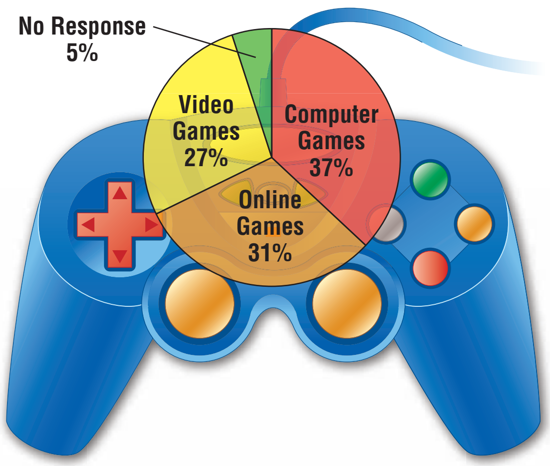
26. $G(-12, 0), H\left(0, 4\sqrt{3}\right), J\left(0, -4\sqrt{3}\right)$

Find the area of each circle given the measure of its circumference. Round to the nearest tenth.

27. 34π 28. 17π 29. 54.8 30. 91.4

GAMING For Exercises 31–33, refer to the following information.

Students were surveyed about which type of game they play at least once per week. The results are shown in the circle graph.



Source: Pew Internet & American Life Project

31. Suppose the radius of the circle on the graph is 1.3 centimeters. Find the area of the circle on the graph.

32. Francesca wants to use this circle graph for a presentation. She wants the circle to use as much space on a 22" by 28" sheet of poster board as possible. Find the area of the circle.

33. **Make a conjecture** about how you could determine the area of the region representing students who play computer games.

Study Tip

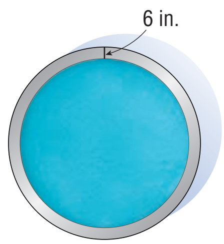
Make a Drawing

When an exercise does not provide a figure, it is helpful to draw one and label it with the given information.

SWIMMING POOL For Exercises 34 and 35, use the following information.

The area of a circular in-ground pool is approximately 7850 square feet. The owner wants to replace the tiling at the edge of the pool.

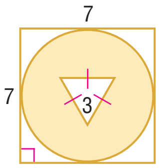
34. The edging is 6 inches wide, so she plans to use 6-inch square tiles to form a continuous inner edge. How many tiles will she need to purchase?



35. Once the square tiles are in place around the pool, there will be extra space between the tiles. What shape of tile will best fill this space? How many tiles of this shape should she purchase?

Find the area of each shaded region. Round to the nearest tenth.

36.



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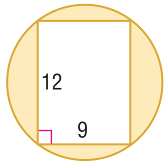

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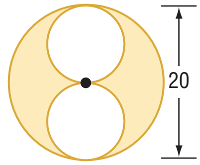

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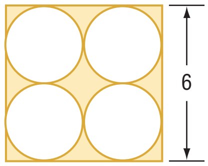

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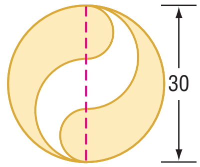
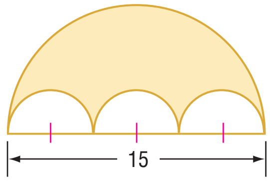


41.



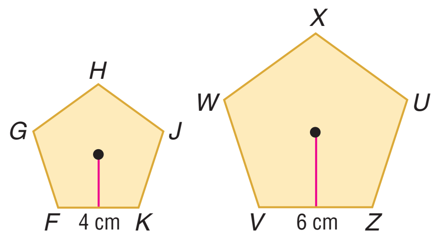
42. A square is inscribed in a circle of area 18π square units. Find the length of a side of the square.

SIMILAR FIGURES **For Exercises 43–47, use the following information.**
Polygons *FGHJK* and *VWXUZ* are similar regular pentagons.

43. Find the scale factor.

44. Find the perimeter of each pentagon. Compare the ratio of the perimeters of the pentagons to the scale factor.

45. Find the area of each pentagon. Compare the ratio of the areas of the pentagons to the scale factor.



46. Compare the ratio of the areas of the pentagons to the ratio of the perimeters of the pentagons.

47. Determine whether the relationship between the ratio of the areas of the pentagons to the scale factor is applicable to all similar polygons. Explain.

EXTRA PRACTICE
See pages 822, 838.
Math Online
Self-Check Quiz at
geometryonline.com

H.O.T. Problems

48. REASONING Explain how to derive the formula for the area of a regular polygon.

49. REASONING Describe the effect on the area and circumference of a circle when the length of the radius is doubled.

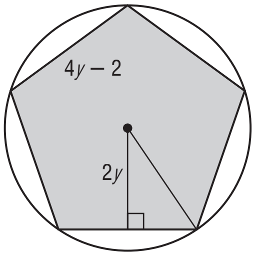
50. OPEN ENDED Draw a polygon inscribed in a circle. Find the area of the space in the interior of the circle and the exterior of the polygon.

51. CHALLENGE A circle inscribes one regular hexagon and circumscribes another. If the radius of the circle is 10 units long, find the ratio of the area of the smaller hexagon to the area of the larger hexagon.

52. ***Writing in Math*** Refer to the Geometry Lab on page 651. What shape does a regular polygon approximate when the number of sides is increased infinitely? Explain how the formula for the area of a regular polygon can approximate the formula for the area of a circle.

STANDARDIZED TEST PRACTICE

53. Which polynomial best represents the area of the regular pentagon shown at the right?



A $10y^2 - 5$

B $10y^2 + 5y$

C $20y^2 + 10$

D $20y^2 - 10y$

54. **REVIEW** In the system of equations $-\frac{5}{4}x + \frac{1}{3}y = 7$ and $2x - 6y = 8$, which expression can be correctly substituted for y in the equation $2x - 6y = 8$?

F $21 + \frac{15}{4}x$

G $7 - 3y$

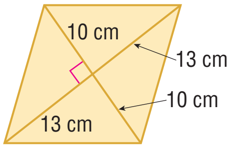
H $\frac{4}{3} + \frac{1}{3}x$

J $4 + 3y$

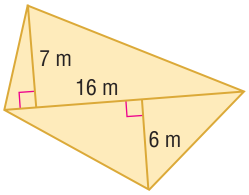
Spiral Review

Find the area of each quadrilateral. (Lesson 11-2)

55.



56.



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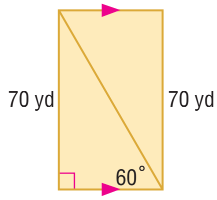


COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.** (Lesson 11-1)

58. $A(-3, 2), B(4, 2), C(2, -1), D(-5, -1)$

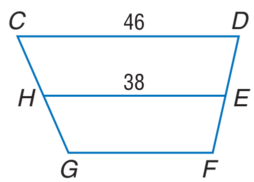
59. $F(4, 1), G(4, -5), H(-2, -5), J(-2, 1)$

COORDINATE GEOMETRY **Draw the rotation image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation.** (Lesson 9-3)

60. $\triangle ABC$ with vertices $A(-1, 3)$, $B(-4, 6)$, and $C(-5, 1)$, reflected in y-axis and then in x-axis

61. $\triangle FGH$ with vertices $F(0, 4)$, $G(-2, 2)$, and $H(2, 2)$, reflected in $y = x$ and then in y-axis

Refer to trapezoid *CDFG* with median $\overline{HE}$. (Lesson 6-6)



62. Find GF.

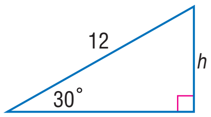
63. Let $\overline{WX}$ be the median of $CDEH$. Find WX.

64. Let $\overline{YZ}$ be the median of $HEFG$. Find YZ.

GET READY for the Next Lesson

PREREQUISITE SKILL **Find *h*.** (Lesson 8-3)

65.



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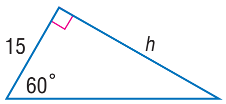
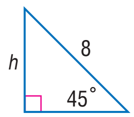


67.



CHAPTER 11

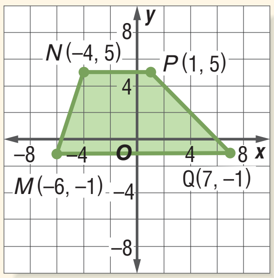
Mid-Chapter Quiz

Lessons 11-1 through 11-3

The coordinates of the vertices of quadrilateral *JKLM* are *J*(−8, 4), *K*(−4, 0), *L*(0, 4), and *M*(−4, 8). (Lesson 11-1)

1. Determine whether *JKLM* is a square, a rectangle, or a parallelogram.
2. Find the area of *JKLM*.

Find the area of each trapezoid. (Lesson 11-2)

3.


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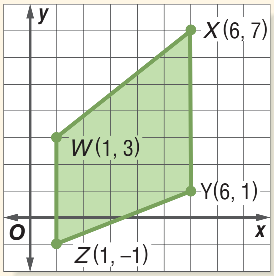
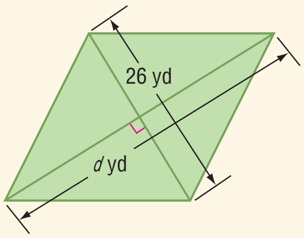


5. **MULTIPLE CHOICE** The area of the rhombus is 546 square yards. What is *d*? (Lesson 11-2)

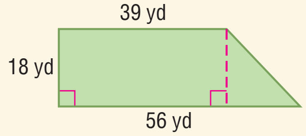


A 21 **C** 42

B 26 **D** 52

6. Find the area of an isosceles trapezoid that has a perimeter of 90 meters. The longer base is 5 meters less than twice the length of the shorter base. The length of each leg is 3 meters less than the length of the shorter base. (Lesson 11-2)

Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 11-2)

7.


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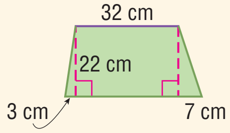

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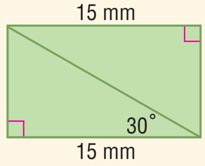
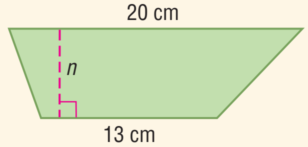
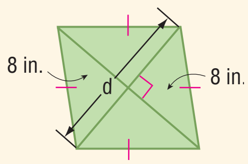


Find the missing measure for each quadrilateral. (Lesson 11-2)

10.


$A = 99 \text{ cm}^2$

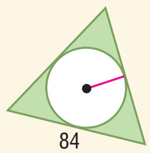
11.


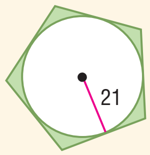
$A = 104 \text{ in}^2$

Find the area of each polygon. Round to the nearest tenth. (Lesson 11-3)

12. regular hexagon with apothem length of 14 millimeters
13. regular octagon with a perimeter of 72 inches

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth. (Lesson 11-3)

14.


15.


16. **CRAFTS** Lori is making a circular pillow. She wants the diameter of the finished pillow to be 12 inches. When cutting the fabric, she allows a $1\frac{1}{2}$ inch border for sewing. What is the total area of fabric needed for one pillow? (Lesson 11-3)

11-4 Areas of Composite Figures

Main Ideas

- Find areas of composite figures.
- Find areas of composite figures on the coordinate plane.

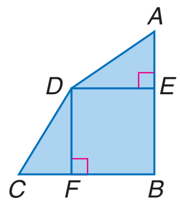
New Vocabulary

composite figure

GET READY for the Lesson

The sail for a windsurf board cannot be classified as a triangle or a parallelogram. However, it can be separated into figures that can be identified, such as trapezoids and a triangle.

Composite Figures A **composite figure** is a figure that can be separated into regions that are basic figures. To find the area of a composite figure, separate the figure into basic figures of which we can find the area. The sum of the areas of the basic figures is the area of the composite figure.



Auxiliary lines are drawn in quadrilateral $ABCD$. $\overline{DE}$ and $\overline{DF}$ separate the figure into $\triangle ADE$, $\triangle CDF$, and rectangle $BEDF$.

POSTULATE 11.2

The area of a region is the sum of the areas of all of its nonoverlapping parts.

A STANDARDIZED TEST EXAMPLE — **Area of a Composite Figure**

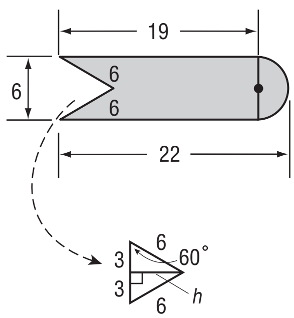
1 Which is closest to the area of this composite figure?

A 112.5 units^2

B 116.4 units^2

C 126.7 units^2

D 132.0 units^2



Read the Test Item

The figure can be separated into a rectangle with dimensions 6 units by 19 units, an equilateral triangle with sides each measuring 6 units, and a semicircle with a radius of 3 units.

Concepts in Motion
Animation
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Solve the Test Item

Use 30°-60°-90° relationships to find that the height of the triangle is $3\sqrt{3}$.

area of composite figure = area of rectangle − area of triangle + area of semicircle

$$= \ell w - \frac{1}{2}bh + \frac{1}{2}\pi r^2 \quad \text{Area formulas}$$

$$= 19 \cdot 6 - \frac{1}{2}(6)\left(3\sqrt{3}\right) + \frac{1}{2}\pi(3^2) \quad \text{Substitution}$$

$$= 114 - 9\sqrt{3} + \frac{9}{2}\pi \quad \text{Simplify.}$$

$$\approx 112.5 \quad \text{Use a calculator.}$$

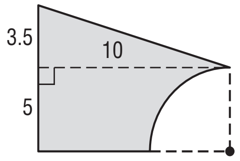
The area of the composite figure is 112.5 square units to the nearest tenth. The correct answer is A.

CHECK Your Progress

1. Which is closest to the area of the composite figure?

F 45.7 units^2 **H** 67.5 units^2

G 47.9 units^2 **J** 87.1 units^2



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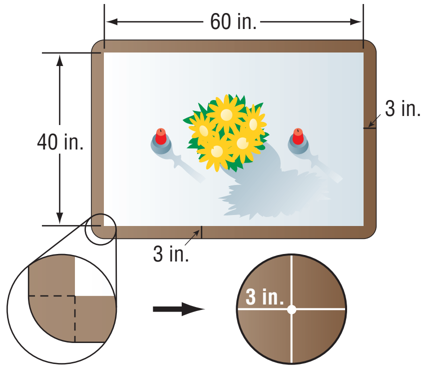
EXAMPLE Find the Area of a Composite Figure to Solve a Problem

2 **FURNITURE Melissa's dining room table has hardwood around the outside. Find the area of wood around the edge of the table.**



First, draw auxiliary lines to separate the figure into regions. The table can be separated into four rectangles and four corners.

The four corners of the table form a circle with radius 3 inches.

area of wood edge = area of rectangles + area of circle

$$= 2\ell w + 2\ell w + \pi r^2 \quad \text{Area formulas}$$

$$= 2(3)(60) + 2(3)(40) + \pi(3^2) \quad \text{Substitution}$$

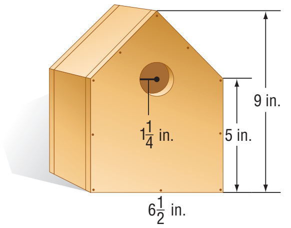
$$= 360 + 240 + 9\pi \quad \text{Simplify.}$$

$$\approx 628.3 \quad \text{Use a calculator.}$$

The area of the wood edge is 628.3 square inches to the nearest tenth.

CHECK Your Progress

2. BIRDHOUSES Ramon is building a birdhouse. He is going to paint the front side. What is the area to be painted? Round to the nearest tenth.



Cross-Curricular Project

Math Online Identifying the polygons forming a region such as a tessellation will help you determine the type of tessellation. Visit geometryonline.com to continue work on your project.

Composite Figures on the Coordinate Plane To find the area of a composite figure on the coordinate plane, separate the figure into basic figures, the areas of which can be determined.

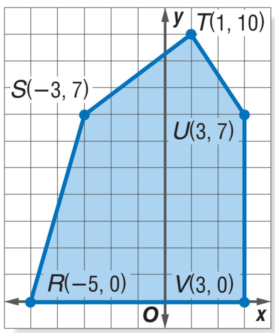
Study Tip

Estimation

Estimate the area of a composite figure on the coordinate plane by counting the unit squares. Use the estimate to determine if your answer is reasonable.

EXAMPLE Coordinate Plane

3 **COORDINATE GEOMETRY** **Find the area of the composite figure.**



First, separate the figure into regions. Draw an auxiliary line from S to U. This divides the figure into triangle STU and trapezoid $RSUV$.

Find the difference between x-coordinates to find the length of the base of the triangle and the lengths of the bases of the trapezoid. Find the difference between y-coordinates to find the heights of the triangle and trapezoid.

area of $RSTUV$ = area of $\triangle STU$ + area of trapezoid $RSUV$

$= \frac{1}{2}bh + \frac{1}{2}h(b_1 + b_2)$ Area formulas

$= \frac{1}{2}(6)(3) + \frac{1}{2}(7)(8 + 6)$ Substitution

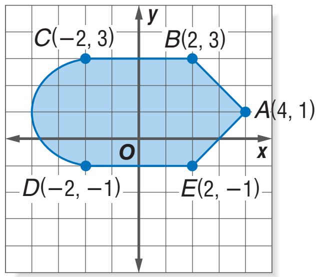
$= 9 + 49$ Multiply.

$= 58$ Simplify.

The area of $RSTUV$ is 58 square units.

CHECK Your Progress

3. **COORDINATE GEOMETRY** Find the area of the composite figure.



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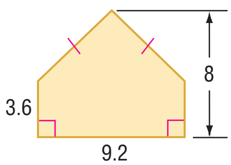
CHECK Your Understanding

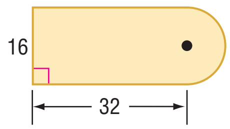
Example 1 (p. 658)

Find the area of each figure. Round to the nearest tenth if necessary.

1.

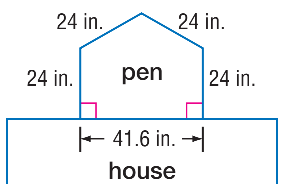


2.



Example 2 (p. 659)

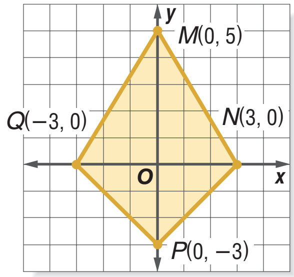
3. DOGS Owen's family has a system of interlocking gates that attach to the wall of the house to form a pen for their dog. Find the area enclosed by the wall and gates.



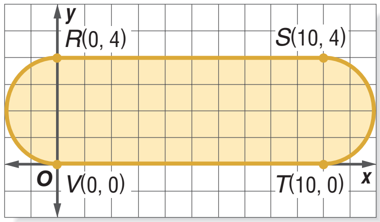
Example 3 (p. 660)

COORDINATE GEOMETRY Find the area of each figure.

4.



5.



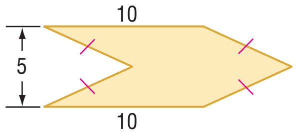
Exercises

HOMEWORK HELP

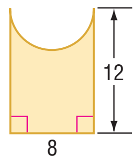
For Exercises	See Examples
6–11	1
12, 13	2
14–20	3

Find the area of each figure. Round to the nearest tenth if necessary.

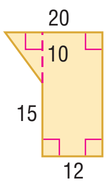
6.



7.



8.



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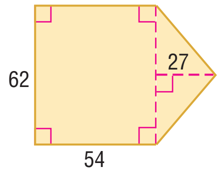

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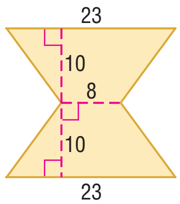
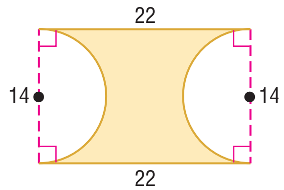
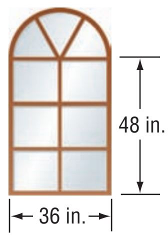


11.



WINDOWS **For Exercises 12 and 13, use the following information.**

Mr. Frazier needs to replace this window in his house. The window panes are rectangles and sectors.

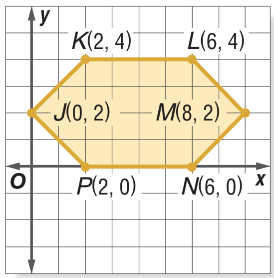


12. Find the perimeter of the window.

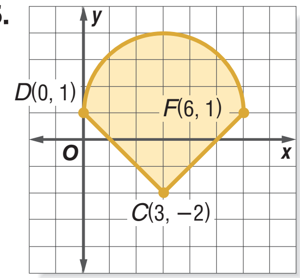
13. Find the area of the window.

COORDINATE GEOMETRY **Find the area of each figure. Round to the nearest tenth if necessary.**

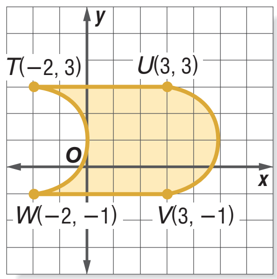
14.



15.



16.



COORDINATE GEOMETRY **The vertices of a composite figure are given. Find the area of each figure.**

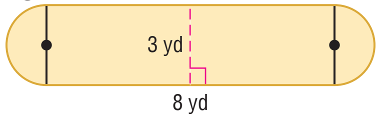
17. $M(-4, 0)$, $N(0, 3)$, $P(5, 3)$, $Q(5, 0)$

18. $T(-4, -2)$, $U(-2, 2)$, $V(3, 4)$, $W(3, -2)$

19. $G(-3, -1)$, $H(-3, 1)$, $I(2, 4)$, $J(5, -1)$, $K(1, -3)$

20. $P(-8, 7)$, $Q(3, 7)$, $R(3, -2)$, $S(-1, 3)$, $T(-11, 1)$

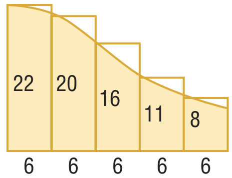
PAINTING **For Exercises 21 and 22, use the following information.**
The senior class of Westwood High School wants to paint the entrance hallway floor of their school as shown at the right.



21. Find the area of the floor to be painted.

22. Paint costs $20 per gallon. Five gallons of paint covers 2000 square feet. How much will paint cost if the students use four coats of paint?

CALCULUS **For Exercises 23–25, use the following information.**
In the branch of mathematics called *calculus*, you can find the area of an *irregular* shape by approximating the shape with rectangles of equal width. This is called a *Riemann sum*.

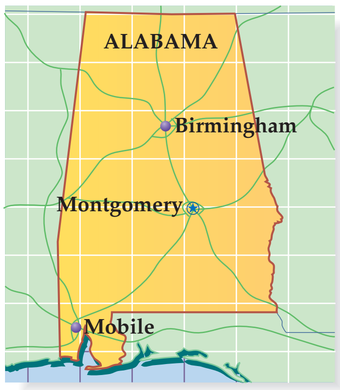


23. Use the rectangles to approximate the area of the region.

24. Analyze the estimate. Do you think the actual area is larger or smaller than your estimate? Explain.

25. How could the irregular region be separated to give an estimate of the area that is more accurate?

26. **GEOGRAPHY** Estimate the area of the state of Alabama. Each square on the grid represents 2500 square miles.



EXTRA PRACTICE
See pages 822, 838.
Math Online
Self-Check Quiz at geometryonline.com

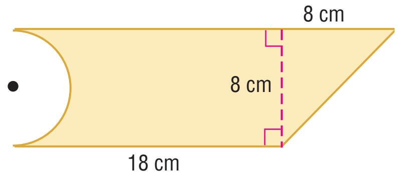
27. **RESEARCH** Find a map of your state or a state of your choice. Estimate the area. Then use the Internet or other source to check the accuracy of your estimate.

H.O.T. Problems

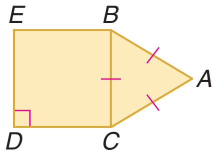
28. **OPEN ENDED** Sketch a composite polygon on a coordinate plane and find its area.

29. **RESEARCH** Use a dictionary or other Internet resource to find the definition of *composite*. Describe below how the definition of composite relates to composite figures.

30. **REASONING** Describe two different methods to find the area of the composite figure at right. Then find the area of the figure. Round to the nearest tenth.



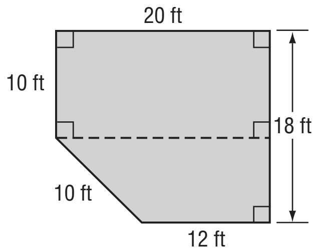
31. **CHALLENGE** Find the ratio of the area of $\triangle ABC$ to the area of square $BCDE$.



32. ***Writing in Math*** Describe how to find the area of a composite figure.

STANDARDIZED TEST PRACTICE

33. A landscape architect gives the diagram of a yard to a fencing company.



What is the area of the yard to be fenced, in square feet?

A 70

B 264

C 328

D 360

34. REVIEW Tammy borrowed money from her parents to pay for a trip. The data in the table show the remaining balance b of Tammy's loan after each payment p.

Number of Payments	1	2	3	4	5
Loan Balance ($)	2142	1989	1836	1683	1530

If payments were graphed on the horizontal axis and balances were graphed on the vertical axis, what would be the equation of a line that fits the data?

F $b = 1530 + 153p$

G $b = 2142 - 153p$

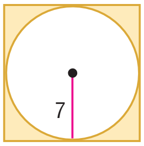
H $b = 2295 - 153p$

J $b = 2448 + 153p$

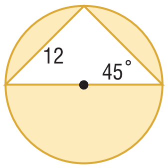
Spiral Review

Find the area of each shaded region. Round to the nearest tenth. (Lesson 11-3)

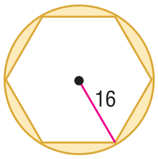
35.



36.



37.



Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 11-2)

38. equilateral triangle with perimeter of 57 feet

39. rhombus with a perimeter of 40 yards and a diagonal of 12 yards

40. COORDINATE GEOMETRY The point (6, 0) is rotated 45° clockwise about the origin. Find the exact coordinates of its image. (Lesson 9-3)

Find the range for the measure of the third side of a triangle given the measures of two sides. (Lesson 5-4)

41. 16 and 31 **42.** 26 and 40 **43.** 11 and 23

GET READY for the Next Lesson

PREREQUISITE SKILL **Express each fraction as a decimal to the nearest hundredth.**

44. $\frac{5}{8}$ **45.** $\frac{13}{16}$ **46.** $\frac{9}{47}$ **47.** $\frac{10}{21}$

READING MATH

Prefixes

Many of the words used in mathematics use the same prefixes as other everyday words. Understanding the meaning of the prefixes can help you understand the terminology better.

Prefix	Meaning	Everyday Words	Meaning
bi-	2	bicycle	a 2-wheeled vehicle
		bipartisan	involving members of 2 political parties
tri-	3	triangle	closed figure with 3 sides
		tricycle	a 3-wheeled vehicle
		triplet	one of 3 children born at the same time
quad-	4	quadrilateral	closed figure with 4 sides
		quadriceps	muscles with 4 parts
		quadruple	four times as many
penta-	5	pentagon	closed figure with 5 sides
		pentathlon	athletic contest with 5 events
hexa-	6	hexagon	closed figure with 6 sides
hept-	7	heptagon	closed figure with 7 sides
oct-	8	octagon	closed figure with 8 sides
		octopus	animal with 8 legs
dec-	10	decagon	closed figure with 10 sides
		decade	a period of 10 years
		decathlon	athletic contest with 10 events

Several pairs of words in the chart have different prefixes, but the same root word. *Pentathlon* and *decathlon* are both athletic contests. *Heptagon* and *octagon* are both closed figures. Knowing the meaning of the root of the term as well as the prefix can help you learn vocabulary.

Reading to Learn

Use a dictionary to find the meanings of the prefix and root for each term. Then write a definition of the term.

1. bisector
2. polygon
3. equilateral
4. concentric
5. circumscribe
6. collinear

7. **RESEARCH** Use a dictionary to find the meanings of the prefix and root of *circumference*.

8. **RESEARCH** Use a dictionary or the Internet to find as many words as you can with the prefix *poly-* and the definition of each.

11-5 Geometric Probability and Areas of Sectors

Main Ideas

- Solve problems involving geometric probability.
- Solve problems involving sectors and segments of circles.

New Vocabulary

geometric probability
sector
segment

GET READY for the Lesson

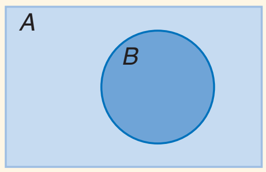
To win at darts, you have to throw a dart at either the center or the part of the dartboard that earns the most points. In games, probability can sometimes be used to determine chances of winning. Probability that involves a geometric measure such as length or area is called **geometric probability**.

Geometric Probability In Chapter 1, you learned that the probability that a point lies on a part of a segment can be found by comparing the length of the part to the length of the whole segment. Similarly, you can find the probability that a point lies in a part of a two-dimensional figure by comparing the area of the part to the area of the whole figure.

KEY CONCEPT — *Probability and Area*

If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is

$$P(B) = \frac{\text{area of region } B}{\text{area of region } A}$$



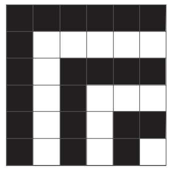
When determining geometric probability with targets, we assume
- that the object lands within the target area, and
- it is equally likely that the object will land anywhere in the region.

EXAMPLE — Probability with Area

1 **A square game board has black and white stripes of equal width, as shown. What is the chance that a dart that strikes the board will land on a white stripe?**

You want to find the probability of landing on a white stripe, not a black stripe.

(continued on the next page)



We need to divide the area of the white stripes by the total area of the game board. Extend the sides of each stripe. This separates the square into 36 small unit squares.

The white stripes have an area of 15 square units. The total area is 36 square units.

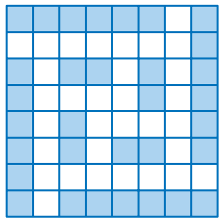
The probability of tossing a chip onto the white stripes is $\frac{15}{36}$ or $\frac{5}{12}$.

Study Tip

Probability
The probability of an event can be expressed as a decimal, a fraction, or a percent.

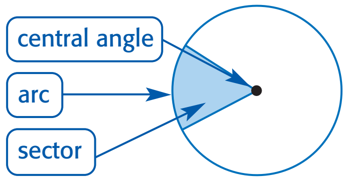
CHECK Your Progress

1. Find the probability that a point chosen at random from the figure lies in the shaded region.



Sectors and Segments of Circles

Sometimes you need to know the area of a sector of a circle in order to find a geometric probability. A **sector** of a circle is a region of a circle bounded by a central angle and its intercepted arc.

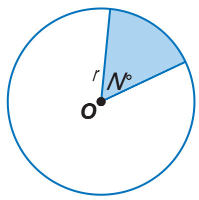


Proportional reasoning can be used to derive the formula for the area of a sector.

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{N°}{360°} \quad \begin{matrix}\leftarrow \text{Degrees in the sector} \\ \leftarrow \text{Degrees in the circle}\end{matrix}$$

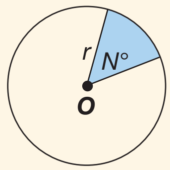
$$\text{area of sector} = \frac{N \cdot \text{area of circle}}{360°} \qquad \text{Multiply.}$$

$$\text{area of sector} = \frac{N}{360}\pi r^2 \qquad \text{area of circle} = \pi r^2$$



KEY CONCEPT — *Area of a Sector*

If a sector of a circle has an area of A square units, a central angle measuring $N°$, and a radius of r units, then $A = \frac{N}{360}\pi r^2$.



EXAMPLE 2 Probability with Sectors

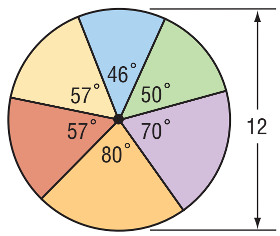
a. Find the area of the blue sector.

Use the formula to find the area of the sector.

$$A = \frac{N}{360}\pi r^2 \qquad \text{Area of a sector}$$

$$= \frac{46}{360}\pi(6^2) \qquad N = 46, r = 6$$

$$= 4.6\pi \qquad \text{Simplify.}$$



b. Find the probability that a point chosen at random lies in the blue region.

To find the probability, divide the area of the sector by the area of the circle. The area of the circle is πr^2 with a radius of 6.

$P(\text{blue}) = \dfrac{\text{area of sector}}{\text{area of circle}}$ Geometric probability formula

$= \dfrac{4.6\pi}{\pi \cdot 6^2}$ Area of sector $= 4.6\pi$, area of circle $= \pi \cdot 6^2$

≈ 0.13 Use a calculator.

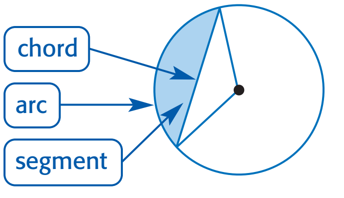
The probability that a random point is in the blue sector is about 0.13 or 13%.

CHECK Your Progress

2A. Find the area of the orange sector.

2B. Find the probability that a point chosen at random lies in the orange region.

The region of a circle bounded by an arc and a chord is called a **segment** of a circle. To find the area of a segment, subtract the area of the triangle formed by the radii and the chord from the area of the sector containing the segment.



EXAMPLE 3 Probability with Segments

A regular hexagon is inscribed in a circle with a diameter of 14.

a. Find the area of the red segment.

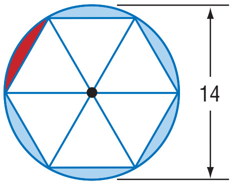
Area of the sector:

$A = \dfrac{N}{360}\pi r^2$ Area of a sector

$= \dfrac{60}{360}\pi(7^2)$ $N = 60, r = 7$

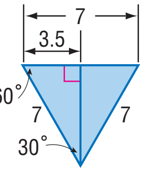
$= \dfrac{49}{6}\pi$ Simplify.

≈ 25.66 Use a calculator.



Area of the triangle:

Since the hexagon was inscribed in the circle, the triangle is equilateral, with each side 7 units long. Use properties of 30°-60°-90° triangles to find the apothem. The value of x is 3.5, the apothem is $x\sqrt{3}$ or $3.5\sqrt{3}$ which is approximately 6.06.



$A = \dfrac{1}{2}bh$ Area of a triangle

$= \dfrac{1}{2}(7)(6.06)$ $b = 7, h = 6.06$

≈ 21.22 Simplify.

(continued on the next page)

Study Tip

Look Back

To review the **properties of special right triangles**, see Lesson 8-3.

Area of the segment:

area of segment = area of sector − area of triangle

$\approx 25.66 - 21.22$ Substitution

≈ 4.44 Simplify.

b. Find the probability that a point chosen at random lies in the red region.

Divide the area of the sector by the area of the circle to find the probability. First, find the area of the circle. The radius is 7, so the area is $\pi(7^2)$ or about 153.94 square units.

$P(\text{red}) = \dfrac{\text{area of segment}}{\text{area of circle}}$ Geometric probability formula

$\approx \dfrac{4.44}{153.94}$ Substitution

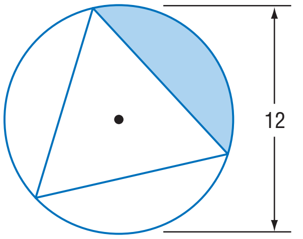
≈ 0.03 Use a calculator.

The probability that a random point is on the red segment is about 0.03 or 3%.

CHECK Your Progress

3A. Find the area of the shaded region.

3B. Find the probability that a point chosen at random will be in the shaded region.



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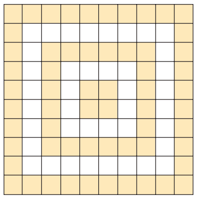
CHECK Your Understanding

Example 1 (p. 665)

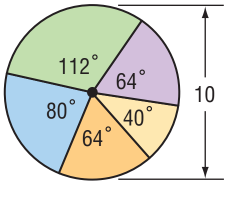
1. Find the probability that a point chosen at random lies in the shaded region.



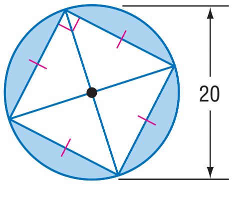
Examples 2 and 3 (pp. 666–667)

Find the area of the blue region. Then find the probability that a point chosen at random will be in the blue region.

2.



3.

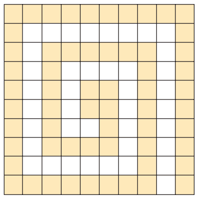


Exercises

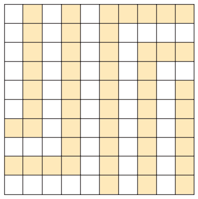
HOMEWORK HELP	
For Exercises	See Examples
4–7	1
8–13	2
14–16	3

Find the probability that a point chosen at random lies in the shaded region.

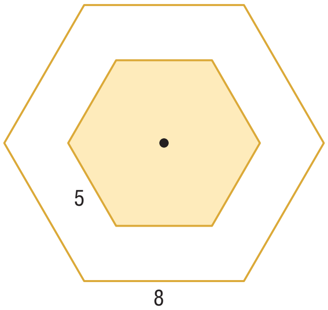
4.



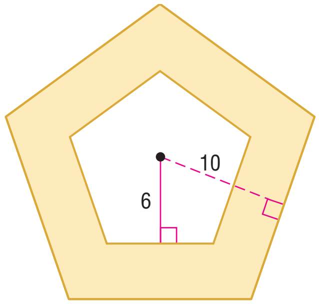
5.



6.

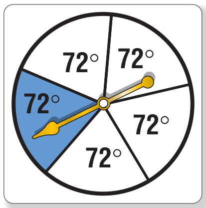


7.

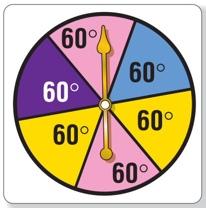


Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of each spinner is 15 centimeters.

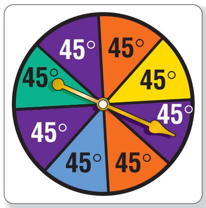
8. blue



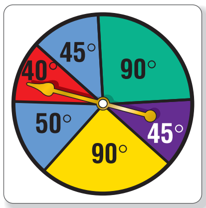
9. pink



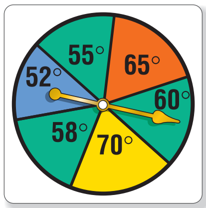
10. purple



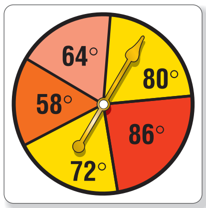
11. red



12. green

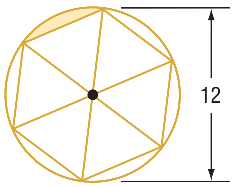


13. yellow

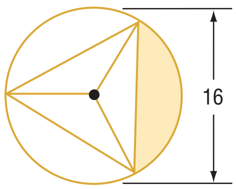


Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume all inscribed polygons are regular.

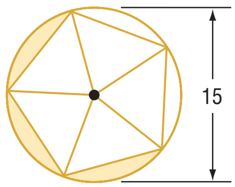
14.



15.



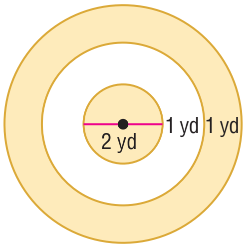
16.



17. GEOGRAPHY The land area of the state of Alaska is 571,951 square miles. The land area of the United States is 3,537,438 square miles. If a point is chosen at random in the United States, what is the probability that it is in Alaska?

EXTRA PRACTICE
See pages 823, 838.
Math Online
Self-Check Quiz at
geometryonline.com

18. PARACHUTES A skydiver must land on a target of three concentric circles. The diameter of the center circle is 2 yards, and the circles are spaced 1 yard apart. Find the probability that she will land on the shaded area.



Real-World Link

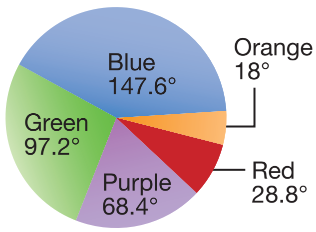
In tennis, the linesperson determines whether the hit ball is in or out. The umpire may only overrule the linesperson if he or she immediately thinks the call was wrong without a doubt and never as a result of a player's protest.

Source: www.usta.com

SURVEYS **For Exercises 19–22, use the following information.**

The circle graph shows the results of a survey of high school students about favorite colors. The measurement of each central angle is shown. If a person is chosen at random from the school, find the probability of each response.

What's Your Favorite Color?



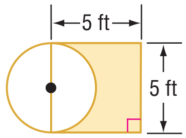
19. Favorite color is red.

20. Favorite color is blue or green.

21. Favorite color is not red or blue.

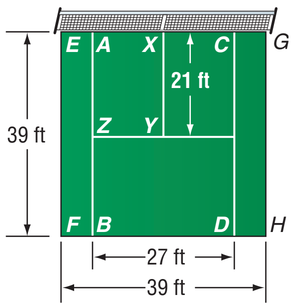
22. Favorite color is not orange or green.

23. One side of a square is a diameter of a circle. The length of one side of the square is 5 feet. To the nearest hundredth, what is the probability that a point chosen at random is in the shaded region?



TENNIS **For Exercises 24 and 25, use the following information.**

A tennis court has stripes dividing it into rectangular regions. For singles play, the inbound region is defined by segments $\overline{AB}$ and $\overline{CD}$. The doubles court is bound by the segments $\overline{EF}$ and $\overline{GH}$.



24. Find the probability that a ball in a singles game will land inside the court but out of bounds.

25. When serving, the ball must land within $AXYZ$, the service box. Find the probability that a ball will land in the service box, relative to the court.

H.O.T. Problems

26. OPEN ENDED List three games that involve geometric probability.

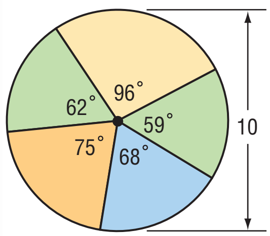
27. FIND THE ERROR Rachel and Taimi are finding the probability that a point chosen at random lies in the green region. Who is correct? Explain your answer.

Rachel

$$A = \frac{N}{360}\pi r^2$$
$$= \frac{59 + 62}{360}\pi(5^2)$$
$$\approx 26.4$$
$$P(\text{green}) \approx \frac{26.4}{25\pi} \approx 0.34$$

Taimi

$$A = \frac{N}{360}\pi r^2$$
$$= \frac{59}{360}\pi(5^2) + \frac{62}{360}$$
$$\approx 13.0$$
$$P(\text{green}) \approx \frac{13.0}{25\pi} \approx 0.17$$



28. REASONING Explain how to find the area of a sector of a circle.

29. **Which One Doesn't Belong?** Identify the term that does not belong with the others. Explain your answer.

chord | radius | apothem | segment

CHALLENGE **Study each spinner in Exercises 11–13.**

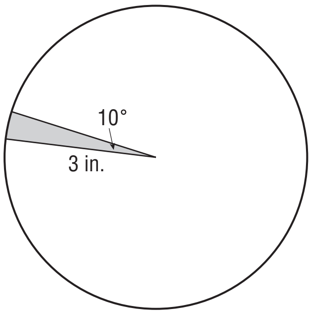
30. Are the chances of landing on each color equal? Explain.

31. Would this be considered a fair spinner to use in a game? Explain.

32. *Writing in Math* Explain how geometric probability can help a person design a dartboard and assign values to spaces.

STANDARDIZED TEST PRACTICE

33. A grocery store is slicing a wheel of cheese into slivers for free samples.



What is the area, in square inches, of one sliver?

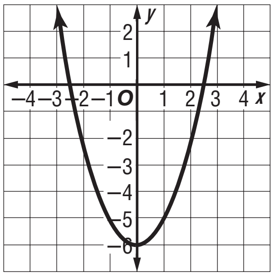
A $\frac{9\pi}{10}$

B $\frac{3\pi}{5}$

C $\frac{\pi}{4}$

D $\frac{\pi}{6}$

34. **REVIEW** Which is the equation of the function graphed below?



F $y = x^2 - 2.5$

G $y = -x^2 + 2.5$

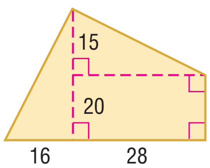
H $y = x^2 - 6$

J $y = -x^2 - 6$

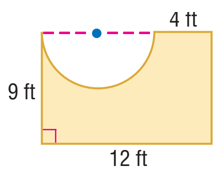
Spiral Review

Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 11-4)

35.



36.



Find the area of each polygon. Round to the nearest tenth if necessary. (Lesson 11-3)

37. a regular triangle with a perimeter of 48 feet

38. a square with a side length of 21 centimeters

39. a regular hexagon with an apothem length of 8 inches

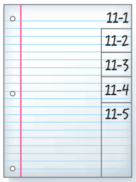
Study Guide and Review

Download Vocabulary Review from geometryonline.com

FOLDABLES™ Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Area of Parallelograms (Lesson 11-1)

- The area of a parallelogram is the product of the base and the height.

Areas of Triangles, Rhombi, and Trapezoids (Lesson 11-2)

- The formula for the area of a triangle can be used to find the areas of many different figures.
- Congruent figures have equal areas.

Areas of Regular Polygons and Circles (Lessons 11-3)

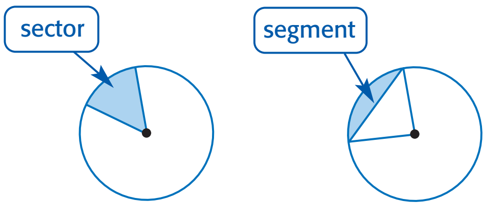
- A regular n-gon is made up of n congruent isosceles triangles.
- The area of a circle of radius r units is πr^2 square units.

Areas of Composite Figures (Lessons 11-4)

- The area of a composite figure is the sum of the areas of its nonoverlapping parts.

Geometric Probability and Areas of Sectors (Lessons 11-5)

- To find a geometric probability, divide the area of a part of a figure by the total area.
- A sector is a region of a circle bounded by a central angle and its intercepted arc.
- The area of a sector is given by the formula, $A = \frac{N}{360}\pi r^2$.
- A segment of a circle is a region bounded by an arc and a chord.



Key Vocabulary

apothem (p. 649)
composite figure (p. 658)
geometric probability (p. 665)
sector (p. 666)
segment (p. 667)

Vocabulary Check

Choose the term that best matches each phrase. Choose from the list above.

1. A figure that cannot be classified as a single polygon is a(n) ______________.
2. The region of a circle bounded by an arc and a chord is called a(n) ______________ of a circle.
3. ______________ uses the principles of length and area to find the probability of an event.
4. A(n) ______________ is a segment that is drawn from the center of a regular polygon perpendicular to a side of the polygon.
5. A(n) ______________ of a circle is a region of a circle bounded by a central angle and its intercepted arc.
6. To find the ______________, divide the area of a part of a figure by the total area.
7. A circle graph is separated into ______________(s).
8. To find the area of a(n)______________, find the area of the triangle and subtract it from the area of the sector.
9. The area of a rectangular polygon is one-half the product of the perimeter and the ______________.
10. A(n)______________ can be separated into basic shapes.

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Mixed Problem Solving
For mixed problem-solving practice, see page 838.

Lesson-by-Lesson Review

11-1 Area of Parallelograms (pp. 630–636)

COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.**

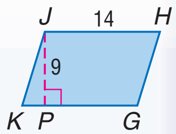
11. $A(-6, 1)$, $B(1, 1)$, $C(1, -6)$, $D(-6, -6)$

12. $E(7, -2)$, $F(1, -2)$, $G(2, 2)$, $H(8, 2)$

13. $J(-1, -4)$, $K(-5, 0)$, $L(-5, 5)$, $M(-1, 1)$

14. $P(-7, -1)$, $Q(-3, 3)$, $R(-1, 1)$, $S(-5, -3)$

Example 1 **Find the area of parallelogram *GHJK*.**



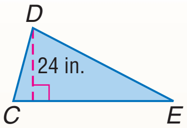
$A = bh$ Area of a parallelogram

$= 14(9)$ or 126 $b = 14, h = 9$

The area of *GHJK* is 126 square units.

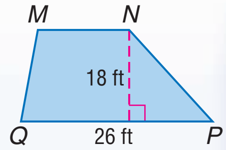
11-2 Areas of Triangles, Rhombi, and Trapezoids (pp. 638–647)

15. Triangle *CDE* has an area of 336 square inches. Find *CE*.



16. **FUND-RAISER** The school marching band is selling pennants. Each pennant is cut in the shape of a triangle 3 feet long and 1 foot high. How many square feet of fabric are needed to make 150 pennants, assuming no waste?

Example 2 **Trapezoid *MNPQ* has an area of 360 square feet. Find the length of $\overline{MN}$.**



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$360 = \frac{1}{2}(18)(b_1 + 26)$ $A = 360, h = 18, b_2 = 26$

$360 = 9b_1 + 234$ Multiply.

$14 = b_1$ Solve for b_1.

The length of $\overline{MN}$ is 14 feet.

11-3 Areas of Regular Polygons and Circles (pp. 649–656)

Find the area of each polygon. Round to the nearest tenth.

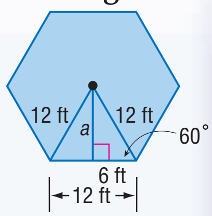
17. a regular pentagon with perimeter of 100 inches

18. an octagon with a perimeter of 96 feet

19. **BAKING** Todd wants to make a cheesecake for a birthday party. The recipe calls for a 9 inch diameter round pan. Todd only has square pans. He has an 8 inch square pan, a 9 inch square pan, and a 10 inch square pan. Which pan comes closest in area to the one that the recipe suggests?

Example 3 **Find the area of the regular hexagon.**

The central angle of a hexagon is 60°. Use the properties of 30°-60°-90° triangles to find that the apothem is $6\sqrt{3}$ feet.



$A = \frac{1}{2}Pa$ Area of a regular polygon

$= \frac{1}{2}(72)(6\sqrt{3})$ $P = 72, a = 6\sqrt{3}$

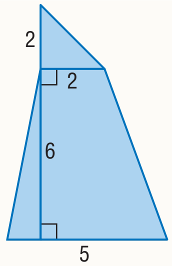
$= 216\sqrt{3} \approx 374.1$

The area of the regular hexagon is 374.1 square feet to the nearest tenth.

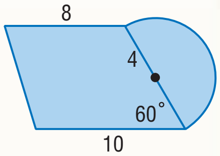
11-4 Areas of Composite Figures (pp. 658–663)

Find the area of each figure to the nearest tenth.

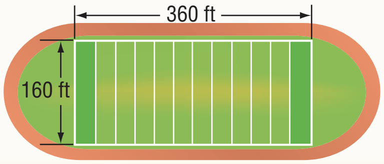
20.



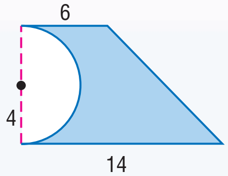
21.



22. RECREATION The football field in the back of the high school is surrounded by a track. The football field has dimensions 160 feet by 360 feet. Find the area of the figure inside the track to the nearest tenth.



Example 4 Find the area of the figure.



Separate the figure into a rectangle and a triangle.

Area of composite figure = area of rectangle − area of semicircle + area of triangle.

$$= \ell w - \frac{1}{2}\pi r^2 + \frac{1}{2}bh$$

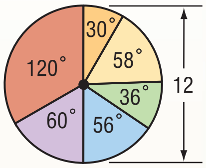
$$= (6)(8) - \frac{1}{2}\pi(4^2) + \frac{1}{2}(8)(8)$$

$$= 48 - 8\pi + 32 \text{ or about } 54.9$$

The area of the composite figure is 54.9 square units to the nearest tenth.

11-5 Geometric Probability and Areas of Sectors (pp. 665–671)

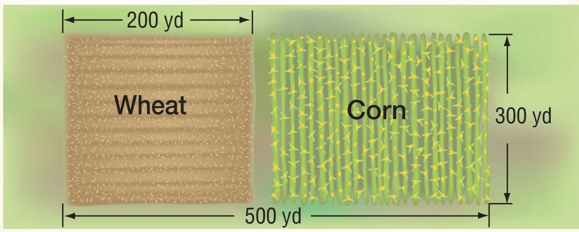
Find the probability that a point chosen at random will be in the sector of the given color.



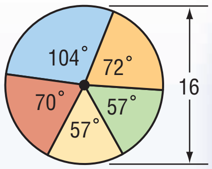
23. red

24. purple or green

25. FARMING A farmer grows corn and wheat in a field shown below. What is the probability that a lightning bolt that strikes will hit the wheat field?



Example 5 Find the probability that a point chosen at random will be in the blue sector.



First find the area of the blue sector.

$A = \frac{N}{360}\pi r^2$ Area of a sector

$= \frac{104}{360}\pi(8^2)$ or about 58.08 Substitute and simplify.

To find the probability, divide the area of the sector by the area of the circle.

$P(\text{blue}) = \frac{\text{area of sector}}{\text{area of circle}}$ Geometric probability formula

$= \frac{58.08}{\pi(8^2)}$ or about 0.29

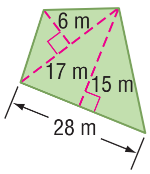
The probability is about 0.29 or 29%.

Chapter 11 Practice Test

COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.**

1. $R(-6, 8)$, $S(-1, 5)$, $T(-1, 1)$, $U(-6, 4)$
2. $R(7, -1)$, $S(9, 3)$, $T(5, 5)$, $U(3, 1)$
3. $R(2, 0)$, $S(4, 5)$, $T(7, 5)$, $U(5, 0)$
4. $R(3, -6)$, $S(9, 3)$, $T(12, 1)$, $U(6, -8)$

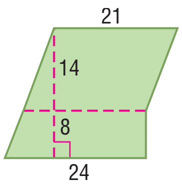
Find the area of each figure. Round to the nearest tenth if necessary.

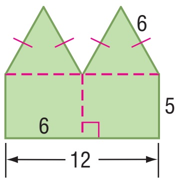
5. 

6. a regular octagon with apothem length of 3 ft
7. a regular pentagon with a perimeter of 115 cm

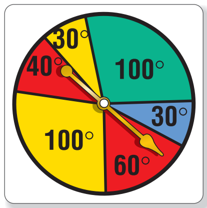
8. **SOCCER BALLS** The surface of a soccer ball is made of a pattern of regular pentagons and hexagons. If each hexagon on a soccer ball has a perimeter of 9 inches, what is the area of a hexagon?

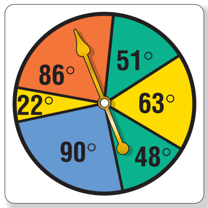
Find the area of each figure. Round to the nearest tenth.

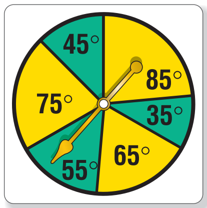
9. 

10. 

Each spinner has a diameter of 12 inches. Find the probability of spinning the indicated color.

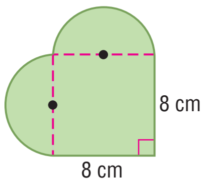
11. red


12. orange


13. green


14. **COORDINATE GEOMETRY** Find the area of *CDGHJ* with vertices $C(-3, -2)$, $D(1, 3)$, $G(5, 5)$, $H(8, 3)$, and $J(5, -2)$.

15. **MULTIPLE CHOICE** What is the area of the figure in square centimeters?



A $64 + 64\pi$

B 80π

C 64π

D $64 + 16\pi$

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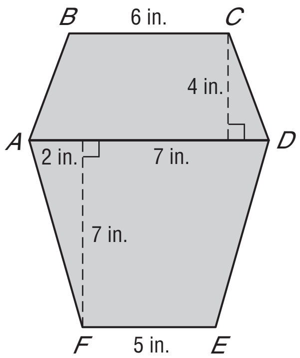
CHAPTER 11

Standardized Test Practice

Cumulative, Chapters 1–11

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the area of figure *ABCDEF* in square inches?



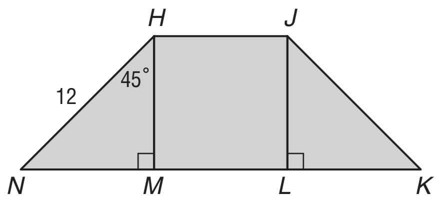
A 158 in^2

B 100 in^2

C 79 in^2

D 68 in^2

2. If *HJKN* is an isosceles trapezoid, what is the area of $\triangle JKL$?



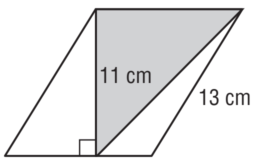
F $144\sqrt{2}$

G 144

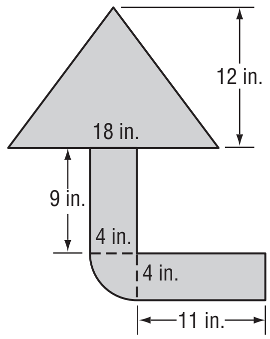
H $72\sqrt{2}$

J 36

3. GRIDDABLE The rhombus below has side length of 13 centimeters and height of 11 centimeters. If the shaded area is removed, what is the area of the remaining figures in square centimeters?



4. Henry is painting directional arrows in the school parking lot. He needs to know the area of each arrow in order to calculate the amount of paint he needs to buy. Find the area using the diagram below.



A $188 + 4\pi \text{ in}^2$

B $188 + 8\pi \text{ in}^2$

C $188 + 16\pi \text{ in}^2$

D $296 + 4\pi \text{ in}^2$

5. Which statement is *always* true?

F When an angle is inscribed in a circle, the angle's measure equals one-half of the measure of the intercepted arc.

G In a circle, an inscribed quadrilateral will have consecutive angles that are supplementary.

H In a circle, an inscribed angle that intercepts a semicircle is obtuse.

J If two inscribed angles of a circle intercept congruent arcs, then the angles are complementary.

6. ALGEBRA The width of a parallelogram can be represented using the expression $\frac{x^2 + 2x - 48}{x + 8}$, where the numerator represents the area and the denominator represents the length. What is the width of the parallelogram?

A $x - 4$

B $x + 4$

C $x - 6$

D $x + 6$

Math Online Standardized Test Practice at geometryonline.com

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 841–856.

7. Which of the segments described could be a secant of a circle?

F has its endpoints on a circle

G intersects exactly one point on a circle

H intersects exactly two points on a circle

J one endpoint at the center of the circle

8. Two triangles are drawn on a coordinate grid. One has vertices at (0, 1), (0, 7), and (6, 4). The other has vertices at (7, 7), (10, 7), and (8.5, 10). What scale factor can be used to compare the smaller triangle to the larger?

A 2.5

B 2

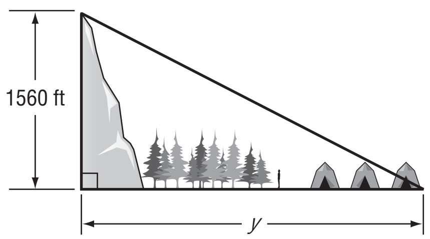
C 1.5

D 0.5

TEST-TAKING TIP

Question 8 If a question does not provide you with a figure that represents the problem, draw one yourself. By recording the information that you know, the problem becomes more understandable.

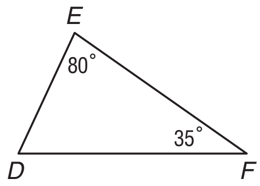
9. Lori and her family are camping near a mountain. Their campground is in a clearing next to a stretch of forest. The angle of elevation from the far edge of the campground to the top of the mountain is 35°. Find the distance y from the base of the mountain to the far edge of the campground.



F 2719 ft **H** 1904 ft

G 2228 ft **J** 1092 ft

10. Which of the following lists the sides of $\triangle DEF$ from greatest to least length?



A $\overline{DE}, \overline{EF}, \overline{DF}$ **C** $\overline{DF}, \overline{EF}, \overline{DE}$

B $\overline{DE}, \overline{DF}, \overline{EF}$ **D** $\overline{DF}, \overline{DE}, \overline{EF}$

11. Ms. Lee told her students, "If you do not get enough rest, you will be tired. If you are tired, you will not be able to concentrate." Which of the following is a logical conclusion that could follow Ms. Lee's statements?

F If you get enough rest, you will be tired.

G If you are tired, you will be able to concentrate.

H If you do not get enough rest, you will be able to concentrate.

J If you do not get enough rest, you will not be able to concentrate.

Pre-AP

Record your answer on a sheet of paper. Show your work.

12. Quadrilateral $ABCD$ has vertices $A(1, 2)$, $B(5, 5)$, and $D(5, 0)$.

a. Find the coordinates of point C such that $ABCD$ is a parallelogram and plot the parallelogram on a coordinate plane.

b. Using the plot you created, find the midpoint of $\overline{CD}$ and label it M. Draw a segment from point B to M and from point A to M. Find the area of triangle AMB.

c. What is the area of each of the other triangles formed by the construction?

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	11-4	11-2	11-1	11-4	10-4	796	10-6	9-5	8-5	5-3	2-4	11-2